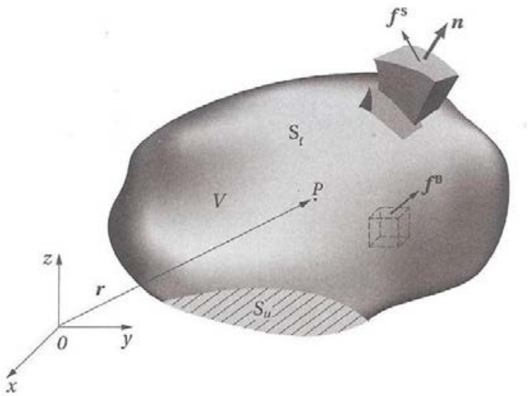
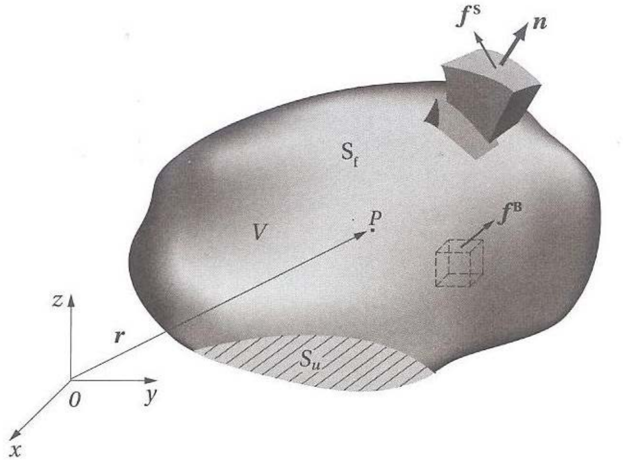
Formulação do Problema Estado Triplo de Tensão

- Equilibrio
- **Estudo das tensões**
- **Relações constitutivas**
- Compatibilidade
- **Estudo das deformações**



Campo de tensões estaticamente admissíveis



$$\frac{\partial \sigma_{xx}^e}{\partial x} + \frac{\partial \sigma_{xy}^e}{\partial y} + \frac{\partial \sigma_{xz}^e}{\partial z} + f_x^B = 0$$

$$\frac{\partial \sigma_{yx}^e}{\partial x} + \frac{\partial \sigma_{yy}^e}{\partial y} + \frac{\partial \sigma_{yz}^e}{\partial z} + f_y^B = 0 \qquad \text{em}\, V$$

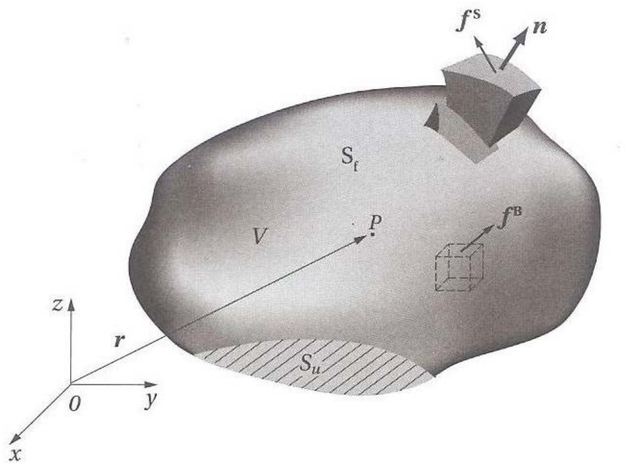
$$\frac{\partial \sigma_{zx}^e}{\partial x} + \frac{\partial \sigma_{zy}^e}{\partial y} + \frac{\partial \sigma_{zz}^e}{\partial z} + f_z^B = 0$$

$$\sigma_{xx}^e n_x + \sigma_{xy}^e n_y + \sigma_{xz}^e n_z = f_x^S$$

$$\sigma_{yx}^e n_x + \sigma_{yy}^e n_y + \sigma_{yz}^e n_z = f_y^S \qquad \text{em}\, S_f$$

