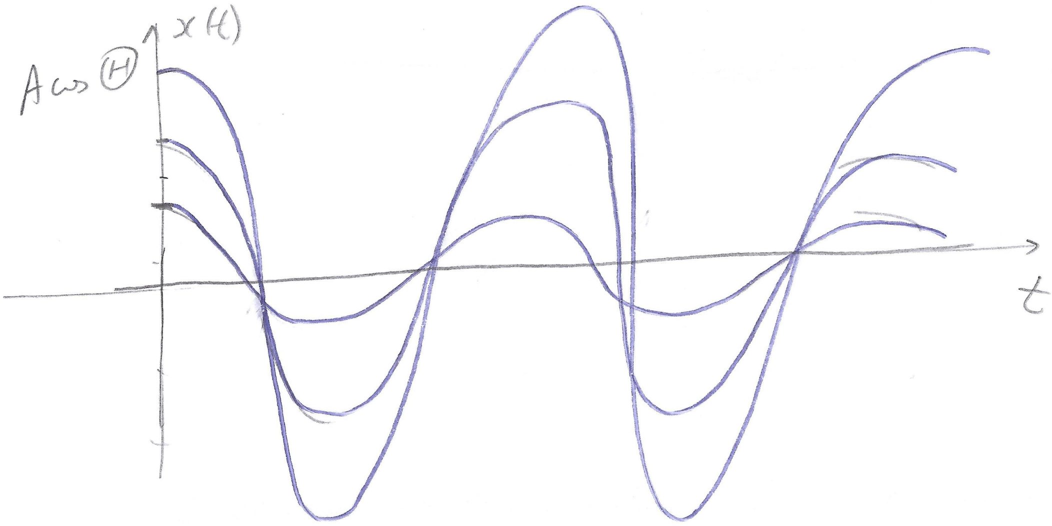
Aula 05 - Gabarito

9.1-1

$x(t) = a\cos(\omega_c t + \Theta)$ $\qquad \omega_c t + \theta = \pi/2$

$t =$



(b) Claramente não estacionário. Por exemplo, em

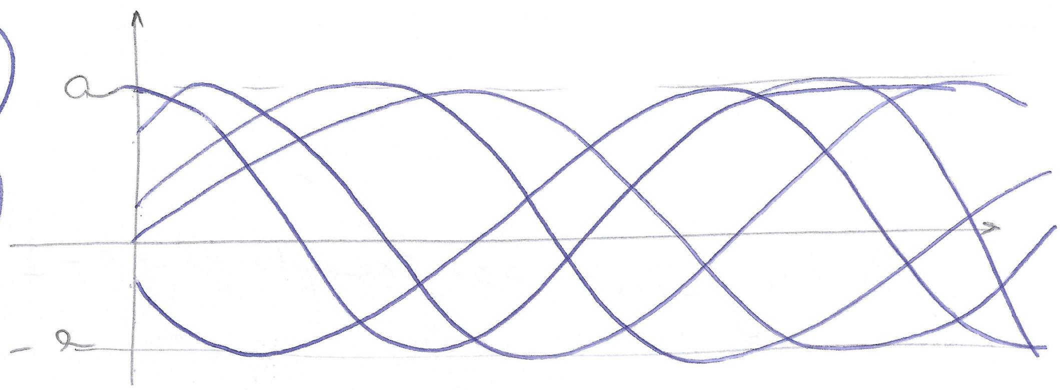
$$t = \frac{\frac{\pi}{2} - \Theta}{\omega_c},$$

todas as funções-amostras se anulam.

Já em $t=0$, $x(0)$ assume valores uniformemente distribuídos entre $(0, A\cos\Theta)$.

