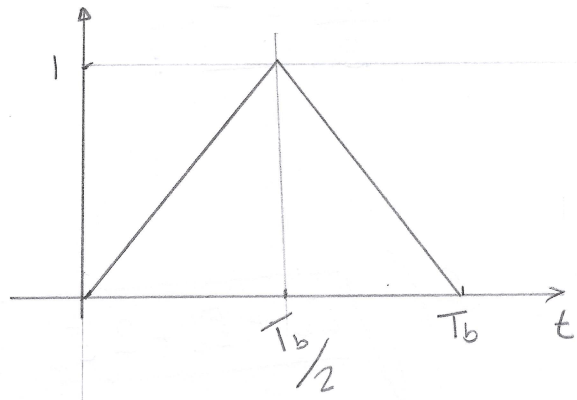
Aula 06 · Gabarito

11.1.1 (a)

$$h_o(t) = p(T_b - t) = 1 - \frac{|T_b - 2T_b + 2t|}{T_b} =$$

$$= 1 - \frac{|2t - T_b|}{T_b}, \quad 0 \le t \le T_b$$



(b)

$$E_p = \int_0^{T_b} \left[1 - \frac{|T_b - 2t|}{T_b}\right]^2 dt = \int_0^{T_b/2} \left[1 - \frac{(T_b - 2t)}{T_b}\right]^2 dt +$$

$$+ \int_{T_b/2}^{T_b} \left[1 + \left(\frac{T_b - 2t}{T_b}\right)\right]^2 dt = \frac{1}{T_b^2}\int_0^{T_b/2} 4t^2\, dt + \frac{1}{T_b^2}\int_{T_b/2}^{T_b} \underbrace{(-2t + 2T_b)^2}_{(T_b - t)^2} dt$$

$$= \frac{4}{T_b^2}\left[\frac{t^3}{3}\Big|_0^{T_b/2} - \frac{(T_b - t)^3}{3}\Big|_{T_b/2}^{T_b}\right] = \frac{4}{3T_b^{\cancel{2}}}\left(\frac{T_b^{\cancel{3}}}{8} + \frac{T_b^{\cancel{3}}}{8}\right) =$$

$$= \frac{4}{3} \cdot \frac{T_b}{4} = \frac{T_b}{3} \qquad \boxed{E_p = \frac{T_b}{3}}$$

$$E_1 = A^2 \cdot \frac{T_b}{3} \quad \Big| \quad E_2 = A^2 \frac{T_b}{3}$$

$$\boxed{E_b = A^2 \frac{T_b}{3}}$$

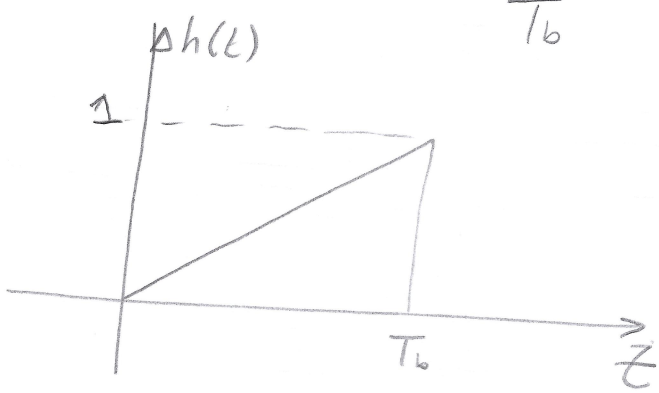
$$E_{12} = -A^2 \int p^2(t)\,dt = -A^2 \frac{T_b}{3} = -E_b$$

$$P_e = Q\left(\sqrt{\frac{E_1 + E_2 - 2E_{12}}{2N}}\right) = Q\left(\sqrt{\frac{E_b + E_b + 2E_b}{2N}}\right) =$$

$$= Q\left(\sqrt{\frac{2E_b}{N}}\right)$$

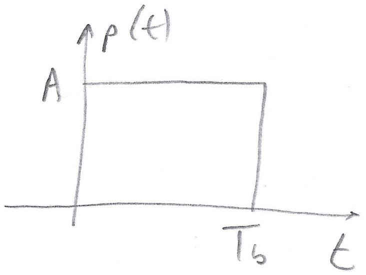
(c) Resultado da parte (b) não mudam (sinalização polar)

