Vertical Drain Consolidation with Parabolic Distribution of Permeability in Smear Zone

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Abstract: A vertical drain radial consolidation equation based on a parabolic reduction in permeability toward the drain is presented. The proposed equation, based on Hansbo's equal strain theory, is compared with settlement data from a laboratory test in a large scale consolidometer.

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Introduction

Vertical drains are used to improve soft soil by providing a horizontal drainage path along which excess pore water pressures, caused by a surcharge, can dissipate faster than by a vertical drainage path alone. Installation of vertical drains results in disturbance of the soil zone adjacent to the drain. A number of researchers (Chai and Miura 1999; Sharma and Xiao 2000; Hawlader et al. 2002) have noted that the disturbance in this "smear zone" increases toward the drain. However, to date, most analytical models (e.g., Hansbo 1981; Zhu and Yin 2004) have included smear effects by incorporating a reduced horizontal permeability that is held constant throughout the smear zone. Typically, the size of smear zone is assumed as a function of mandrel or drain size with the smear zone permeability set equal to the undisturbed vertical permeability. The extent of smearing depends on the mandrel size and soil type (Lo 1998; Eriksson et al. 2000). Assumed radii for a constant permeability smear zone range in size from 1.6 to 4 times the equivalent drain or mandrel radius (Hansbo 1981; Indraratna and Redana 1998b). If sufficient measurements are available, then the smear zone properties do not have to be assumed. Fig. 1 shows that for laboratory work on reconstituted clays (Onoue et al. 1991; Indraratna and Redana 1998b) a parabolic decay in horizontal permeability towards the drain is appropriate. The model proposed in this technical note is an extension of Hansbo (1981) theory incorporating a parabolic permeability distribution. The model predictions are compared with observed settlement data using a large scale consolidation cell.

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Analytical Solution

Vertical drains, installed in a square or triangular pattern, are usually modeled analytically by considering an equivalent axisymmetric system. Pore water flows from a soil cylinder to a single central vertical drain with simplified boundary conditions. Fig. 2 shows a unit cell with an external radius r_e, and an initial drainage path length l. The radius of the vertical drain and smear zone are r_w and r_s, respectively. According to Hansbo (1981), for axisymmetric flow, the average degree of consolidation, $\bar{U}_h$, on a horizontal plane at a depth z and at time t is

$$\bar{U}_h = 1 - \exp[-8T_h/\mu_{hs}] \tag{1a}$$

where the value of μ_{hs} for smear effect, assuming no well resistance, is given by

$$\mu_{hs} = \ln(n/s) + (k_h/k'_h)\ln(s) - 0.75 \tag{1b}$$

Including smear and well resistance

$$\mu_{hs} = \ln(n/s) + (k_h/k'_h)\ln(s) - 0.75 + \pi z(2l - z)(k_h/q_w) \tag{1c}$$

Neglecting both smear and well resistance

$$\mu_h = \ln(n) - 0.75 \tag{1d}$$

In the preceding, $n = r_e/r_w$ and $s = r_s/r_w$ (see Fig. 2); q_w= drain discharge capacity; k_h= horizontal coefficient of permeability; and k'_h= horizontal coefficient of permeability in the smear zone, which is assumed constant throughout the smear zone in the Hansbo (1981) theory.

Parabolic Smear Zone Permeability

This section presents a revised radial consolidation equation based on a parabolic decay in permeability towards the drain. The calculation steps in Hansbo's (1981) axisymmetric analysis are followed with modification, in order to incorporate the decay of permeability in the smear zone. Real soil properties may change during consolidation but in this analysis, for simplicity, the coefficient of consolidation is assumed constant (horizontal permeability varies spatially but not with time). The velocity of water flow in the undisturbed zone (Darcy's law) is given by

