5. $f_{XY}(x,y) = \dfrac{1}{2\pi\sigma^2} \cdot e^{-\frac{(x^2+y^2)}{2\sigma^2}}$; $\begin{cases} R = \sqrt{X^2+Y^2} \\ \theta = \operatorname{arctg}\left(\frac{Y}{X}\right) \end{cases}$

$f_{R\theta}(r,\theta) = \, ?$

$X_1, X_2 \Rightarrow$ VA's , $Y_1 = g_1(X_1, X_2)$

f_{X_1,X_2} conhecida $\quad Y_2 = g_2(X_1, X_2)$

$$f_{Y_1Y_2}(y_1,y_2) = f_{X_1X_2}(g_1^{-1}(y_1,y_2),\, g_2^{-1}(y_1,y_2)) \cdot |\det(J^{-1})| \qquad J^{-1} = \begin{bmatrix} \frac{\partial x_1}{\partial g_1} & \frac{\partial x_1}{\partial g_2} \\ \frac{\partial x_2}{\partial g_1} & \frac{\partial x_2}{\partial g_2} \end{bmatrix}$$

I) Funções inversas:

$$\left.\begin{aligned} R^2 &= x^2 + y^2 \\ tg\,\theta &= \frac{Y}{X} \Rightarrow \frac{sen\,\theta}{cos\,\theta} = \frac{Y}{X} \end{aligned}\right\} \Rightarrow \begin{cases} X = R\cos\theta \\ Y = R\,sen\,\theta \end{cases}$$

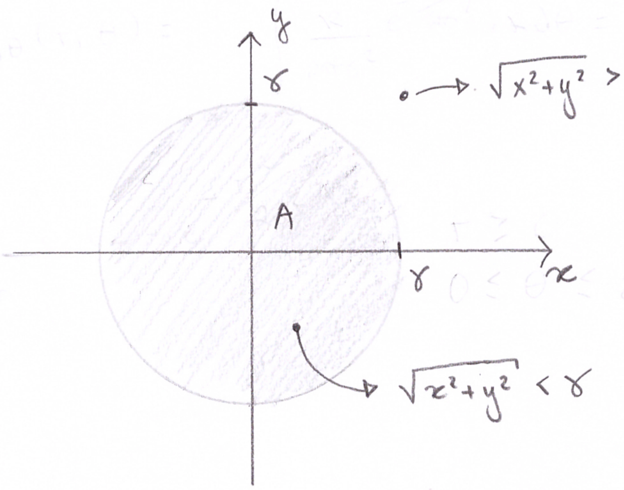
II) Jacobiano:

$$J^{-1} = \begin{bmatrix} \cos\theta & -R\,sen\,\theta \\ sen\,\theta & R\cos\theta \end{bmatrix} \Rightarrow \det J^{-1} = R\cos^2\theta + R\,sen^2\theta = R$$

III) pdf: $f_{R,\theta}(r,\theta) = f_{XY}(r\cos\theta,\, r\,sen\,\theta) \cdot R$

$$\Rightarrow \quad = \frac{r}{2\pi\sigma^2} \cdot e^{-\frac{(r^2\cos^2\theta + r^2 sen^2\theta)}{2\sigma^2}} = \begin{cases} \dfrac{r}{2\pi\sigma^2} \cdot e^{-\frac{r^2}{2\sigma^2}} & r \geq 0, \; 0 \leq \theta \leq 2\pi \\ 0, & c.c. \end{cases}$$

Extra: $P[R \leq \gamma] = ? = P[\sqrt{x^2+y^2} \leq \gamma] = P[(x,y) \in A]$



$$= \iint_A f_{XY}(x,y)\,dy\,dx =$$

transformação
$R = \sqrt{x^2+y^2}$
$\theta = \operatorname{arctg}(\theta)$

$$= \int_0^{2\pi}\int_0^{\gamma} \frac{r}{2\pi\sigma^2} \cdot e^{-\frac{r^2}{2\sigma^2}}\,dr\,d\theta =$$

$$= \begin{cases} 1 - e^{-\frac{\gamma^2}{2\sigma^2}}, & \gamma \geq 0 \\ 0, & \gamma < 0 \end{cases}$$