

# ***PMR 5237***

## Modelagem e Design de Sistemas

### Discretos em Redes de Petri

#### Aula 12: Introduzindo o tempo: Timed Petri Nets

Prof. José Reinaldo Silva  
[reinaldo@poli.usp.br](mailto:reinaldo@poli.usp.br)

## Petri Nets Modeling Revisited

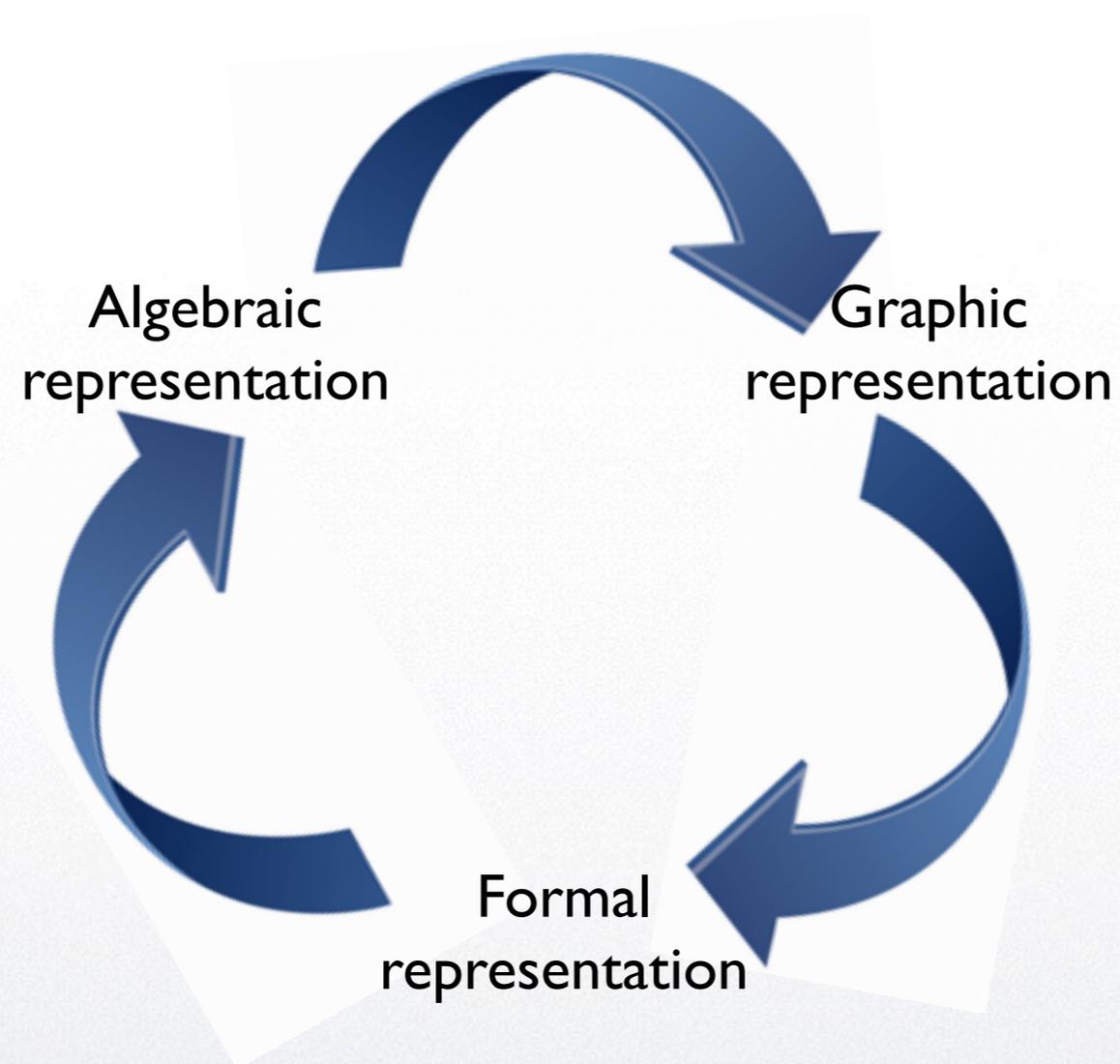
Petri Nets modeling is based on three basic concepts that are a basis for its representation power:

- ★ concurrency or parallelism;
- ★ synchronization
- ★ non-determinism

Shortly:

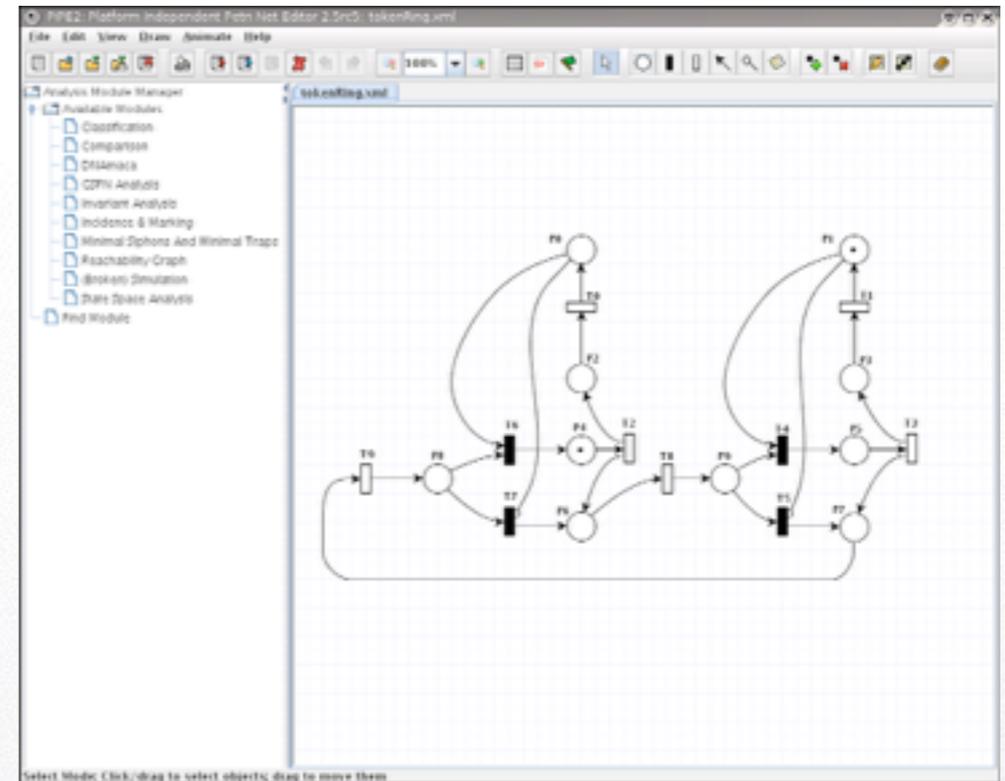
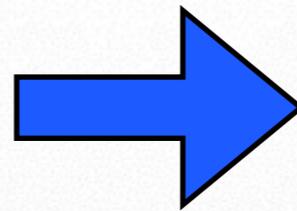
- concurrency allow that state and transitions (activities) would split in multiple parts preserving the ordering;
- synchronization would stand to that fact that transitions (activities) would have to wait for others before firing
- non-determinism means that events enabled could not be fired;

# Representing the model





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## Modeling in Classic Petri Nets

In classic systems a PN structure  $(P, T; F)$  can be used to represent the dynamic of a system combining (literally) all combinations of states that could enable transitions starting from a initial state  $M_0$ . However, even if simple and intuitive there are really a few (real) systems that could be modeled by classic nets.

## Petri Nets Extensions

We could divide extensions of Petri Net in two groups:

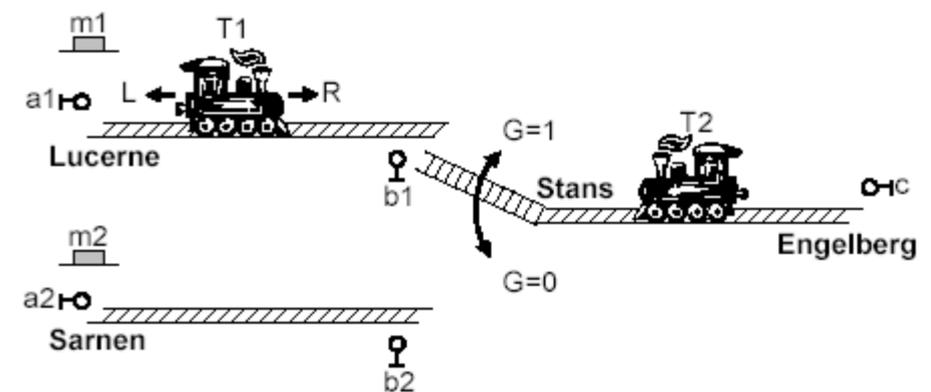
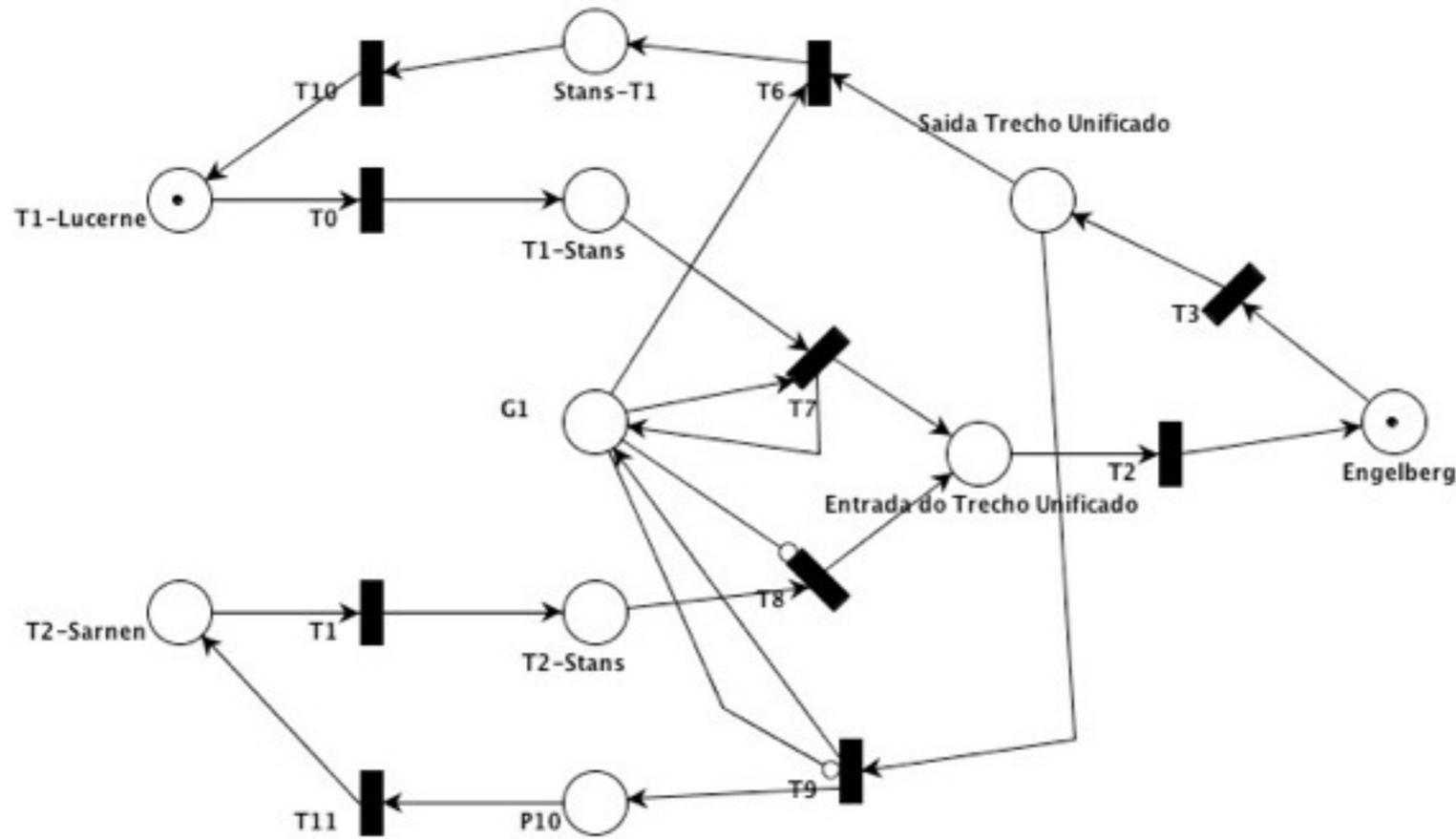
- ◆ those that do not change the strategy of modeling and analysis;
- ◆ those that change significantly the modeling approach;

## Introducing Timed Nets

Among extensions that modify the modeling approach figure the introduction of hierarchy (abstraction), objects and Timed Petri Nets.

## Deterministic Time Petri Nets

Classic Nets just analyze a possible order between actions or events that could lead the evolution of states from the initial marking  $M_0$ . However, that is not really true that all events (transitions) are instantaneous. Therefore, it would be a very simple and intuitive approach to associate to each one a deterministic time.



