Questão 1 (4,0 pontos)

Considere-se a estrutura submetida às ações de carregamento externo, de variação de temperatura e de recalques de apoio, conforme se indica na figura 1. A flexão ocorre no plano de maior rigidez. Despreza-se a parcela de deslocamento devida à força normal.

Nessas condições, pedem-se:

1. O diagrama de momentos fletores na estrutura
2. Deslocamento vertical do nó C
3. Rotação do nó D

Dados:

$\alpha = 10^{-5}\ {}^0C^{-1}$ $EI = 16.000\ \text{kNm}^2$ seção retangular $= (0{,}15 \times 0{,}40)\ \text{m}^2$

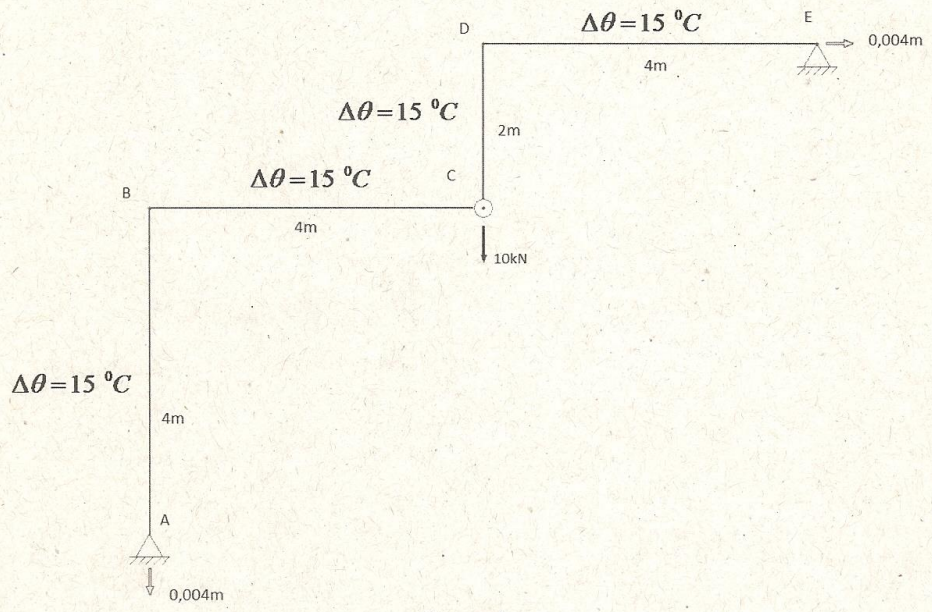
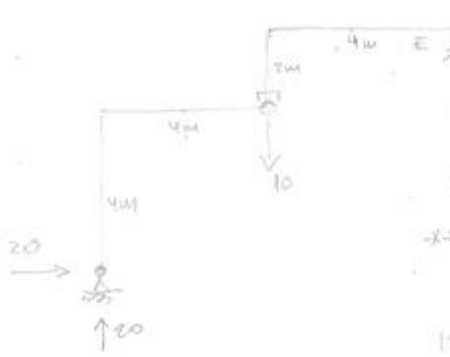


Figura 1

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Q1

1.