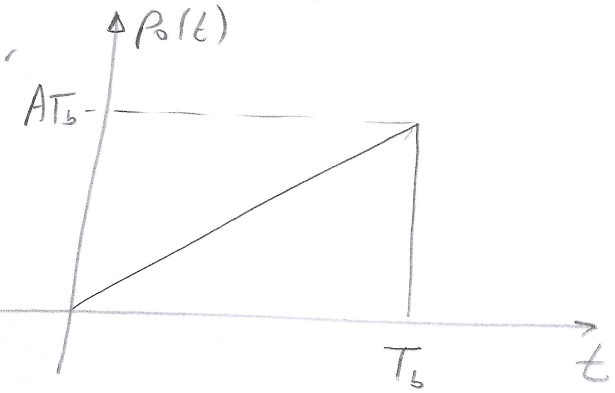
$$h(t) = p(T_b - t) = 1 - \frac{(T_b - t)}{T_b} = \frac{t}{T_b}, \quad 0 \le t \le T_b$$



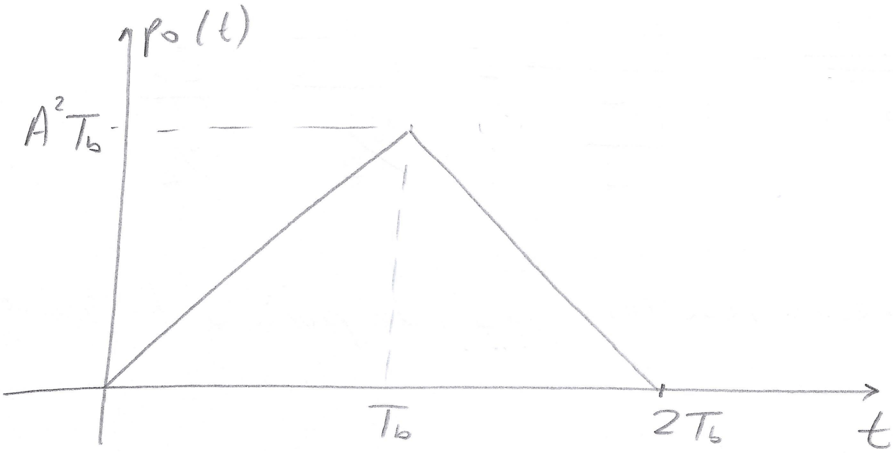
11.1-3

a) Integrador





b) Filtro casado



c) Para o filtro "integrate-and-dump", a saída é a integral de $p(t)$. Assim, em $t = T_b$, $p_o(T_b) = A T_b$.

Se aplicarmos $\delta(t)$ na entrada desse filtro, a saída será $\boxed{h(t) = u(t) - u(t - T_b)}$

Portanto, $H(f) = T_b \operatorname{sinc}(\pi f T_b)\, e^{-j\pi f T_b}$ e

$$\overline{n_o^2(t)} = \int_{-\infty}^{\infty} \underbrace{\frac{N}{2}}_{S_n} \cdot \underbrace{T_b^2 \operatorname{sinc}^2(\pi f T_b)}_{|H(f)|^2} df = \frac{N T_b^2}{2} \int_{-\infty}^{\infty} \operatorname{sinc}^2(\pi f T_b)\, df$$

Fazendo $u = \pi f T_b$, $du = \pi T_b \, df$ ou $df = \frac{du}{\pi T_b}$

$$\overline{n_o^2(t)} = \frac{N T_b^2}{2} \int_{-\infty}^{\infty} \frac{\text{sinc}^2(u)}{\pi T_b} \, du = \frac{N T_b}{2\pi} \cancel{\int_{-\infty}^{\infty} \text{sinc}^2 u \, du} \; \pi$$

$$= \frac{N T_b}{2}$$

e

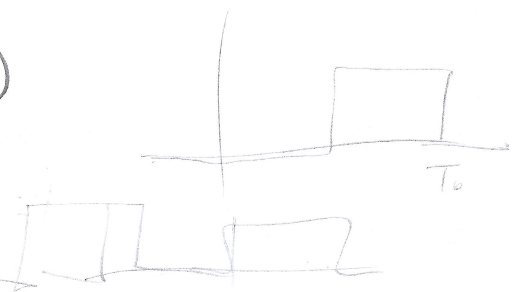
$$\rho^2 = \frac{p_o^2(T_b)}{\overline{n_o^2(t)}} = \frac{A^2 T_b^2}{\frac{N T_b}{2}} = \frac{A^2 T_b}{N/2} = \frac{E_p}{N/2}$$

que é o mesmo resultado obtido para o filtro casado!!