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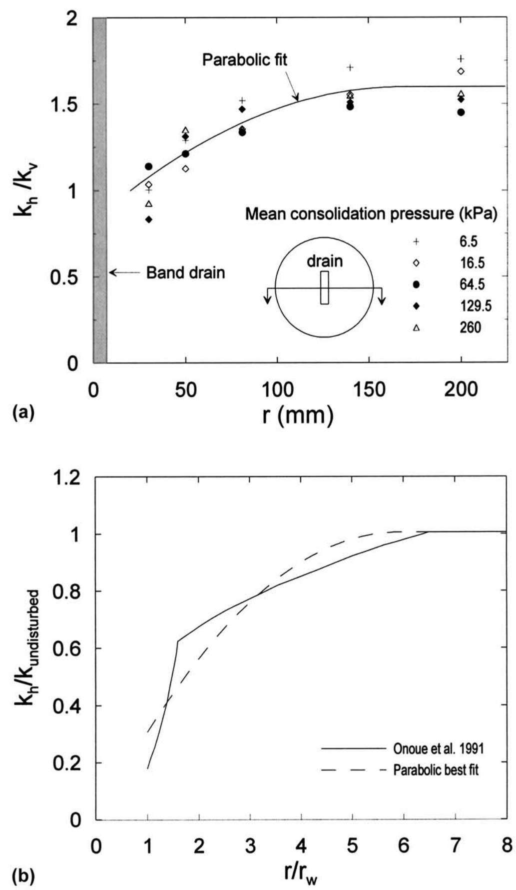

Fig. 1. (a) Ratio of horizontal to vertical permeability along radial distance from drain in large scale consolidometer [original data from Indraratna and Redana (1998b)]; (b) proposed horizontal permeability distribution for isotropic soil [original data from Onoue et al. (1991)]

$$v_r = \left(\frac{k_h}{\gamma_w}\right)\left(\frac{\partial u}{\partial r}\right) \tag{2}$$

where γ_w=unit weight of water; u=pore water pressure; and r=radial coordinate. A similar relation exists in the smear zone, hence

$$v_r = \left(\frac{k'_h}{\gamma_w}\right)\left(\frac{\partial u'}{\partial r}\right) \tag{3}$$

where u'=pore water pressure in the smear zone.

Hansbo (1981) used a constant value of k'_h. In the writers' model, k'_h is a function of r determined by the conditions

$$k'_h(r_w) = k_0 \tag{4a}$$

$$k'_h(r_s) = k_h \tag{4b}$$

$$\partial k'_h(r_s)/\partial r = 0 \tag{4c}$$

The parabolic curve that satisfies the above conditions, shown schematically in Fig. 3, is given by

$$k'_h(r) = k_0(\kappa - 1)(A - B + Cr/r_w)(A + B - Cr/r_w) \tag{5}$$

where $\kappa = k_h/k_0$; $A = \sqrt{\kappa/(\kappa - 1)}$; $B = s/(s-1)$; and $C = 1/(s-1)$.

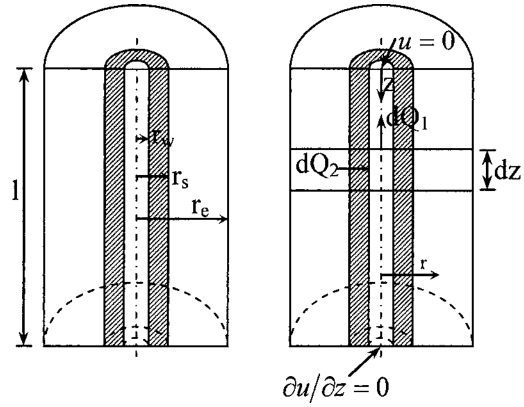

Fig. 2. Axisymmetric unit cell

The flow of pore water through the boundary of the cylinder with radius r is equal to the change in volume of the hollow cylinder with outer radius r_e and inner radius r, which gives

$$2\pi r v_r = \pi(r_e^2 - r^2)\left(\frac{\partial \varepsilon}{\partial t}\right) \tag{6}$$

where $\partial\varepsilon/\partial t$=depth averaged vertical strain rate. Substituting Eq. (2) into Eq. (6) and rearranging gives the pore pressure gradient in the undisturbed zone

$$\left(\frac{\partial u}{\partial r}\right) = \left(\frac{\gamma_w}{2k_h}\right)\left(\frac{\partial \varepsilon}{\partial t}\right)\left(\frac{r_e^2}{r} - r\right) \quad r_s \leqslant r \leqslant r_e \tag{7}$$

Similarly in the smear zone the pore pressure gradient is

$$\left(\frac{\partial u'}{\partial r}\right) = \left(\frac{\gamma_w}{2k'_h}\right)\left(\frac{\partial \varepsilon}{\partial t}\right)\left(\frac{r_e^2}{r} - r\right) \quad r_w \leqslant r \leqslant r_s \tag{8}$$

For vertical flow in the drain, the change in flow from the entrance to the exit of the slice with thickness dz (Fig. 2) is given by

$$dQ_1 = \frac{\pi r_w^2 k_w}{\gamma_w}\left(\frac{\partial^2 u'(r_w,z)}{\partial z^2}\right)dz\,dt \tag{9}$$

where k_w=drain permeability.