$H_E = 20$

$V_E = 10$

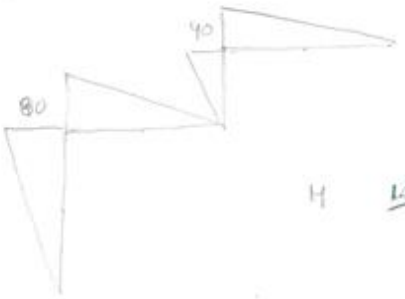
$\curvearrowleft \quad 2H_E - 4V_E = 0$

$\times 4 \quad 6H_E - 8V_E - 10 \times 4 = 0$

$H_E = 2V_E$

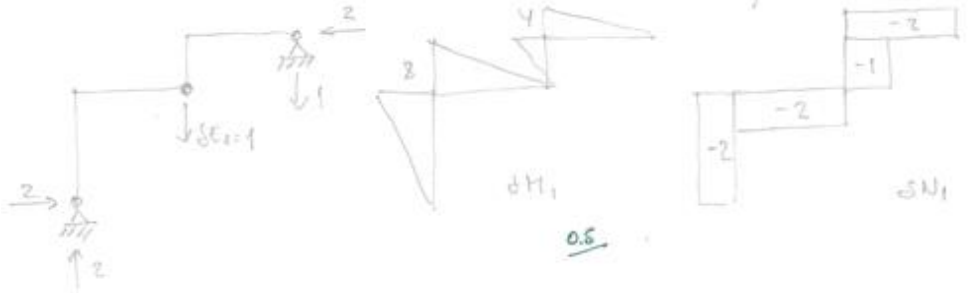
$12V_E - 8V_E - 10 \times 4 = 0$

$4V_E - 40 = 0 \longrightarrow V_E = 10$



M 1.0

2. $\delta E_{qx} V_1 + \sum R_{ki} \hat{D}_n = \int \delta M_1 \frac{M}{EJ} ds + \int \delta M_1 \alpha \frac{(\Delta\theta_1 - \Delta\theta_2)}{h} ds + \int \delta N_1 \alpha \Delta\theta_0 ds$



0.5

$$1 \times U_1 + \left(-2 \times 0{,}004 - 2 \times 0{,}004\right) = \frac{1}{EI}\left[\frac{4}{3} \times 40 \times 4 + \frac{2}{3}\,40 \times 4 + \right.$$

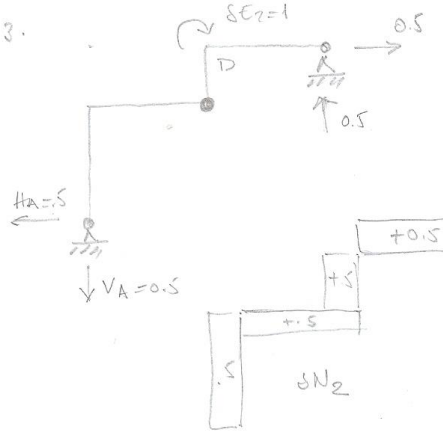
$$\left. + \frac{4}{3} \times 80 \times 8 + \frac{4}{3}\,80 \times 8\right] + \frac{10^{-5} \times 15}{0{,}4} \times \left(16 + 16 + 4 + 8\right) + 10^{-5} \times 7{,}5 \times \left(-8 - 8 - 2 - 8\right)$$

$$U_1 = 0{,}016 + \frac{1}{16000} \times \left[\frac{640}{3} + \frac{320}{3} + \frac{2560}{3} + \frac{2560}{3}\right] + \frac{10^{-5} \times 15}{0{,}4}\left(44\right) + 10^{-5} \times 7{,}5\left(-26\right)$$

$$U_1 = \underbrace{0{,}016}_{\text{R. apoio}} + \overbrace{\boxed{0{,}127}}^{\text{c. ext}} + \underbrace{1650 \times 10^{-5}}_{\Delta\theta} - \underbrace{195 \times 10^{-5}}_{\Delta\theta_0} = 0{,}158\,\text{m} \qquad 1.0$$

* $U_{1,\Delta\theta+R} = 0{,}031$ m

3.

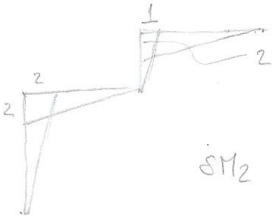


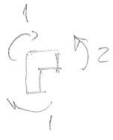
$H_A \times 4 - V_A \times 4 = 0 \longrightarrow H_A = V_A$