$$\sigma_{zx}^e n_x + \sigma_{zy}^e n_y + \sigma_{zz}^e n_z = f_z^S$$

Campo de deslocamentos cinematicamente admissíveis



$$u^c = \hat{u} \qquad v^c = \hat{v} \qquad w^c = \hat{w} \qquad\qquad \text{em } S_u$$

$$\varepsilon^c_{xx} = \frac{\partial u^c}{\partial x} \qquad \varepsilon^c_{yy} = \frac{\partial v^c}{\partial y} \qquad \varepsilon^c_{zz} = \frac{\partial w^c}{\partial z}$$

$$2\varepsilon^c_{xy} = \gamma^c_{xy} = \frac{\partial u^c}{\partial y} + \frac{\partial v^c}{\partial x} \qquad\qquad \text{em } V$$

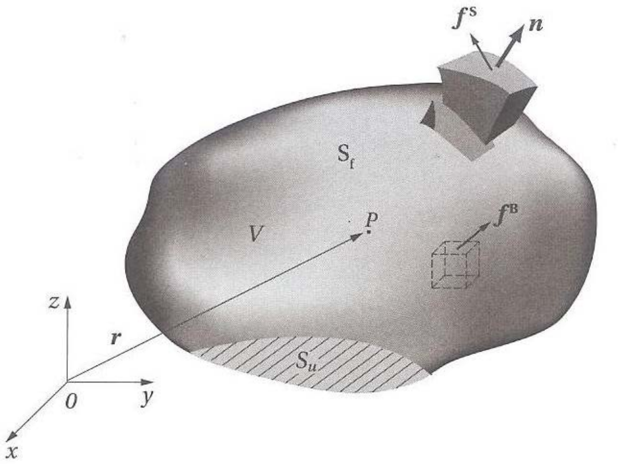
$$2\varepsilon^c_{yz} = \gamma^c_{yz} = \frac{\partial v^c}{\partial z} + \frac{\partial w^c}{\partial y}$$

$$2\varepsilon^c_{zx} = \gamma^c_{zx} = \frac{\partial w^c}{\partial x} + \frac{\partial u^c}{\partial z}$$

Teorema do trabalho

Na Teoria da Elasticidade Linear

$$T_i = T_e$$



$$\mathrm{T}_i = \int_V \{\sigma^e\}^T \{\varepsilon^c\} dV =$$

$$\int_V \left(\sigma_{xx}^e \varepsilon_{xx}^c + \sigma_{yy}^e \varepsilon_{yy}^c + \sigma_{zz}^e \varepsilon_{zz}^c + \sigma_{xy}^e \gamma_{xy}^c + \sigma_{yz}^e \gamma_{yz}^c + \sigma_{zx}^e \gamma_{zx}^c\right) dV$$

$$\mathrm{T}_e = \int_S \{\mathbf{f}^S\}^T \{\mathbf{u}^c\} dS + \int_V \{\mathbf{f}^B\}^T \{\mathbf{u}^c\} dV$$

Regra da derivação
+
Teorema de Gauss ou da Divergência

Na Teoria da Elasticidade Linear

O trabalho interno é dado por:

$$\mathrm{T}_i = \int_V \left\{\sigma^e\right\}^T \left\{\varepsilon^c\right\} dV = \int_V \left(\sigma^e_{xx}\varepsilon^c_{xx} + \sigma^e_{yy}\varepsilon^c_{yy} + \sigma^e_{zz}\varepsilon^c_{zz} + \sigma^e_{xy}\gamma^c_{xy} + \sigma^e_{yz}\gamma^c_{yz} + \sigma^e_{zx}\gamma^c_{zx}\right) dV$$