The radial flow into the slice is determined from

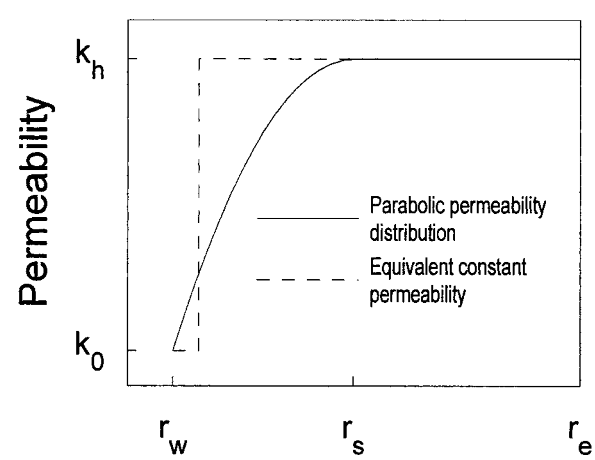

Fig. 3. Permeability distribution

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$$dQ_2 = \frac{2\pi r_w k_0}{\gamma_w}\left(\frac{\partial u'(r_w,z)}{\partial r}\right)dzdt \tag{10}$$

To satisfy continuity of flow

$$dQ_1 = dQ_2 \tag{11}$$

Assuming no sudden drop in pore pressure at the drain-soil boundary (i.e., $u=u'$ at $r=r_w$), substituting Eqs. (9) and (10) into Eq. (11) yields the following:

$$\left(\frac{\partial u'(r_w,z)}{\partial r}\right) + \frac{r}{2}\frac{k_w}{k_0}\left(\frac{\partial^2 u'(r_w,z)}{\partial z^2}\right) = 0 \tag{12}$$

Substituting Eq. (12) into Eq. (8) and integrating in the z direction subject to the boundary conditions: $u'=0$ at $z=0$, $u'=0$ at $z=2l$, and $\partial u'/\partial z=0$ at $z=l$, the pore pressure at the drain-soil boundary is determined as

$$u'(r_w,z) = \left(\frac{\gamma_w}{k_w}\right)\left(\frac{\partial \varepsilon}{\partial t}\right)(n^2-1)\left(lz - \frac{z^2}{2}\right) \tag{13}$$

With the substitution of Eq. (5), Eq. (8) is integrated in the r direction with the same boundary conditions as mentioned previously. In the same manner, Eq. (7) is then integrated assuming $u=u'$ at $r=r_s$. The resulting expressions for pore water pressure on either side of the smear zone boundary are

$$\begin{aligned} u = \frac{\gamma_w r_e^2}{2k_h}\left(\frac{\partial \varepsilon}{\partial t}\right)\Bigg[&\ln\left(\frac{r/r_w}{s}\right) - \frac{1}{2n^2}\left(\frac{r^2}{r_w^2} - s^2\right) \\ &+ A^2\left(\frac{1}{A^2-B^2}\left(\ln(s) - \frac{1}{2}\left\{\ln(\kappa) + \frac{BE}{A}\right\}\right)\right. \\ &\left.+ \frac{1}{2n^2C^2}\left\{\ln(\kappa) - \frac{BE}{A}\right\}\right) + \frac{k_h}{k_w}(n^2-1)(2lz - z^2)\Bigg] \end{aligned} \tag{14}$$

$$\begin{aligned} u' = \frac{\gamma_w r_e^2}{2k_0(\kappa-1)}\left(\frac{\partial \varepsilon}{\partial t}\right)\Bigg[&\frac{1}{A^2-B^2}\left(\ln\left(\frac{r}{r_w}\right)\right. \\ &\left.- \frac{1}{2A}\{(A-B)F + (A+B)G\}\right) \\ &+ \frac{1}{2n^2AC^2}\big((A+B)F + (A-B)G\big) \\ &+ \frac{k_0(\kappa-1)}{k_w}(n^2-1)(2lz - z^2)\Bigg] \end{aligned} \tag{15}$$

where

$$E = \ln\left(\frac{A+1}{A-1}\right)$$

$$F(r) = \ln\left(\frac{A+B-Cr/r_w}{A+1}\right)$$

$$G(r) = \ln\left(\frac{A-B+Cr/r_w}{A-1}\right)$$

If $\bar{u}$ is the average excess pore pressure in the soil cylinder at depth z, then

$$\bar{u}\pi(r_e^2 - r_w^2) = \int_{r_w}^{r_s} 2\pi r u' dr + \int_{r_s}^{r_e} 2\pi r u dr \tag{16}$$