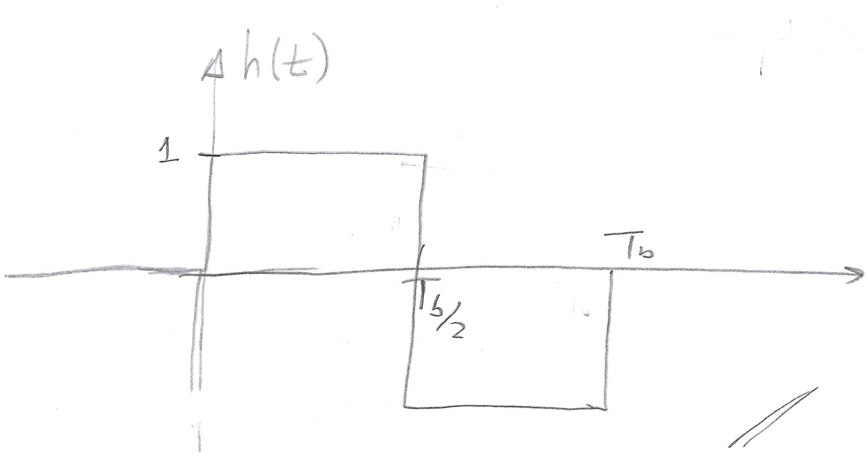
11.2.1

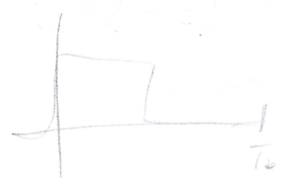
a

$$\begin{cases} \text{"1"} \rightarrow p(t) = p_o(t - T_b/2) \\ \text{"0"} \rightarrow q(t) = p_o(t) \end{cases}$$



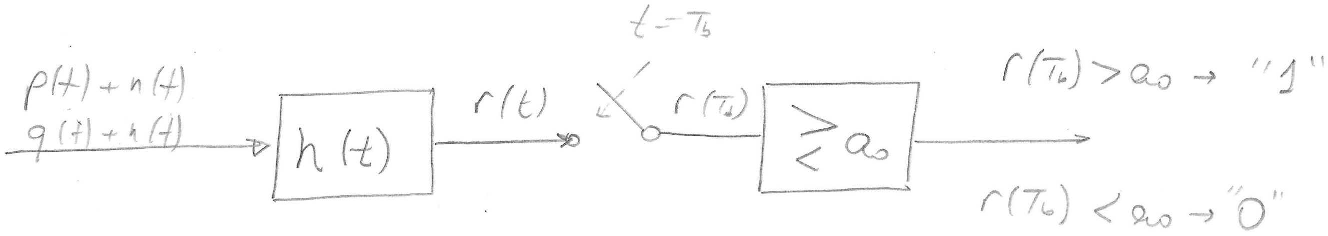
Filtro: $h(t) = p(T_b - t) - q(T_b - t) = p_o(T_b/2 - t) - p_o(T_b - t)$



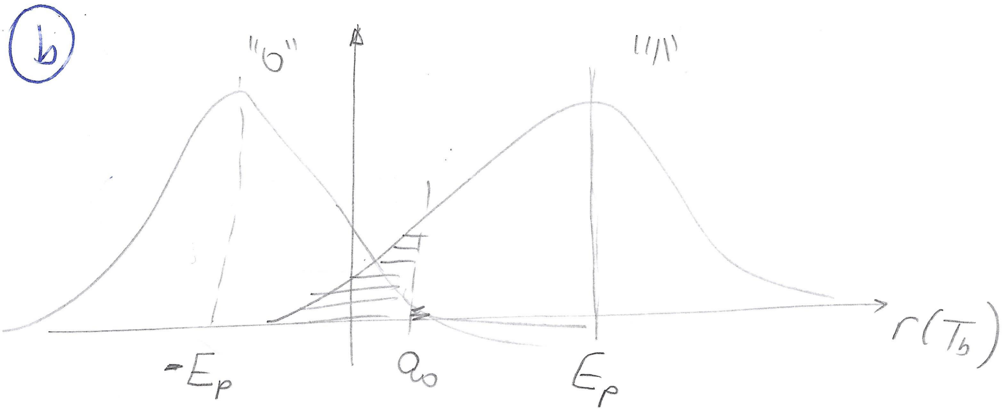


6

$E_p = E_q = T_b/2$ $\quad \boxed{a_0 = 0}$ (Símbolos equiprováveis)