Seja, por exemplo:

$$\sigma^e_{xy}\gamma^c_{xy}$$

$$\sigma^e_{xy}\left(\frac{\partial u^c}{\partial y} + \frac{\partial v^c}{\partial x}\right)$$

$$\sigma^e_{xy}\frac{\partial u^c}{\partial y} + \sigma^e_{xy}\frac{\partial v^c}{\partial x}$$

$$\sigma^e_{xy}\frac{\partial u^c}{\partial y} + \sigma^e_{xy}\frac{\partial v^c}{\partial x}$$

Considerando todos os termos, resulta:

$$\mathrm{T}_i = \int_V \left(\sigma^e_{xx}\frac{\partial u^c}{\partial x} + \sigma^e_{xy}\frac{\partial u^c}{\partial y} + \sigma^e_{xz}\frac{\partial u^c}{\partial z}\right) dV$$

$$+ \int_V \left(\sigma^e_{yx}\frac{\partial v^c}{\partial x} + \sigma^e_{yy}\frac{\partial v^c}{\partial y} + \sigma^e_{yz}\frac{\partial v^c}{\partial z}\right) dV$$

$$+ \int_V \left(\sigma^e_{zx}\frac{\partial w^c}{\partial x} + \sigma^e_{zy}\frac{\partial w^c}{\partial y} + \sigma^e_{zz}\frac{\partial w^c}{\partial z}\right) dV$$

Na Teoria da Elasticidade Linear

Seja:

$$\frac{\partial(\sigma_{xx}^{e}u^{c})}{\partial x}=\frac{\partial\sigma_{xx}^{e}}{\partial x}u^{c}+\sigma_{xx}^{e}\frac{\partial u^{c}}{\partial x}$$

$$\sigma_{xx}^{e}\frac{\partial u^{c}}{\partial x}=\frac{\partial(\sigma_{xx}^{e}u^{c})}{\partial x}-\frac{\partial\sigma_{xx}^{e}}{\partial x}u^{c}$$

$$\begin{aligned}\mathrm{T}_i &= \int_V (\sigma_{xx}^{e}\frac{\partial u^{c}}{\partial x}+\sigma_{xy}^{e}\frac{\partial u^{c}}{\partial y}+\sigma_{xz}^{e}\frac{\partial u^{c}}{\partial z})dV \\ &+\int_V (\sigma_{yx}^{e}\frac{\partial v^{c}}{\partial x}+\sigma_{yy}^{e}\frac{\partial v^{c}}{\partial y}+\sigma_{yz}^{e}\frac{\partial v^{c}}{\partial z})dV \\ &+\int_V (\sigma_{zx}^{e}\frac{\partial w^{c}}{\partial x}+\sigma_{zy}^{e}\frac{\partial w^{c}}{\partial y}+\sigma_{zz}^{e}\frac{\partial w^{c}}{\partial z})dV\end{aligned}$$

Considerando todos os termos, chega-se em:

$$\begin{aligned}\mathrm{T}_i &= \int_V (\frac{\partial(\sigma_{xx}^{e}u^{c})}{\partial x}+\frac{\partial(\sigma_{xy}^{e}u^{c})}{\partial y}+\frac{\partial(\sigma_{xz}^{e}u^{c})}{\partial z})dV-\int_V [(\frac{\partial\sigma_{xx}^{e}}{\partial x}+\frac{\partial\sigma_{xy}^{e}}{\partial y}+\frac{\partial\sigma_{xz}^{e}}{\partial z})u^{c}]dV \\ &+\int_V (\frac{\partial(\sigma_{yx}^{e}v^{c})}{\partial x}+\frac{\partial(\sigma_{yy}^{e}v^{c})}{\partial y}+\frac{\partial(\sigma_{yz}^{e}v^{c})}{\partial z})dV-\int_V [(\frac{\partial\sigma_{yx}^{e}}{\partial x}+\frac{\partial\sigma_{yy}^{e}}{\partial y}+\frac{\partial\sigma_{yz}^{e}}{\partial z})v^{c}]dV \\ &+\int_V (\frac{\partial(\sigma_{zx}^{e}w^{c})}{\partial x}+\frac{\partial(\sigma_{zy}^{e}w^{c})}{\partial y}+\frac{\partial(\sigma_{zz}^{e}w^{c})}{\partial z})dV-\int_V [(\frac{\partial\sigma_{zx}^{e}}{\partial x}+\frac{\partial\sigma_{zy}^{e}}{\partial y}+\frac{\partial\sigma_{zz}^{e}}{\partial z})w^{c}]dV\end{aligned}$$