## Definition 16

Uma rede Place/Transition P/T, é uma n-upla,  $N = (S, T; F, W, K, M_0)$ , onde,

- $S$  é um conjunto finito de lugares;
- $T$  é um conjunto finito de trasições;
- $F = (S \times T) \cup (T \times S)$  representa as relações de fluxo (arcos);
- $W : F \rightarrow \mathbb{N}^+$  representando o peso, isto é, a quantidade de marcas que flui em cada arco;
- $K : S \rightarrow \mathbb{N}$  é um mapeamento que atribui a cada lugar uma capacidade máxima para o armazenamento de marcas.
- $M_0$  é a marcação inicial.

Still insisting in a classic and intuitive approach, how could we simulate a deterministic time Petri Net?  
How the firing would be modified?

A very naive approach would be to reduce time representation to natural numbers and associate it with a process of counting (which is not really a timed approach) even if intuitive and plenty representable by a classic net

## Introducing Timed Petri Nets

A more sensible approach would be to associate to each transition a closed interval  $[EFT \ LFT]$ , where  $EFT$  is the early firing time and means the minimum time a transition can fire (it could not fire before that time), and  $LFT$  is the last firing time or the superior limit of time a transition can fire (it could not fire after that time).

What is the real impact of enhancing a Petri Net with Time intervals, be it deterministic or dense?

In 1974 two similar approaches emerged to introduce time in Petri Nets: one by Ramchandani claimed that it was possible to associate a time interval as the one just showed to PN elements where the mark would be held by this element for a deterministic time inside that interval.

A second approach was proposed by Merlin and could also associate a time interval to each PN component, but, in that case the mark would be held for any time inside the interval. This is called dense time PN and is a more general approach, that is, it could have the previous one as a particular case.

## Petri Net preferences

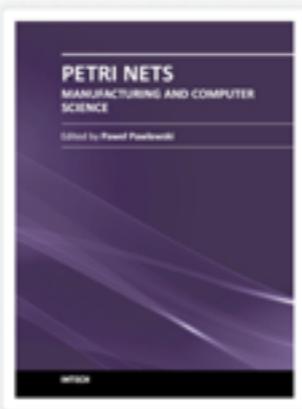
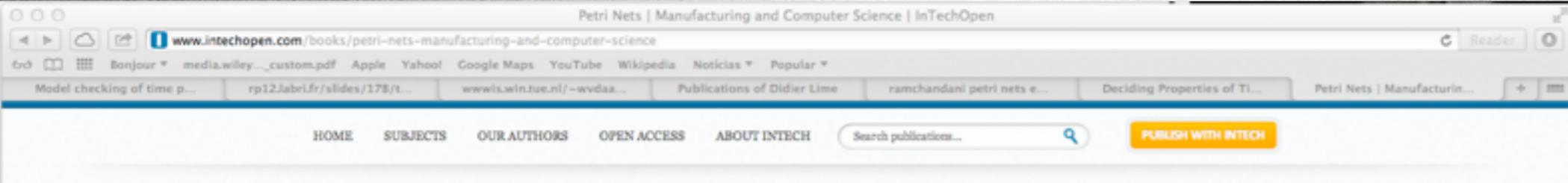
According to different preferences a Time Petri Net could be:

T-Time Petri Net (where time is associate only to transitions)

P-Time Petri Net (where time is associated only to places)

A-Time Petri Nets (where time is associated to arcs)

A different approach was presented in 2006 by J.R.Silva and P.G. del Foyo where time was associated to places and transitions (but not to arcs) as a PT-Time Petri Net.



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Petri Nets were introduced in the doctoral dissertation by K.A. P. "Automaten" and published in 1962 by University of Bonn. Petri Nets modeling language) and mathematical (advanced formal analysis performed actions is the natural phenomenon due to which Petri Nets for modeling concurrent systems. The main idea of this theory was according to their needs, owing to the unusual "flexibility" of this theory on Petri Nets applications in two main areas: manufacturing (see These two areas have still huge influence on our lives and our work developing: some directions of investigations are presented in sections including some interesting facts concerning application of Petri Nets control of public bicycle sharing systems. The monograph shows with use of Petri Nets in science centers all over the world.

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## Chapter 0

### Timed Petri Nets

Silva, José Reinaldo\*,  
del Foyo, Pedro M. G.