b



$$P_e = P_{e|1} \cdot P(1) + P_{e|0} \cdot P(0) =$$

$$= P(r < a_0 \,|\, \text{"1"}) \cdot P(1) + P(r > a_0 \,|\, \text{"0"}) \cdot P(0) \cdot$$

$$= \left[1 - Q\left(\frac{a_0 - E_p}{\sigma_n}\right)\right] \cdot P(1) + Q\left(\frac{a_0 + E_p}{\sigma_n}\right) \cdot P(0) =$$

$$= Q\left(\frac{E_p - a_0}{\sigma_n}\right) \cdot P(1) + Q\left(\frac{E_p + a_0}{\sigma_n}\right) \cdot P(0)$$

Fazendo $\dfrac{dP_e}{da_0} = P(1) \cdot Q'\left(\dfrac{E_p - a_0}{\sigma_n}\right) \cdot \cancel{\left(-\dfrac{1}{\sigma_n}\right)} + Q'\left(\dfrac{E_p + a_0}{\sigma_n}\right) \cdot \cancel{\dfrac{1}{\sigma_n}}\, P(0) = 0$

$$\Rightarrow e^{-\left(\frac{E_p - a_0}{\sigma_n}\right)^2/2} \cdot P(1) = e^{-\left(\frac{E_p + a_0}{\sigma_n}\right)^2/2} \cdot P(0) \Rightarrow$$

$$\Rightarrow e^{\frac{-[E_p - a_0]^2 + [E_p + a_0]^2}{2\sigma_n^2}} = \frac{P(0)}{P(1)} \Rightarrow$$

$$\Rightarrow \frac{-(\cancel{E_p^2} - 2a_0E_p + \cancel{a_0^2}) + \cancel{E_p^2} + 2a_0E_p + \cancel{a_0^2}}{2\sigma_n^2} = \ln\left(\frac{P(0)}{P(1)}\right)$$

$$\frac{\cancel{4}^{2} a_0 E_p}{\cancel{2}\sigma_n^2} = \ln\frac{P(0)}{P(1)} \Rightarrow \boxed{a_0 = \frac{\sigma_n^2}{2E_p} \ln\frac{P(0)}{P(1)}}$$

$$\text{Ma}\quad \sigma_n^2 = \frac{N}{2} \cdot \int_{-\infty}^{\infty} |H(f)|^2 df = N/2 \cdot [E_p + E_p] = N E_p$$

Assim,

$$\boxed{a_0 = \frac{N}{2} \ln\left(\frac{P(0)}{P(1)}\right)} \Rightarrow a_0 = \ln\left(\frac{0.4}{0.6}\right) N/2$$

e

$$\Rightarrow \boxed{a_0 = \ln(2/3)\, N/2}$$

$$P_e = Q\left(\frac{E_p - a_0}{\sqrt{N E_p}}\right) \cdot P(1) + Q\left(\frac{E_p + a_0}{\sqrt{N E_p}}\right) \cdot P(0)$$

$$= 0.6\, Q\left(\frac{E_b - N/2 \ln(2/3)}{\sqrt{N E_b}}\right) + 0.4\, Q\left(\frac{E_b + N/2 \ln(2/3)}{\sqrt{N E_b}}\right)$$

↳ Veja página seguinte!

c) Se $\underline{\underline{a_0 = 0}}$

$$P_e = Q\left(\sqrt{\frac{E_r}{N}}\right)0.4 + 0.6\,Q\left(\sqrt{E_p/N}\right) =$$

$$= Q\left(\sqrt{\frac{E_b}{N}}\right)$$

Fim b)

$$P_e = 0.6\,Q\left(\frac{E_b/N - \frac{1}{2}\ln\left(2/3\right)}{\sqrt{E_b/N}}\right) + 0.4\,Q\left(\frac{E_b/N + \frac{\ln\left(2/3\right)}{2}}{\sqrt{E_b/N}}\right)$$