Aula 04 - Gabarito

8.1-6

a) $P_e = 0{,}4$

$$P_3 = \binom{12}{3}(0{,}4)^3(0{,}6)^9 = 14{,}19\%$$

b) $P \geq 4$

$$P_{\geq 4} = 1 - P_3 - P_2 - P_1 - P_0 =$$

$$= 1 - \overbrace{P_3}^{0{,}1419} - \underbrace{\binom{12}{2}(0{,}4)^2(0{,}6)^{10}}_{0{,}0639} - \underbrace{\binom{12}{1}(0{,}4)^1(0{,}6)^{11}}_{0{,}0174} - \underbrace{\binom{12}{0}(0{,}4)^0(0{,}6)^{12}}_{0{,}0022}$$

$$= 77{,}46\%$$

8.1-15

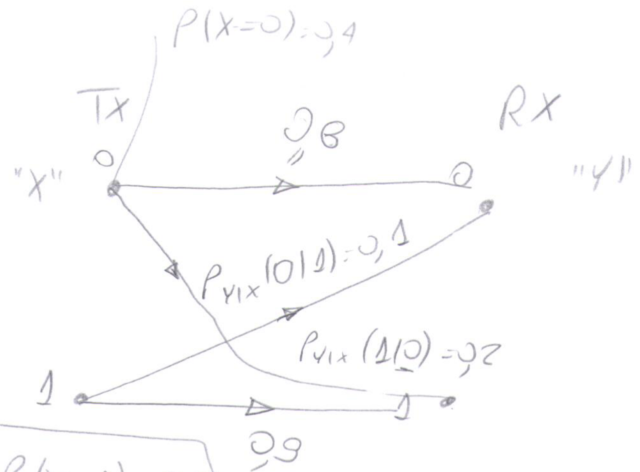
P_e

$$P = \binom{100}{3}(P_e)^3(1-P_e)^{97} + \binom{100}{2}P_e^2(1-P_e)^{98} + \binom{100}{1}P_e(1-P_e)^{99}$$

$$+ \binom{100}{0}P_e^0(1-P_e)^{100} + \binom{100}{4}(P_e)^4(1-P_e)^{96}$$

$$= \sum_{i=0}^{4}\binom{100}{i}P_e^i(1-P_e)^{100-i}$$

8.2-1



$\boxed{P(X=1)=0,6}$

$P(X=0)=0,4$

a) $P_y(0)=P_{Y|X}(0|1)\cdot P(X=1)+P_{Y|X}(0|0)\cdot P(X=0)$

$= 0,1\cdot 0,6+0,8\cdot 0,4 = 0,06+0,32=0,38$

$P_y(1)=P_{Y|X}(1|0)\cdot P(X=0)+P_{Y|X}(1|1)\cdot P(X=1)$

$= 0,2\cdot 0,4+0,9\cdot 0,6=0,08+0,54=0,62$

b) $P=\binom{10}{0}(0,62)^0(0,38)^{10}=6,27\cdot 10^{-5}$

c) $P=\binom{10}{8}(0,62)^8(0,38)^2=0,1419$

d) $P=\sum_{i=0}^{5}\binom{10}{i}(0,62)^i(0,38)^{10-i}=0,3177$

8.2-5 $P_X(x) = \frac{1}{\sqrt{2\pi}\, C} e^{-(x-4)^2/18}$

a

$2\sigma^2 = 18 \Rightarrow \underline{\sigma = 3}$ $\boxed{C = 3}$

b $P(X \geq 2) = 1 - F_X(2) = 1 - \left(1 - Q\left(\frac{2-4}{3}\right)\right) =$

$= Q\left(\frac{-2}{3}\right) = 1 - Q\left(2/3\right) = 1 - 0{,}2514 = 74{,}86\%$

c $P(X \leq -1) = F_X(-1) = 1 - Q\left(\frac{-1-4}{3}\right) = 1 - Q\left(-5/3\right) =$

$= Q\left(5/3\right) = Q(1{,}67) = 0{,}04746$

d $P(X \geq -2) = 1 - F_X(-2) = 1 - \left(1 - Q\left(\frac{-2-4}{3}\right)\right) =$

$= Q(-2) = 1 - Q(2) = 1 - 0{,}02275 = 97{,}72\%$

8.3-2 $P_X(x) = C e^{-2|x-2|}$

$E[X] = \int_{-\infty}^{\infty} x C e^{-2|x-2|} dx = \int_{-\infty}^{\infty} x e^{-2|x-2|} dx = \circledast$

$x-2>0$

x

4

$$\int_{-\infty}^{\infty} C\, e^{-2|x-2|}\,dx = C\left[\int_{-\infty}^{2} e^{+2(x-2)}\,dx + \int_{2}^{\infty} e^{-2(x-2)}\,dx\right] =$$

$$= C\cdot\left[e^{-4}\left[\frac{e^{2x}}{2}\right]_{-\infty}^{2} + e^{4}\left[\frac{e^{-2x}}{-2}\right]_{2}^{\infty}\right] =$$

$$= C\cdot\left[e^{-4}\,\frac{e^{4}}{2} + e^{4}\cdot\frac{e^{-4}}{2}\right] = \underline{\underline{C=1}}$$