\*Corresponding Author: [reinaldo@usp.br](mailto:reinaldo@usp.br)  
Full list of author information is available at the end of the chapter

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#### 1. Introduction

In the early 60's a young researcher in Darmstadt looked for a good representation for communicating systems processes that were mathematically sound has, at the same time, a visual intuitive flavor. This event marked the beginning of a schematic approach that became very important to the modeling of distributed systems in several and distinct areas of knowledge, from Engineering to biologic systems. Carl Adam Petri presented in 1962 his PHD which included the first definition of what is called today a Petri Net. Since its creation Petri Nets evolved as a sound representation to discrete dynamic systems into a general schemata, capable to represent the knowledge about processes and (discrete and distributed) systems according to their internal relations and not to their work domain. Among other advantages



<http://www.intechopen.com/books/petri-nets-manufacturing-and-computer-science>

The screenshot shows a web browser window displaying the IntechOpen website. The page title is "Timed Petri Nets | InTechOpen". The URL in the address bar is "www.intechopen.com/books/petri-nets-manufacturing-and-computer-science/timed-petri-nets". The page features a blue header with the IntechOpen logo and navigation links. Below the header, there is a search bar and a "PUBLISH WITH INTECH" button. The main content area displays the article "Timed Petri Nets" by José Reinaldo Silva and Pedro M. G. Del Foyo. The article is part of the book "Petri Nets - Manufacturing and Computer Science". The page includes a table of contents on the left and the beginning of the article text on the right. The article text starts with "1. Introduction" and describes the early 60's research in Darmstadt.

Definition 1. [Petri Net] A Petri net structure is a directed weighted bipartite graph

$$N = (P, T, A, w)$$

where

$P$  is the finite set of places,  $P \neq \emptyset$

$T$  is the finite set of transitions,  $T \neq \emptyset$

$A \subseteq (P \times T) \cup (T \times P)$  is the set of arcs from places to transitions and from transitions to places

$w : A \rightarrow \{1, 2, 3, \dots\}$  is the weight function on the arcs.

Definition 2. [Marked Petri Net] A marked Petri net is a five-tuple  $(P, T, A, w, M)$  where  $(P, T, A, w)$  is a Petri Net and  $M$  is a marking, defined as a mapping  $M : P \rightarrow \mathbb{N}^+$

Thus, a marking is a row vector with  $|P|$  elements. Figure 1 shows a possible marking for a simple Petri Net.