Substituting Eqs. (14) and (15) into Eq. (16) and subsequent solution gives

$$\bar{u} = \frac{\gamma_w r_e^2}{2k_h}\left(\frac{\partial \varepsilon}{\partial t}\right)\mu_p \tag{17a}$$

where

$$\mu_p = \frac{n^2}{n^2-1}\left(\frac{A^2}{n^2}\mu_1 + \mu_2\right) + \frac{k}{q_h}\pi z(2l-z)\left(1 - \frac{1}{n^2}\right) \tag{17b}$$

$$\begin{aligned} \mu_1 = &\frac{1}{A^2-B^2}\left(s^2\ln(s) - \frac{1}{2}(s^2-1)\right) \\ &- \frac{1}{(A^2-B^2)C^2}\left(\frac{A^2}{2}\ln(\kappa) + \frac{ABE}{2} + \frac{1}{2} - B - (A^2-B^2)\ln(\kappa)\right) \\ &+ \frac{1}{n^2C^4}\left(-\left(\frac{A^2}{2} + B^2\right)\ln(\kappa) + \frac{3ABE}{2} + \frac{1}{2} - 3B\right) \end{aligned} \tag{17c}$$

and

$$\begin{aligned} \mu_2 = &\ln\left(\frac{n}{s}\right) - \frac{3}{4} + \frac{s^2}{n^2}\left(1 - \frac{s^2}{4n^2}\right) + A^2\left(1 - \frac{s^2}{n^2}\right) \\ &\times\left\{\frac{1}{A^2-B^2}\left(\ln\left(\frac{s}{\sqrt{\kappa}}\right) - \frac{BE}{2A}\right) + \frac{1}{n^2C^2}\left(\ln(\sqrt{\kappa}) - \frac{BE}{2A}\right)\right\} \end{aligned} \tag{17d}$$

As n^2 is generally much greater than s^2 in most cases, the insignificant terms can then be ignored, thus Eq. (17b) can be reduced to

$$\begin{aligned} \mu_p = &\ln\left(\frac{n}{s}\right) - \frac{3}{4} + \frac{\kappa(s-1)^2}{(s^2-2\kappa s+\kappa)}\ln\left(\frac{s}{\sqrt{\kappa}}\right) \\ &- \frac{s(s-1)\sqrt{\kappa(\kappa-1)}}{2(s^2-2\kappa s+\kappa)}\ln\left(\frac{\sqrt{\kappa}+\sqrt{\kappa-1}}{\sqrt{\kappa}-\sqrt{\kappa-1}}\right) + \frac{k}{q_h}\pi z(2l-z) \end{aligned} \tag{17e}$$

The final term involving q_w is omitted when ignoring well resistance. As $\kappa \to 1$ and $s \to 1$, Eq. (17) approaches the ideal case in Eq. (1c). Errors using Eq. (17e) rather than Eq. (17b) become significant when $n \leqslant 10$. Eq. (17) may now be combined with the Terzaghi constitutive equation for one-dimensional compression

$$\frac{\partial \varepsilon}{\partial t} = m_v \frac{\partial \bar{\sigma}'}{\partial t} = -m_v \frac{\partial \bar{u}}{\partial t} \tag{18}$$

where m_v=coefficient of volume compressibility (m_v in smear and undisturbed zone assumed equal) and $\bar{\sigma}'$=average effective stress. Combining Eqs. (18) and (17) with the initial condition $\bar{u}=\bar{u}_0$ at $t=0$ gives

$$\bar{u} = \bar{u}_0 \exp\lfloor -8T_h/\mu_p \rfloor \tag{19}$$

where $T_h = c_h t/4r_e^2$=horizontal time factor and $c_h = k_h/m_v\gamma_w$ =horizontal coefficient of consolidation. The average degree of consolidation in the radial direction at a particular depth with well resistance, as in Eq. (1a), is now given by

$$\bar{U}_h = 1 - \frac{\bar{u}}{\bar{u}_0} = 1 - \exp[-8T_h/\mu_p] \tag{20}$$

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Comparison with Laboratory Testing