$a\,e^{ax}(ax-1)$

$a\,e^{ax}(ax-1)$

$+\,e^{ax}\cdot a$

$a\,e^{ax}(ax\ldots$

$$\boxed{C=1}$$

$$\boxed{\int x\,e^{ax}\,dx = \frac{e^{ax}(ax-1)}{a^2}}$$

(*)
$$= e^{-4}\int_{-\infty}^{2} x\,e^{2x}\,dx + e^{4}\int_{2}^{\infty} x\,e^{-2x}\,dx$$

$$= e^{-4}\cdot\left[\frac{e^{2x}(2x-1)}{4}\right]_{-\infty}^{2} + e^{4}\left[\frac{e^{-2x}(-2x-1)}{4}\right]_{2}^{\infty} =$$

$$= \cancel{e^{-4}}\cdot\left(\cancel{e^{4}}\cdot\frac{3}{4}\right) + \cancel{e^{4}}\cdot\left(-\frac{\cancel{e^{-4}}\cdot(-5)}{4}\right) =$$

$$= \frac{3}{4} + \frac{5}{4} = \underline{\underline{2}}$$

$$\boxed{E[X]=2}$$

$$\boxed{\int x^2 e^{ax}dx = \frac{e^{ax}(a^2x^2-2ax+2)}{a^3}}$$

$$E[X^2]=\int_{-\infty}^{\infty} x^2 e^{-2|x-2|}dx = e^{-4}\int_{-\infty}^{2} x^2 e^{+2x} + e^{+4}\int_{2}^{\infty} x^2 e^{-2x}dx$$

$$= e^{-4}\cdot\left[e^{2x}\frac{(4x^2-4x+2)}{8}\right]_{-\infty}^{2} + e^{+4}\left[e^{-2x}\frac{(4x^2+4x+2)}{-8}\right]_{2}^{\infty}$$

$$= \cancel{e^{-4}}\left[\cancel{e^{4}}\frac{(16-8+2)}{8}\right] + \cancel{e^{4}}\left[\frac{\cancel{e^{-4}}(16+8+2)}{8}\right] =$$

$$= \frac{10}{8}+\frac{26}{8}=\frac{36}{8}=\frac{9}{2} \qquad \boxed{E[X^2]=9/2}$$

$$\sigma_x^2 = E[X^2]-\bar{X}^2 = \frac{9}{2}-4 = 1/2$$

8.3-6

$$\int_{-\infty}^{\infty} p_x(x)\,dx = 1 \Rightarrow \frac{4\cdot K}{2} = 1 \Rightarrow \boxed{K=1/2}$$

$$p_x(x) = \frac{1/2}{4}x + \frac{1}{8} \Rightarrow \boxed{p_x(x)=\frac{1}{8}x+\frac{1}{8}}$$

3/8

(6)

$$E[X] = \int_{-1}^{3} x\left(\frac{1}{8}x + \frac{1}{8}\right)dx = \frac{1}{8}\int_{-1}^{3} x^2 + x \, dx = \frac{1}{8}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{3}$$

$$= \frac{1}{8}\left(\frac{27}{3} + \frac{9}{2} - \left(\frac{-1}{3} + \frac{1}{2}\right)\right) = \frac{1}{8}\left(\frac{28}{3} + \overset{4}{\frac{9}{2} - \frac{1}{2}}\right) =$$

$$= \frac{1}{8}\left(\frac{40}{3}\right) = \frac{5}{3}$$

38

243
27
108
51
4
8

273
108
381

382 | 8
62 | 4

$$E[X^2] = \int_{-1}^{3} x^2\left(\frac{1}{8}x + \frac{1}{8}\right)dx = \frac{1}{8}\int_{-1}^{3} (x^3 + x^2)\,dx =$$

352 | 8
32 | 44

27
9
8

$$= \frac{1}{8}\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-1}^{3} = \frac{1}{8}\left(\frac{81}{4} + \frac{27}{3} - \frac{1}{4} + \frac{1}{3}\right) =$$

$$= \frac{1}{8}\left(\frac{243 + 108 - 3 + 4}{12}\right) = \frac{1}{\cancel{8 \cdot 12}_{3}} \cdot \cancel{352}^{44\,11} = \frac{11}{3}$$

$$\sigma_x^2 = E[X^2] - \bar{X}^2 = \frac{11}{3} - \frac{25}{9} = \frac{33 - 25}{9} = \frac{8}{9}$$