$+ H_A \times 6 - V_A \times 8 + 1 = 0$

$-2V_A + 1 = 0 \longrightarrow V_A = 0.5$

0.5





$$1 \times U_2 + \left(0{,}5 \times 0{,}004 + 0{,}5 \times 0{,}004\right) = \frac{1}{EI}\left[-\frac{4}{3} \times 40 \times 2 - \frac{2}{3} \times 40 \times 1 - \frac{4}{2} \times 80 \times 2 \times 2\right] +$$

$$+ \frac{10^{-5} \times 15}{0{,}4}\left(-4 - 1 - 4 - 4\right) + 10^{-5} \times 7{,}5 \times 14 \times 0{,}5$$

$$U_2 = -0{,}004 + \frac{1}{EI}\left[-\frac{320}{3} - \frac{80}{3} - \frac{1280}{3}\right] - \overbrace{487{,}5 \times 10^{-5}}^{\Delta\theta} + \overbrace{52{,}5 \times 10^{-5}}^{\Delta\theta_0}$$

$$U_2 = \underbrace{-0{,}004}_{\text{R.a}} \overbrace{\boxed{-0{,}035}}^{\text{c.e}} - \underbrace{435}_{\cancel{887}} \times 10^{-5} = -{,}043\ \text{rad.} \qquad 1.0$$

* $U_{2,\Delta\theta+R} = -.008$ rad

* sem carregamento externo.

Questão 2 (6,0 pontos)

Considere-se a estrutura solicitada por carregamento externo, conforme se indica na figura 2. As barras são prismáticas com secção transversal quadrada e de mesmo material.

Nessas condições, pedem-se:

- Resolver a estrutura pelo método dos esforços e traçar os diagramas de momentos fletores e momentos torsores;
- Calcular a rotação do ponto A em torno da direção AB;
- Calcular o deslocamento vertical do ponto A.

Dados: $EI = 5{,}5\times10^{4}\ \text{kN/m}^2$ $GI_T = 3{,}8\times10^{4}\ \text{kN/m}^2$ $k = 870\ \text{kN/m}$

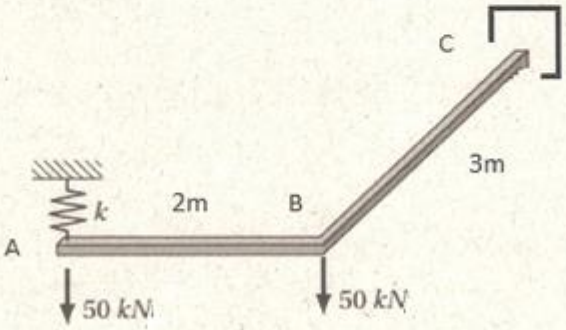
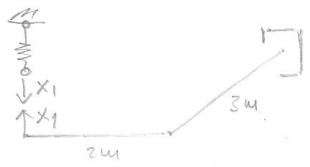


Figura 2

Q2

A.



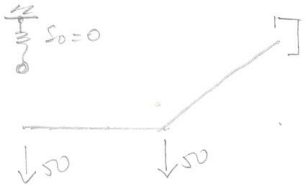
$$U_1 = U_{10} + F_{11} X_1$$

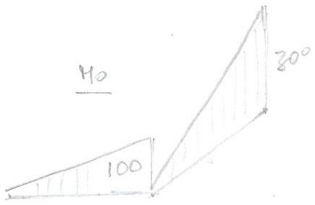
$$U_{10} = \int \frac{M_1 M_0}{EI} ds + \int \frac{T_1 T_0}{GJ_T} + \frac{S_1 S_0}{k}$$

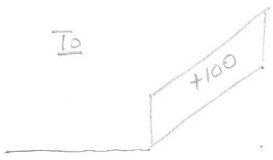
0.5

$$F_{11} = \int \frac{M_1 M_1}{EI} ds + \int \frac{T_1 T_1}{GJ_T} ds + \frac{S_1 S_1}{k}$$

