# Resolução teste nº 6

$x(t) = A\,\text{sen}(\omega t + \theta)$ , $\theta \Rightarrow$ V.A. com certa distribuição

I) $\theta \sim$ uniforme $(0, 2\pi)$ $\Rightarrow$ melhor representa a situação do problema.

II) Para cada $t$ , $x(t)$ é uma V.A.

$$X = g(\theta) = A\,\text{sen}(\omega t + \theta)$$

$$E[X] = E[g(\theta)] = \int_{-\infty}^{+\infty} g(\theta) \cdot f_\theta(\theta)\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} A\,\text{sen}(\omega t + \theta)\, d\theta = 0$$

(integral num período)

III) $$\text{Var}[X] = E[(X - \underbrace{E[X]}_{=0})^2] = E[X^2] = E[h(\theta)]$$

$$E[X^2] = \int_{-\infty}^{+\infty} h(\theta) f_\theta(\theta)\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} A^2 \text{sen}^2(\omega t + \theta)\, d\theta = \frac{A^2}{2\pi}\int_0^{2\pi}\frac{(1 - \cos(2\omega t + 2\theta))}{2}\, d\theta$$

$$= \frac{A^2}{2\pi}\left(\frac{2\pi + 0}{2}\right) = \frac{A^2}{2}$$

IV) $$\text{Cov}(X_1, X_2) = E[X_1 X_2] - E[X_1] \cdot E[X_2]$$

$$E[X_1] = E[X_2] = 0$$

$$X_1 = A\,\text{sen}(\omega t_1 + \theta) \quad \Rightarrow \quad X_1 X_2 = A^2 \text{sen}(\omega t_1 + \theta) \cdot \text{sen}(\omega t_2 + \theta) = l(\theta)$$

$$X_2 = A\,\text{sen}(\omega t_2 + \theta)$$

$$E[X_1 X_2] = E[l(\theta)] = \int_{-\infty}^{+\infty} l(\theta) f_\theta(\theta)\, d\theta = \frac{A^2}{2\pi}\int_0^{2\pi} \text{sen}(\omega t_1 + \theta)\,\text{sen}(\omega t_2 + \theta)\, d\theta =$$

$$= \frac{A^2}{2\pi}\int_0^{2\pi}\left(\frac{\cos(\omega(t_1 - t_2)) - \cos(\omega(t_1 + t_2) + 2\theta)}{2}\right) d\theta = \frac{A^2}{2\pi}\left(\frac{2\pi \cdot \cos(\omega(t_1 - t_2)) + 0}{2}\right)$$

$$= \frac{A^2 \cos(\omega(t_1 - t_2))}{2} = \frac{A^2 \cos(\omega(t_2 - t_1))}{2}$$