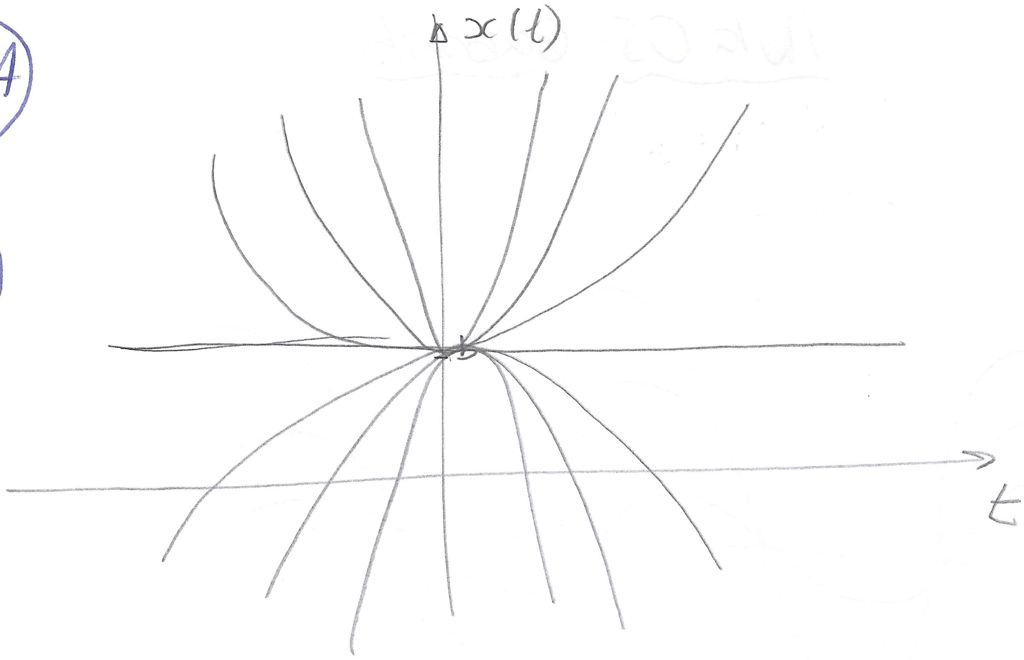
9.1-2

(a)



9.1.4

a



b Naõ estacionário... Variância varia com t!

9.2.3 DEP → real, naõ negativa, par

a $\dfrac{(2\pi f)^2}{(2\pi f)^4+2}$ ✓

b $\dfrac{1}{(2\pi f)^2-2}$ X (negativa em $f=0$, por exemplo)

c $\dfrac{(2\pi f)^2}{(2\pi f)^3+2}$ X (negativo por exemplo p/ $f=-1$)

d $\delta(f-f_0)+\dfrac{1}{(2\pi f)^2+2}$ X Naõ par

e) $\delta(f+f_0) - \delta(f-f_0) \rightarrow$ Não par.

f) $\cos(2\pi(f+f_0)) + j\,\text{sen}(2\pi(f-f_0))$

↳ complexa em $f = f_0/4$, por exemplo

g) $e^{-(2\pi f)^2}$

Somente a) e g) ok!

93-1) Para qualquer a,

$$E[(aX(t_1) - Y(t_2))^2] \geq 0 \Rightarrow$$

$$\Rightarrow a^2 E[X^2(t_1)] - 2a\,E[X(t_1)Y(t_2)] + E[Y^2(t_2)] \geq 0$$

$$\Rightarrow \Delta < 0 \Rightarrow 4 \cdot R_{XY}(\tau)^2 - 4R_X(0)R_Y(0) \geq 0$$

$\tau = t_2 - t_1$

$$\Rightarrow \boxed{R_{XY}(\tau) > R_X(0)R_Y(0)}$$

9.4-1

$$S_{x_1} = \frac{2\alpha}{\alpha^2 + (\cancel{j}2\pi f)^2} \rightarrow \text{Não é DEP!} \qquad S_{x_2} = K$$

$S_y = ?$ $\quad P_y = ?$

Como a rede é linear, a tensão de saída $y(t)$ pode ser expressa como

$$y(t) = y_1(t) + y_2(t)$$

Do exemplo,

$$H_1(f) = \frac{1}{3(3 \cdot j2\pi f + 1)} \quad \Bigg| \quad H_2(f) = \frac{1}{2(3 \cdot j2\pi f + 1)}$$

$$S_{y_1}(f) = |H_1(f)|^2 \cdot S_{x_1}(f) = \frac{2\alpha}{9(1 + 9 \cdot (2\pi f)^2)(\alpha^2 + (\cancel{j}2\pi f)^2)}$$

$$S_{y_2} = |H_2(f)|^2 \cdot S_{x_2}(f) = \frac{K}{4 \cdot (1 + 9(2\pi f)^2)}$$

$x_1(t)$ e $x_2(t)$ independentes $\rightarrow$ $y_1(t)$ e $y_2(t)$ também são.

PSDs não tem impulsos em zero

558
580

$$S_y(f) = S_{y_1}(f) + S_{y_2}(f)$$

$$\boxed{S_y(f) = \frac{36\alpha + 9K(\alpha^2 + (2\pi f)^2)}{36(1+9(2\pi f)^2)\cdot(\alpha^2+(2\pi f)^2)}}$$

$$S_{y_2}(f) = \frac{K}{36\left[\frac{1}{9} + (2\pi f)^2\right]} = \frac{K/36}{\left[\underbrace{1/9}_{a^2} + (2\pi f)^2\right]} =$$

$$= \frac{2/3}{\left[1/9 + (2\pi f)^2\right]} \cdot \frac{K}{\cancel{36}_{12}} \cdot \frac{\cancel{3}}{2}$$

$\Downarrow$

$$R_2(\tau) = \frac{K}{24} \cdot e^{-\frac{1}{3}|\tau|}$$

$$S_{y_1}(f) = \frac{2\alpha}{9\left[1+9(2\pi f)^2\right]\left[\alpha^2 + (2\pi f)^2\right]} =$$

$$= \frac{\alpha}{2\left[9(2\pi f)^2+1\right]\left[\alpha^2+(2\pi f)^2\right]} \cdot \frac{2}{\cancel{\alpha}} \cdot \frac{2\cancel{\alpha}}{9} =$$