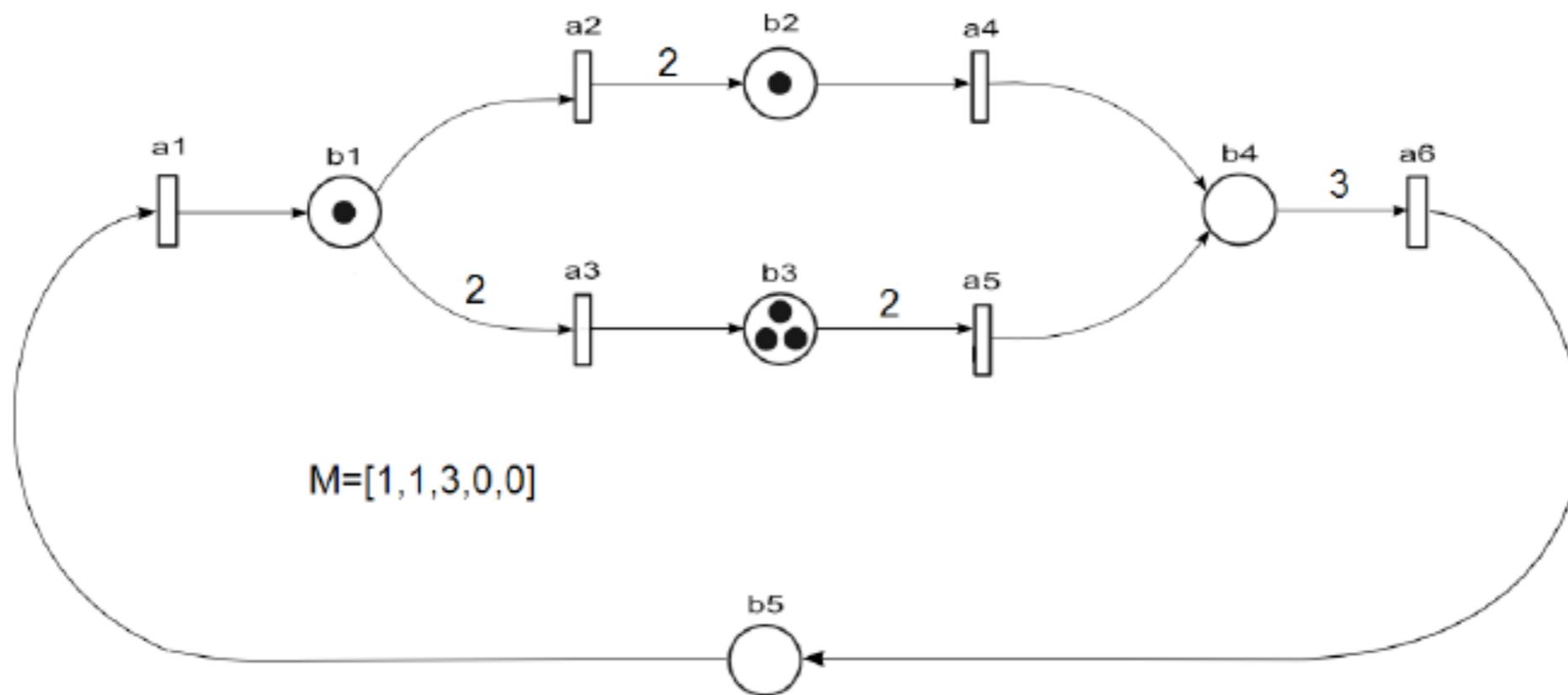


Figure 1. A marked Petri net and its respective marking vector  $M$

Definition 3. [Labeled Petri Net] A labeled Petri net is a seven-tuple

$$N = (P, T, A, w, E, l, M_0)$$

where

$(P, T, A, w)$  is a Petri net structure

$E \subseteq \mathbb{P}(T)$  ,  $E \neq \emptyset$

$l : T \rightarrow E$  is the transition labeling function

$M_0 : P \rightarrow \mathbb{N}^+$  is the initial state of the net

Definition 4. [Enabled Transition] A transition  $t_j \in T$  in a Petri net is said to be enabled if

$$\forall p \in P, \quad M(p) \geq Pre(p, t_j)$$

Definition 5. [State Transition] A Petri net evolves from marking  $M$  to a marking  $M'$  through the firing of transition  $t_f \in T$  only if  $t_f \in enb(M)$ . The new marking  $M'$  can be obtained by

$$\forall p \in P | (p, t_f) \vee (t_f, p) \in A, M'(p) = M(p) - Pre(p, t_f) + Pos(p, t_f)$$

Definition 6. [Timed Petri Net] A timed Petri net is a six-tuple

$$N = (P, T, A, w, M_0, f)$$

where

$(P, T, A, w, M_0)$  is a marked Petri net

$f : T \rightarrow \mathbb{R}^+$  is a firing time function that assigns a positive real number to each transition on the net

Therefore, the firing rule has to be modified in order to consider time elapses in the transition firing. An enabled transition  $t_j \in enb(M)$  then will fire after  $f(t_j)$  times units since it become enabled. The system state is not only determined by the net marking but also by a timer attached to every enabled transition in the net.

**Definition 7. [Clock State]** The clock state is a pair  $(M, V)$ , where  $M$  is a marking and  $V$  is a clock valuation function,  $V : enb(M) \rightarrow \mathbb{R}^+$

For a clock state  $(M, V)$  and  $t \in enb(M)$ ,  $V(t)$  is the value of the clock associated with a transition  $t$ . The initial clock state is  $s_0 = (M_0, V_0)$  where  $V_0(t) = f(t), \forall t \in enb(M_0)$ .

Definition 8. [New Enabled Transition] A transition  $t \in T$  is said new enabled after firing transition  $t_f$  at marking  $M$  leading to marking  $M'$  if it is enabled at marking  $M'$  and it was not enabled at  $M$  or, if it was enabled at  $M$ , it is the former fired transition  $t_f$ . Formally:

$$new(M') = \{t \in enb(M') \mid t = t_f \vee \exists p, (M'(p) - Pos(p, t_f)) \leq Pre(p, t)\}$$

Definition 9. [Firable Transition] A transition  $t_f \in T$  can fire in a marking  $M$  yielding marking  $M'$  if:

$$t_f \in \text{enb}(M)$$

$$V_M(t_f) \leq V_M(t_i) \quad \forall t_i \in \text{enb}(M)$$

So far we can state that it is possible to enhance a classic net (no matter which one but we reinforce a P/T net) to be a new extension including time (deterministic or dense). In the case of deterministic time we will call that a Timed Petri Net and in the case of a dense time a Time Petri Net.