Soil properties, testing procedures, settlement, and pore pressure data for the laboratory test described here are described fully in Indraratna and Redana (1998a,b). The relevant data (summarized in the following) from this test is reanalyzed here with the proposed consolidation equations. Predicted and measured settlement data are compared. Reconstituted alluvial clay from Moruya [40–50% clay sized particles ($<2\ \mu$m), 40% saturated water content, liquid limit of 70, plastic limit of 30, and 17 kN/m^3 saturated unit weight] was thoroughly mixed and placed in the steel consolidation cell, which is a stainless steel cylinder (height=950 mm and diameter=450 mm), where drainage is provided at the top of the soil. The height of the sample can be shortened by using an internal "riser." The ring friction expected with such a large height/diameter ratio (1.5–2) is almost eliminated by using an ultrasmooth Teflon membrane around the cell boundary (friction coefficient less than 0.03). The soil was subjected to an initial preconsolidation pressure, $\sigma'_p=35$ kPa until the settlement rate became negligible. The load was then removed and a single vertical drain (Flowdrain 75 mm$\times$4 mm) was installed using a rectangular steel mandrel (80 mm$\times$10 mm).

In order to measure the disturbance of the soil due to insertion of the mandrel, small horizontal and vertical specimens were cored from the consolidometer. These samples were subject to one-dimensional consolidation using conventional (50 mm diameter) oedometers. The measured soil properties are as follows: compression index $C_c=0.34$, recompression index $C_r=0.14$, vertical coefficient of consolidation $c_v=1.5\times10^{-8}$ m^2/s (c_v in smear and undisturbed zone assumed equal), vertical coefficient of permeability $k_v=2.25\times10^{-10}$ m/s, and the horizontal permeability distribution is shown in Fig. 1(a). The high C_r/C_c ratio exceeding 0.4 is due to remolding, and a similar value for remolded Winipeg clay was determined by Graham and Li (1985). The equivalent radius of the band drain [after Rixner et al. (1986)] is $r_w=(75+4)/4=20$ mm. The fitted parabolic curve in Fig. 1(a) is described by $k_h/k_0=1.6$ (at $r=r_w$, k_0 is assumed equal to k_v), $r_e/r_w=11.25$, and $r_s/r_w=8.4$. These parameters give $k_h=3.60\times10^{-10}$ m/s, $k_0=2.25\times10^{-10}$ m/s, $\mu_p=2.25$, and $c_h=2.4\times10^{-8}$ m^2/s.

Owing to the short vertical drainage length, the well resistance was ignored, and vertical degree of consolidation was considered by Terzaghi's one-dimensional equation

$$\bar{U}_z = 1 - \sum_{m=0}^{\infty}\frac{2}{M}\exp[-M^2T_z] \quad (21)$$

where $M=\pi(2m-1)/2$; $m=1,2,\ldots$; and $T_z=c_vt/l^2$=vertical time factor. Consolidation by vertical and horizontal drainage are combined with Carillo's (1942) relationship

$$(1-\bar{U}) = (1-\bar{U}_z)(1-\bar{U}_h) \quad (22)$$

After drain installation an overconsolidated initial state ($\sigma'_0<\sigma'_p$) for the soil was first induced with the application of $\sigma'_0=20$ kPa. When the settlement rate became negligible, the surcharge pressure was increased to 50, 100, and 200 kPa (three-stage loading). Using Eqs. (20)–(22), the average excess pore pressure was calculated using the parabolic permeability distribution within the smear zone. For an initial void ratio of $e_0=0.95$, the settlement, ρ, was calculated using:

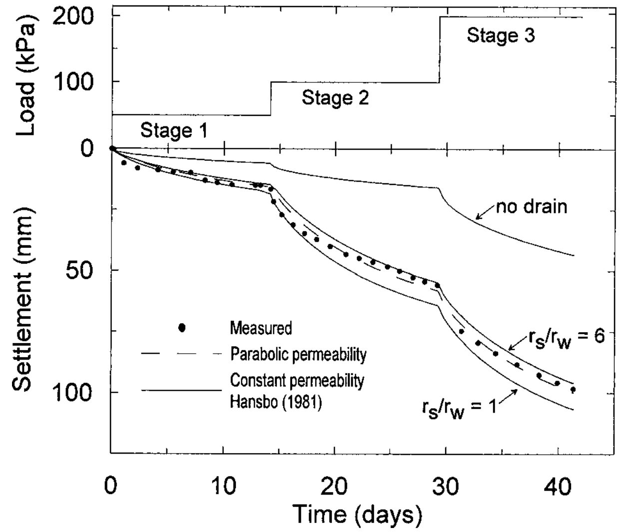

Fig. 4. Predicted and measured settlements for large scale consolidometer

$$\rho = \begin{cases} \dfrac{HC_r}{1+e_0}\log\left(\dfrac{\sigma'}{\sigma'_0}\right), & \sigma' < \sigma'_p \\[2ex] \dfrac{HC_r}{1+e_0}\log\left(\dfrac{\sigma'_p}{\sigma'_0}\right) + \dfrac{HC_c}{1+e_0}\log\left(\dfrac{\sigma'}{\sigma'_p}\right), & \sigma' > \sigma'_p \end{cases} \quad (23)$$

Settlement curves are shown in Fig. 4. Also shown in Fig. 4 are the corresponding settlement plots with constant permeability throughout the smear zone [Eq. (1*b*), k_h/k_0 is the same as for the parabolic case, i.e., 1.6] for the ideal drain (no smear: $r_s/r_w=1$) and an assumed upper bound for maximum smear ($r_s/r_w=6$). The writers' solution with parabolic permeability decay, $r_s/r_w=8.4$, and Hansbo (1981) with constant permeability are identical only in the case of Hansbo's $r_s/r_w=2.62$. This shows that the extent of smearing is much greater than that assumed when considering a smear zone with constant reduced permeability smear zone. Fig. 4 confirms that the effects of smear can be assessed by using existing assumptions about the size of a constant permeability smear zone with a radius of 1.6–4 times the equivalent drain or mandrel radius (Hansbo 1981; Indraratna and Redana 1998b). However, more meaningful interpretations of the extent of smear can be made using the proposed parabolic change of lateral permeability within the smear zone, justifiable based on laboratory observations.

By using the measured parabolic permeability distribution, which gives good agreement with the model analysis as shown in Fig. 4, the need for assuming a smear zone radius and the consequent uncertainty in the analysis are reduced. It is acknowledged that μ_{hs} is easier to calculate than μ_p, but in soils where the rate of consolidation is dependent on the properties of the smear zone, the current model provides enhanced reliability, in spite of the more rigorous computational procedure.

Conclusion

In this study, a modified vertical drain radial consolidation equation based on the parabolic reduction of permeability towards a vertical drain is presented. The parabolic reduction in permeability is based on laboratory evidence. Hansbo's (1981) well known radial consolidation equations (where a constant coefficient of

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consolidation is assumed) are modified, without increasing the number of variables. The validity of the method has been examined by comparison with settlement data using a large scale consolidometer. Greater confidence in predicted results is gained by basing the analysis on a measured permeability distribution rather than the past conventional approach of assuming the size of smear zone with a constant permeability.

Notation

The following symbols are used in this technical note:

A, B, C, E, F, G = parameters in μ_p;
C_c = compression index;
C_r = recompression index;
c_h = radial consolidation coefficient;
c_v = radial consolidation coefficient;
H = depth of soil;
k_h = horizontal soil permeability in undisturbed zone;
k'_h = horizontal soil permeability in smear zone;
k_w = permeability in drain;
k_0 = horizontal soil permeability at r_w;
l = vertical drainage length;
M = summation term;
m = summation index;
m_v = soil volume compressibility;
n = ratio of influence radius to drain radius;
$Q_{1,2}$ = pore water flow volumes;
q_w = drain discharge capacity;
r = radial coordinate;
r_e = radius of influence;
r_s = radius of smear zone;
r_w = radius of drain;
s = ratio of smear radius to drain radius;
T_h = radial time factor;
T_z = vertical time factor;
t = time;
$\bar{U}_h$ = average degree of radial consolidation;
$\bar{U}_z$ = average degree of vertical consolidation;
u = pore water pressure in undisturbed zone;
u' = pore water pressure in smear zone;
$\bar{u}$ = average excess pore water pressure;
v_r = radial pore water velocity;
z = depth;
γ_w = unit weight of water;
ε = strain;
κ = smear zone permeability ratio;
μ_{hs} = drain and soil parameter for constant k'_h;
μ_p = drain and soil parameter for parabolic k'_h;
$\mu_{1,2}$ = parameters in μ_p; and
σ' = effective stress.

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