Na Teoria da Elasticidade Linear

Pelo teorema do divergente:

$$\int_V (div\,\mathbf{A})dV = \int_S (\mathbf{A}\cdot\mathbf{n})dS \qquad \begin{cases} \mathbf{A} = A_x(x,y,z)\mathbf{e_x} + A_y(x,y,z)\mathbf{e_y} + A_z(x,y,z)\mathbf{e_z} \\ div\,\mathbf{A} = \dfrac{\partial A_x}{\partial x} + \dfrac{\partial A_y}{\partial y} + \dfrac{\partial A_z}{\partial z} \end{cases}$$

Pode-se definir:

$$A_x = \sigma^e_{xx}u^c + \sigma^e_{xy}v^c + \sigma^e_{xz}w^c$$

$$A_y = \sigma^e_{yx}u^c + \sigma^e_{yy}v^c + \sigma^e_{yz}w^c$$

$$A_z = \sigma^e_{zx}u^c + \sigma^e_{zy}v^c + \sigma^e_{zz}w^c$$

Então:

$$A_x n_x = (\sigma^e_{xx}u^c + \sigma^e_{xy}v^c + \sigma^e_{xz}w^c)n_x$$

$$A_y n_y = (\sigma^e_{yx}u^c + \sigma^e_{yy}v^c + \sigma^e_{yz}w^c)n_y$$

$$A_z n_z = (\sigma^e_{zx}u^c + \sigma^e_{zy}v^c + \sigma^e_{zz}w^c)n_z$$

Resulta:

$$\mathrm{T}_i = \int_S (\sigma^e_{xx}n_x + \sigma^e_{xy}n_y + \sigma^e_{xz}n_z)u^c dS + \int_S (\sigma^e_{yx}n_x + \sigma^e_{yy}n_y + \sigma^e_{yz}n_z)v^c dS +$$

$$+ \int_S (\sigma^e_{zx}n_x + \sigma^e_{zy}n_y + \sigma^e_{zz}n_z)w^c dS - \int_V \left(\frac{\partial \sigma^e_{xx}}{\partial x} + \frac{\partial \sigma^e_{xy}}{\partial y} + \frac{\partial \sigma^e_{xz}}{\partial z}\right)u^c dV$$

$$- \int_V \left(\frac{\partial \sigma^e_{yx}}{\partial x} + \frac{\partial \sigma^e_{yy}}{\partial y} + \frac{\partial \sigma^e_{yz}}{\partial z}\right)v^c dV - \int_V \left(\frac{\partial \sigma^e_{zx}}{\partial x} + \frac{\partial \sigma^e_{zy}}{\partial y} + \frac{\partial \sigma^e_{zz}}{\partial z}\right)w^c dV$$

Na Teoria da Elasticidade Linear

Como $\underline{T}\,\underline{n} = \underline{f}^{S}$, obtém-se:

$$\mathrm{T}_i = \int_S (f_x{}^S u^c + f_y{}^S v^c + f_z{}^S w^c)\,dS + \int_V (f_x{}^B u^c + f_y{}^B v^c + f_z{}^B w^c)\,dV$$

Ou:

$$\mathrm{T}_i = \int_S \left\{f_x{}^S\right\}^T \left\{u^c\right\} dS + \int_V \left\{f_x{}^B\right\}^T \left\{u^c\right\} dV$$

$$\mathrm{T}_i = T_e$$