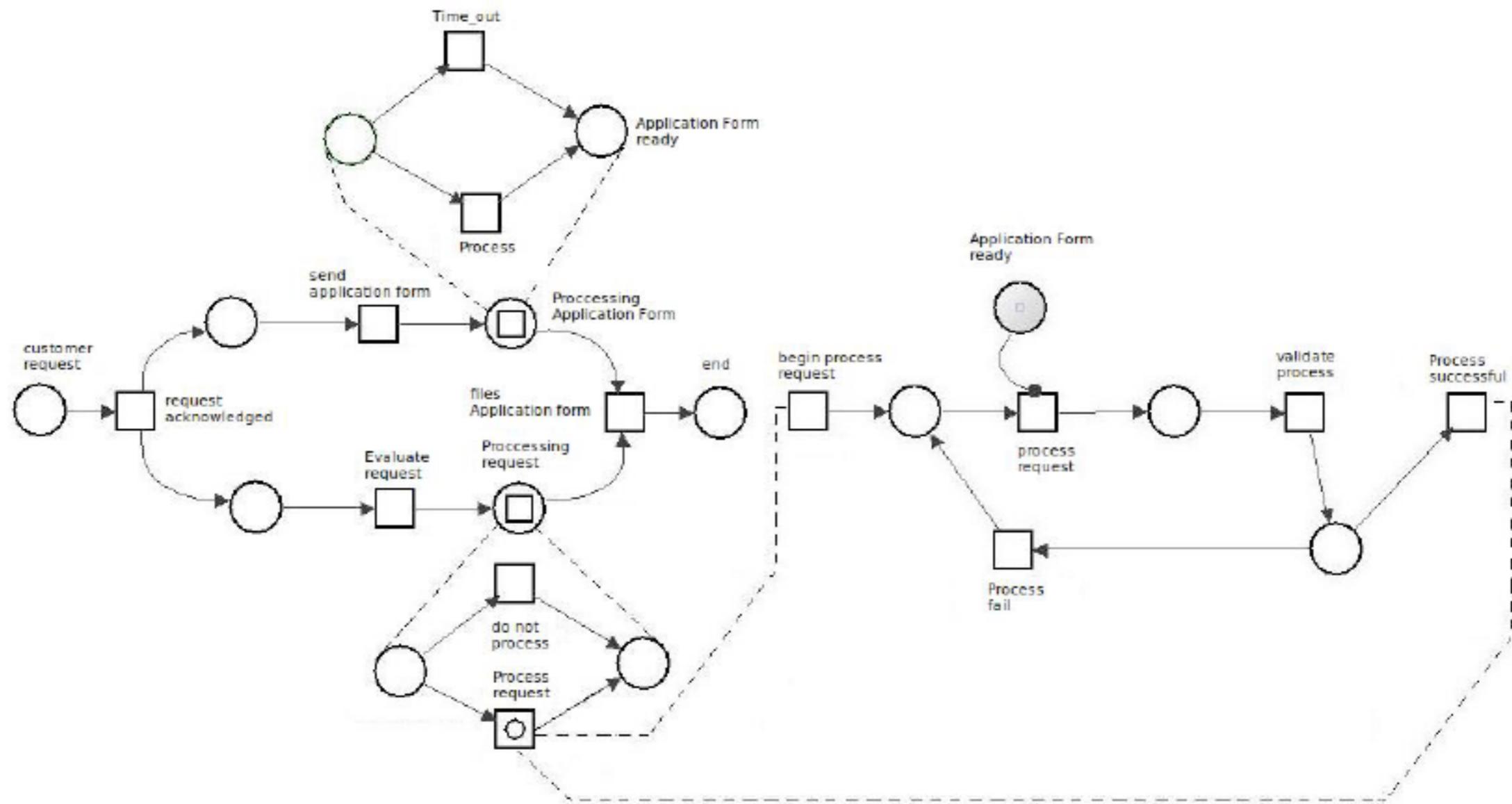
# Time extensions in GHENeSys

Keeping the general proposal of having a unified Petri Net environment we can associate a (general) time extension to GHENeSys. In this particular case we associate time to both macro boxes and macro activities, in a PT-Time Petri Net approach.