8.4-1

$$\boxed{p_{XY}(x,y) = Axy\, e^{-x^2} e^{-y^2/2} u(x)u(y)}$$

$$\rho_{x,y} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

$$p_X(x) = \int_{-\infty}^{\infty} Axy\, e^{-x^2} e^{-y^2/2} u(x)u(y)\, dy =$$

$$= Ax\, e^{-x^2} u(x) \int_0^{+\infty} y\, e^{-y^2/2} dy = Ax\, e^{-x^2} u(x) \left[e^{-y^2/2} \right]_0^{\infty}$$

$$= Ax\, e^{-x^2} u(x)$$

$$A \cdot \frac{A}{2} = A \qquad A^2 = 2A$$

$$p_Y(y) = \int_{-\infty}^{\infty} Axy\, e^{-x^2} e^{-y^2/2} u(x)u(y)\, dx =$$

$$= Ay\, e^{-y^2/2} u(y) \int_0^{\infty} x\, e^{-x^2} dx = Ay\, e^{-y^2/2} u(y) \left[-\frac{e^{-x^2}}{2} \right]_0^{\infty}$$

$$= \frac{A}{2} y\, e^{-y^2/2} u(y)$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_{XY}(x,y)\, dx\, dy = 1 \Rightarrow A \int_0^{\infty} x\, e^{-x^2} dx \cdot \int_0^{\infty} y\, e^{-y^2/2} dy = 1$$

$$\Rightarrow A\left[\frac{-e^{-x^2}}{2}\right]_0^{\infty}\cdot\left[-e^{-y^2/2}\right]_0^{\infty}=1\Rightarrow$$

$$\Rightarrow A\cdot\left(\frac{1}{2}\right)\cdot 1=1\Rightarrow \boxed{A=2}$$

Assim, $\rho_{X,Y}(x,y)=2xy\,e^{-x^2}e^{-y^2/2}\mu(x)\mu(y)$

$\rho_X(x)=2x\,e^{-x^2}\mu(x)$

$\rho_Y(y)=y\,e^{-y^2/2}\mu(y)$

e

$\boxed{\rho_{X,Y}(x,y)=\rho_X(x)\cdot\rho_Y(y)}\Rightarrow$ X e Y saõ independentes!

Assim, $\sigma_{XY}=0$ e $\boxed{\rho_{XY}=0}$

8.4-4 $\rho_{X,Y}(x,y)=K\,e^{-(x^2+xy+y^2)}$

$$\rho_X(x)=\int_{-\infty}^{\infty}K\,e^{-(x^2+xy+y^2)}dy=K\,e^{-x^2}\int_{-\infty}^{\infty}e^{-y(x+y)}dy=$$

$$=K\,e^{-x^2}\left[\sqrt{\pi}\,e^{x^2/4}\right]=K\sqrt{\pi}\,e^{-3x^2/4}$$

9

$$\rho_Y(y) = \int_{-\infty}^{\infty} K \, e^{-(x^2+xy+y^2)} dx = K e^{-y^2} \int_{-\infty}^{\infty} e^{-x^2-xy} dx$$

$$= K e^{-y^2} \sqrt{\pi} \, e^{y^2/4} = K \sqrt{\pi} \, e^{-3y^2/4}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho_{X,Y}(x,y)\, dx\, dy = 1 \Rightarrow K \cdot 2\pi \frac{\sqrt{3}}{3} = 1 \Rightarrow$$

$$\Rightarrow K = \frac{1}{2\pi} \frac{3}{\sqrt{3}} \Rightarrow \boxed{K = \frac{\sqrt{3}}{2\pi}}$$

$$\begin{cases} \rho_X(x) = \sqrt{\dfrac{3}{4\pi}} \, e^{-3x^2/4} \\ \rho_Y(y) = \sqrt{\dfrac{3}{4\pi}} \, e^{-3y^2/4} \end{cases}$$

(Não são independentes)

$$E[X] = \int_{-\infty}^{\infty} x \, \rho_X(x) \, dx = \sqrt{\frac{3}{4\pi}} \underbrace{\int_{-\infty}^{\infty} x \, e^{-3x^2/4} dx}_{\text{função ímpar}} = 0$$

$E[Y] = 0$ (da mesma forma)

$$E[X^2] = \int x^2 \rho_X(x) dx = \sqrt{\frac{3}{4\pi}} \int_{-\infty}^{\infty} x^2 \, e^{-3x^2/4} dx = \sqrt{\frac{3}{4\pi}} \cdot \frac{4\sqrt{3\pi}}{9}$$

$$= \frac{3\sqrt{4}}{9} = \frac{2}{3}$$

$E[Y^2] = 2/3$ (por simetría)

$\sigma_X^2 = 2/3$; $\sigma_Y^2 = 2/3$

$$\sigma_{XY} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, K\, e^{-(x^2+xy+y^2)}\, dx\, dy = \frac{\sqrt{3}}{\cancel{2\pi}}\left(-\frac{\cancel{2\pi}\sqrt{3}}{9}\right) =$$

$$= -1/3$$

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = \frac{-1/3}{\frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{2}}{\sqrt{3}}} = \frac{-1/3}{2/3} = -1/2$$