Do exemply

$$R_1(\tau) = \cancel{\frac{4}{9}} \cdot \left[\frac{3\alpha - e^{-\alpha|\tau|}}{\cancel{4} \cdot (9\alpha^2 - 1)} \right] =$$

$$= \frac{3\alpha - e^{-\alpha|\tau|}}{9 \cdot (9\alpha^2 - 1)}$$

e

$$R_y(\tau) = \frac{3\alpha - e^{-\alpha|\tau|}}{9 \cdot (9\alpha^2 - 1)} + \frac{K}{24} e^{-1/3|\tau|}$$

e

$$\boxed{P_y = R_y(0) = \frac{K}{24} + \frac{3\alpha - 1}{9(9\alpha^2 - 1)}}$$

9.6-1

$$S_m(f) = \frac{20}{900 + (2\pi f)^2} \quad \Big| \quad S_n(f) = 6\Pi\left(\frac{f}{200}\right)$$

a

$$H_{opt}(f) = \frac{S_m(f)}{S_m(f) + S_n(f)} = \begin{cases} \dfrac{\dfrac{20}{900 + (2\pi f)^2}}{\dfrac{20}{900 + (2\pi f)^2} + 6} & (f < 100) \\ 1, & f > 100 \end{cases}$$

$$H_{opt}(f) = \frac{20}{20+5400+6(2\pi f)^2} = \frac{20}{5420+6(2\pi f)^2} \quad (|f|<100)$$

$$\boxed{H_{opt}(f) = \begin{cases} \dfrac{10}{2710+3\cdot(2\pi f)^2}, & |f|<100 \\ 1, & |f|>100 \end{cases}}$$

(b) No entrada

$$P_n = \int_{-\infty}^{\infty} S_n(f)\,df = 6\cdot 200 = 1200$$

$$S_n \cdot \frac{S_m^2}{(S_m+S_n)^2}$$

$$P_{no} = \int_{-\infty}^{\infty} \frac{S_m(f)\,S_n(f)}{S_m(f)+S_n(f)}\,df = \int_{-100}^{100} \frac{\frac{20}{500+(2\pi f)^2}\cdot 6}{\frac{20}{500+(2\pi f)^2}+6}$$

$$= \int_{-100}^{100} \frac{120}{20+5400+6(2\pi f)^2}\,df =$$

$$= \int_{-100}^{100} \frac{120}{5420+6\cdot(2\pi f)^2}\,df = \int_{-100}^{100} \frac{60}{2710+3\cdot(2\pi f)^2}\,df$$

$$= 0{,}3226$$

(c) Entrada: $P_n = 6 \cdot 200 = 1200$

$$P_m = \int_{-\infty}^{\infty} \frac{20}{300 + (2\pi f)^2} \, df = 1/3$$

$$SNR: \frac{P_m}{P_n} = \frac{1/3}{1200} = 2{,}778 \cdot 10^{-4}$$

Saída:

$$SNR = \frac{P_m}{P_{n_0}} = \frac{1/3}{0{,}3226} = 1{,}033$$

$\hookrightarrow$ ganho: $3719{,}5$

96.2 $S_m(f) = \dfrac{4}{4 + (2\pi f)^2}$ / $S_n(f) = \dfrac{32}{64 + (2\pi f)^2}$

(a)

$$H_{opt}(f) = \frac{S_m(f)}{S_m(f) + S_n(f)} = \frac{\dfrac{4}{(2\pi f)^2 + 4}}{\dfrac{4}{4 + (2\pi f)^2} + \dfrac{32}{64 + (2\pi f)^2}} =$$

$$= \frac{\cancel{4}}{\cancel{\omega^2+4}} \cdot \frac{\cancel{(4+\omega^2)}(64+\omega^2)}{\cancel{4}\cdot[64+\omega^2]+\cancel{32}^{8}[4+\omega^2]} =$$

$$= \frac{64+\omega^2}{96+9\omega^2} \rightarrow \boxed{H_{opt}(f) = \frac{\omega^2+64}{9\omega^2+96}}$$

6 Ruído na entrada

$$S_n(f) = \frac{32}{64+(2\pi f)^2} = 2 \cdot \frac{16}{64+(2\pi f)^2}$$

$$\Updownarrow \text{TF}$$

$$R_n(\tau) = 2 \cdot e^{-8|t|} \rightarrow \boxed{P_n = 2}$$

Ruído na saída

$$N_o = \int_{-\infty}^{\infty} \underbrace{\frac{S_m(f)}{S_m(f)+S_n(f)}}_{H_{opt}} S_n(f)\, df = \int_{-\infty}^{\infty} \frac{\cancel{\omega^2+64}}{9\omega^2+36} \cdot \frac{32}{\cancel{64+\omega^2}}\, df =$$

$$= \frac{1}{9}\int_{-\infty}^{\infty} \frac{32}{\omega^2+36/9}\, df = 0{,}5443$$

(C) A potência do sinal é

$$P_{\bar{m}} = P_{m_0} = \int_{-\infty}^{\infty} \frac{4}{4+\omega^2}\, df = 1$$

$$P_n = \int_{-\infty}^{\infty} \frac{32}{64+\omega^2}\, df = 2$$

$(SNR)_{in} = 1/2$

$$(SNR)_{out} = \frac{1}{0{,}5443} = 1{,}8372$$

Ganho: 3,6744