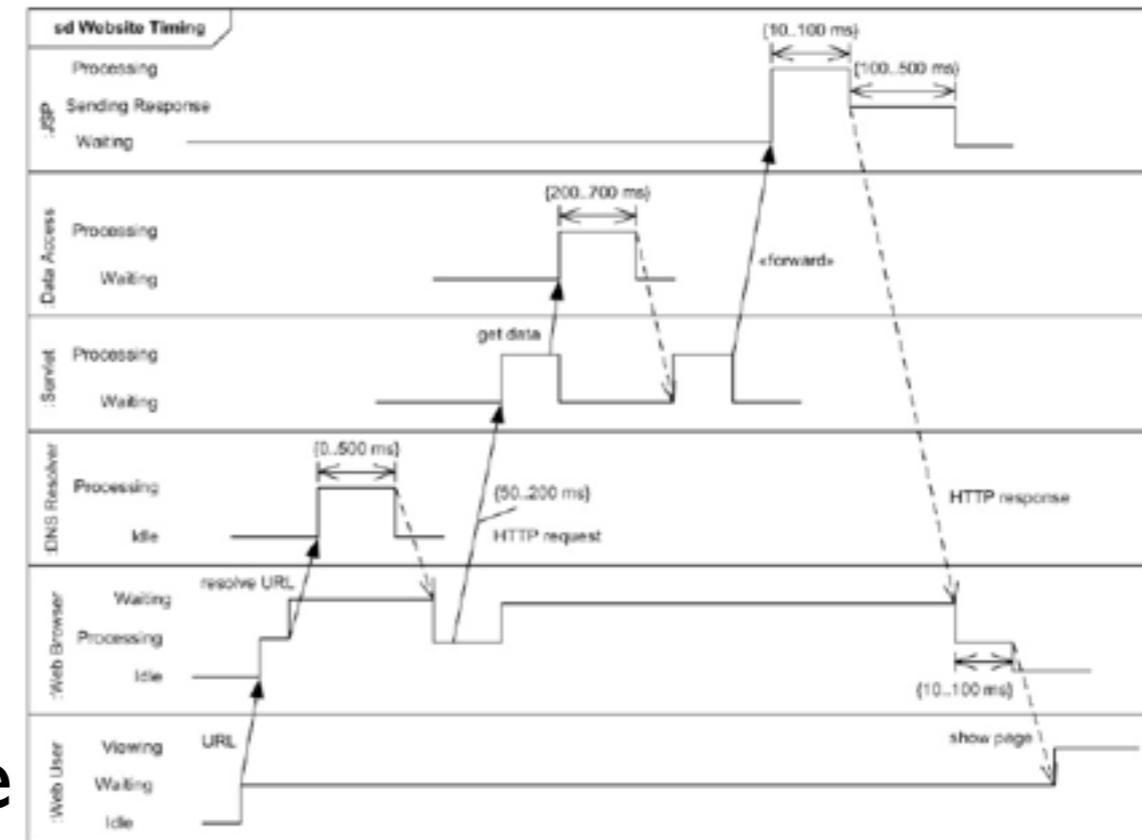
Definition 14.[GHENeSys] GHENeSys is tuple  $G = (L, A, F, K, \Pi, C_0, \tau)$  where  $(L, A, F, K, \Pi)$  represents a net structure,  $C_0$  is a set of multisets representing the initial marking, and  $\tau$  is a function that maps time intervals to each element of the net.

- $L = B \cup P$ , are sets of places denoted by *Boxes* and *pseudo-boxes*;
- $A$  is a set of activities;
- $F \subseteq (L \times A \rightarrow N) \cup (A \times L \rightarrow N)$  is the flux relation;
- $K : L \rightarrow N^+$  is a capacity function;
- $\Pi : (B \cup A) \rightarrow \{0, 1\}$  is a mapping that identifies the macro elements;
- $C_0 = \{(l, \sigma_j) | l \in L, \sigma_j \in R^+ |l| \leq K(l)\}$  is the marking of the initial state;
- $\tau : (B \cup A) \longrightarrow \{\mathbb{R}^+, \mathbb{R}^+ \cup \{\infty\}\}$  is a mapping that associates time intervals to each element of the net



# Using Timelines

Since Petri Nets are being used to model and analyze requirements in the design of dynamic systems we can also think that UML Timelines can be used to capture requirements that can later on be inserted in the Petri Net model.



Time Petri Nets is certainly the topic more discussed and the one where more research effort is spend today, in the domain of Petri Net modeling and analysis. Applications are devoted to design and system verification which is a key topic in automation.

*Fim*