PROBLEM SET

5.1 Security A pays $30 if state 1 occurs and $10 if state 2 occurs. Security B pays $20 if state 1 occurs and $40 if state 2 occurs. The price of security A is $5 and the price of security B is $10.

a) Set up the payoff table for securities A and B.

b) Determine the prices of the two pure securities.

5.2 You are given the following information:

	Payoff		
	State 1	State 2	Security Prices
Security j	$12	$20	$p_j = \$22$
Security k	24	10	$p_k = 20$

a) What are the prices of pure security 1 and pure security 2?

b) What is the initial price of a third security i, for which the payoff in state 1 is $6 and the payoff in state 2 is $10?

5.3 Interplanetary starship captain José Ching has been pondering the investment of his recent pilot's bonus of 1000 stenglers. His choice is restricted to two securities: Galactic Steel, selling for 20 stenglers per share, and Nova Nutrients, at 10 stenglers per share. The future state of his solar system is uncertain. If there is a war with a nearby group of asteroids, Captain Ching expects Galactic Steel to be worth 36 stenglers per share. However, if peace prevails, Galactic Steel will be worth only 4 stenglers per share. Nova Nutrients should sell at a future price of 6 stenglers per share in either eventuality.

a) Construct the payoff table that summarizes the starship captain's assessment of future security prices, given the two possible future states of the solar system. What are the prices of the pure securities implicit in the payoff table?

b) If the captain buys only Nova Nutrients shares, how many can he buy? If he buys only Galactic Steel, how many shares can he buy? What would be his final wealth in both cases in peace? At war?

c) Suppose Captain Ching can issue (sell short) securities as well as buy them, but he must be able to meet all claims in the future. What is the maximum number of Nova Nutrients shares he could sell short to buy Galactic Steel? How many shares of Galactic Steel could he sell short to buy Nova Nutrients? What would be his final wealth in both cases and in each possible future state?

d) Suppose a third security, Astro Ammo, is available and should be worth 28 stenglers per share if peace continues and 36 stenglers per share if war breaks out. What would be the current price of Astro Ammo?

e) Summarize the results of (a) through (d) on a graph with axes W_1 and W_2.

f) Suppose the captain's utility function can be written $U = W_1^{.8} W_2^{.2}$. If his investment is restricted to Galactic Steel and/or Nova Nutrients, what is his optimal portfolio, i.e. how many shares of each security should he buy or sell?

5.4 Ms. Mary Kelley has initial wealth $W_0 = \$1200$ and faces an uncertain future that she partitions into two states, $s = 1$ and $s = 2$. She can invest in two securities, j and k, with initial prices of $p_j = \$10$ and $p_k = \$12$, and the following payoff table:

	Payoff	
Security	$s = 1$	$s = 2$
j	$\$10$	$\$12$
k	20	8

a) If she buys only security j, how many shares can she buy? If she buys only security k, how many can she buy? What would her final wealth, W_s, be in both cases and each state?

b) Suppose Ms. Kelley can issue as well as buy securities; however, she must be able to meet all claims under the occurrence of either state. What is the maximum number of shares of security j she could sell to buy security k? What is the maximum number of shares of security k she could sell to buy security j? What would her final wealth be in both cases and in each state?

c) What are the prices of the pure securities implicit in the payoff table?

d) What is the initial price of a third security i for which $Q_{i1} = \$5$ and $Q_{i2} = \$12$?

e) Summarize the results of (a) through (d) on a graph with axes W_1 and W_2.

f) Suppose Ms. Kelley has a utility function of the form $U = W_1^{.6} W_2^{.4}$. Find the optimal portfolio, assuming the issuance of securities is possible, if she restricts herself to a portfolio consisting only of j and k. How do you interpret your results?

5.5 Two securities have the following payoffs in two equally likely states of nature at the end of one year:

	Payoff	
Security	$s = 1$	$s = 2$
j	$\$10$	$\$20$
k	30	10

Security j costs $8 today, whereas k costs $9, and your total wealth is currently $720.

a) If you wanted to buy a completely risk-free portfolio (i.e., one that has the same payoff in both states of nature), how many shares of j and k would you buy? (You may buy fractions of shares.)

b) What is the one-period risk-free rate of interest?

c) If there were two securities and three states of nature, you would not be able to find a completely risk-free portfolio. Why not?

5.6 Suppose there are only two possible future states of the world, and the utility function is logarithmic. Let the probability of state 1, π_1, equal $\frac{2}{3}$, and the prices of the pure securities, p_1 and p_2, equal \$0.60 and \$0.40, respectively. An individual has an initial wealth or endowment, W_0, of \$50,000.

a) What amounts will the risk-averse individual invest in pure securities 1 and 2?

b) How will the individual divide his or her initial endowment between current and future consumption?

[*Hint:* Use the wealth constraint instead of the Lagrange multiplier technique.][20]

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