* obtenção de M_0, T_0 e S_0

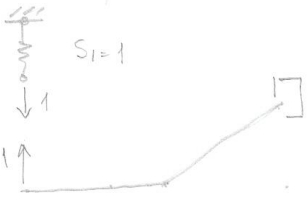


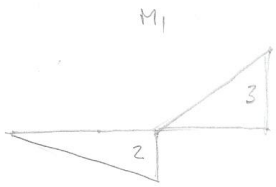


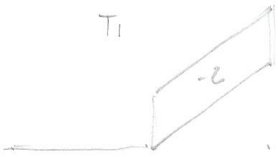


1.0

* obtenção de M_1, T_1 e S_1







1.0

* Cálculo das integrais

$$U_{10} = \frac{1}{EI}\left[-\frac{2}{3}\,100\times 2 - \frac{3}{3}\,300\times 3\right] + \frac{1}{GJ_T}\left[100\times(-2)\times 3\right] + 0$$

$$U_{10} = -\frac{3100}{3EI} - \frac{600}{GJ_T} = -187.88\times 10^{-4} - 157.89\times 10^{-4} = -345{,}77\times 10^{-4}$$

$$F_{11} = \frac{1}{EI}\left[\frac{2}{3}\times 2\times 2 + \frac{3}{3}\times 3\times 3\right] + \frac{1}{GI_T}\left[(-2)(-2)\times 3\right] + \frac{1\times 1}{k}$$

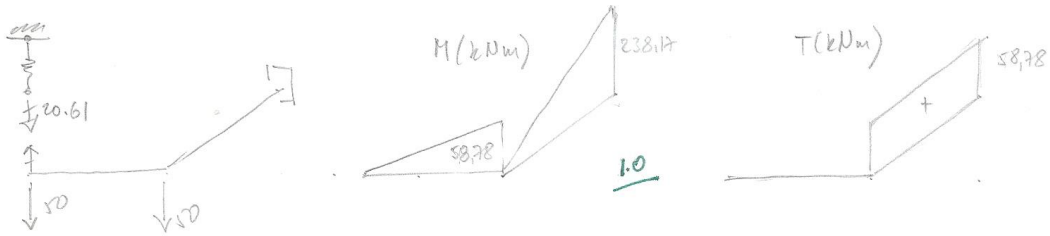
$$F_{11} = \frac{35}{3EI} + \frac{12}{GI_T} + \frac{1}{k} = 2.121\times 10^{-4} + 3.158\times 10^{-4} + 11.494\times 10^{-4}$$

$$F_{11} = 16.793\times 10^{-4}$$

* Calculo de X_1

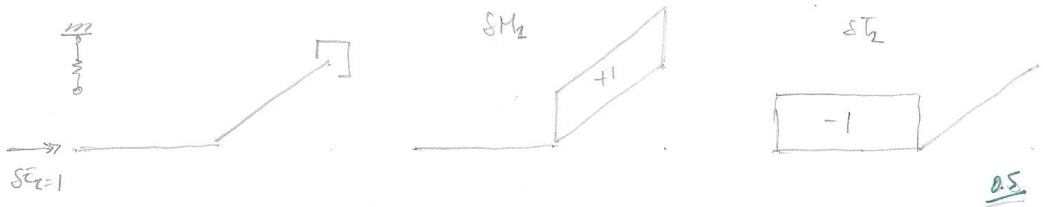
$$U_{10} + F_{11} X_1 = 0 \rightsquigarrow X_1 = 20.61\ kN \quad 0.5$$

* Diagramas de esfuerzos solicitantes



1.0

B. Calculo de U_2



0.5

$$\delta E_2 \times U_2 = \int \delta M_2 \frac{M}{EI} ds + \int \delta T_2 \frac{T}{GI_T} ds + \delta S_2 \times \frac{S}{k}$$

$$U_2 = \frac{1}{EI}\left[\frac{1}{2} 238,17 \times 3\right] = 6,496\times 10^{-3}\ rad \quad 1.0$$

C. Calculo de U_3

$$U_3 = \frac{S}{k} = \frac{20,61}{870} = 0,024\ m \quad 0.5$$