Aula 5 - Controle Robusto

SEM5875 - Controle de Sistemas Robóticos

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Torque Calculado

- Valores estimados de $M(\mathbf{q})$ e $b(\mathbf{q}, \dot{\mathbf{q}})$

$$\tau = \hat{M}(\mathbf{q})(\ddot{\mathbf{q}}^d - u) + \hat{b}(\mathbf{q}, \dot{\mathbf{q}})$$

- Torque Calculado + PD

$$\tau = \hat{M}(\mathbf{q})(\ddot{\mathbf{q}}^d + K_v\dot{\mathbf{e}} + K_p\mathbf{e}) + \hat{b}(\mathbf{q}, \dot{\mathbf{q}})$$

- Equação dinâmica do erro

$$\ddot{\mathbf{e}} + K_v\dot{\mathbf{e}} + K_p\mathbf{e} = \Delta(K_v\dot{\mathbf{e}} + K_p\mathbf{e}) + \Delta\ddot{\mathbf{q}}^d + M(\mathbf{q})^{-1}\delta$$

sendo

$$\Delta = M(\mathbf{q})^{-1}[M(\mathbf{q}) - \hat{M}(\mathbf{q})] = I - M(\mathbf{q})^{-1}\hat{M}(\mathbf{q})$$

$$\delta = b(\mathbf{q}, \dot{\mathbf{q}}) - \hat{b}(\mathbf{q}, \dot{\mathbf{q}})$$

Torque Calculado+PD Robusto

- Hipóteses:
$$\frac{1}{\mu_2} \leq \left\| M^{-1}(\mathbf{q}) \right\| \leq \frac{1}{\mu_1}, \quad \mu_1 \neq 0$$
$$\|\Delta\| \leq a < 1$$
$$\|\delta\| \leq \beta_0 + \beta_1 \|\mathbf{e}\| + \beta_2 \|\mathbf{e}\|^2$$
$$\left\| \ddot{\mathbf{q}}^d \right\| \leq c$$
$$\mathbf{e}(0) = \dot{\mathbf{e}}(0) = 0$$

- Controlador:
$$\tau = \hat{M}(\mathbf{q})(\ddot{\mathbf{q}}^d + 2a\dot{\mathbf{e}} + 4a\mathbf{e}) + \hat{b}(\mathbf{q}, \dot{\mathbf{q}})$$
$$\hat{M}(\mathbf{q}) > \mu_2 I$$
$$a > 1 + \frac{1}{\mu_1}[\beta_1 + 2(\beta_2\beta_0 + \beta_2(\mu_1 + \mu_2)c)^{1/2}]$$

Controle combinado: Torque Calculado + PD + Robusto

- Torque Calculado + PD + Robusto

$$\tau = \hat{M}(\mathbf{q})(\ddot{\mathbf{q}}^d + K_v\dot{\mathbf{e}} + K_p\mathbf{e} + \mathbf{u}_R) + \hat{b}(\mathbf{q}, \dot{\mathbf{q}})$$

- Equação no espaço de estados com distúrbio $\mathbf{w}$

$$\begin{bmatrix} \dot{\mathbf{e}} \\ \ddot{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix} \begin{bmatrix} \mathbf{e} \\ \dot{\mathbf{e}} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \mathbf{u}_R + \begin{bmatrix} 0 \\ I \end{bmatrix} \mathbf{w}$$

$$\mathbf{y} = \mathbf{e} = [I \quad 0]\mathbf{x}$$

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B_1\mathbf{w} + B_2\mathbf{u}_R \\ \mathbf{y} &= C_2\mathbf{x} \end{aligned}$$

- Considere o sistema linear

$$
\begin{aligned}
\dot{\mathbf{x}} &= A\mathbf{x} + B_1\mathbf{w} + B_2\mathbf{u} \\
\mathbf{z} &= C_1\mathbf{x} + D_{12}\mathbf{u} \\
\mathbf{y} &= C_2\mathbf{x} + D_{21}\mathbf{w}
\end{aligned}
$$

- Controlador K

$$
\begin{aligned}
\dot{\mathbf{x}} &= A_K\mathbf{x} + B_K\mathbf{y} \\
\mathbf{u} &= C_K\mathbf{x} + D_K\mathbf{y}
\end{aligned}
$$

- Controle sub-ótimo $\mathcal{H}_\infty$ linear

$$\|T_{\mathbf{zw}}\|_\infty < \gamma$$

Controle $\mathcal{H}_\infty$ linear

- Hipóteses simplificadoras
  - $(A,\ B_1)$ é estabilizável e $(C_1,\ A)$ é detectável
  - $(A,\ B_2)$ é estabilizável e $(C_2,\ A)$ é detectável
  - $D_{12}^T[C_1 \quad D_{12}] = [0 \quad I]$
  - $\begin{bmatrix} B_1 \\ D_{21} \end{bmatrix} D_{21}^T = \begin{bmatrix} 0 \\ I \end{bmatrix}$

- Solução

$$
\begin{aligned}
A_K &= A - B_2 B_2^T X + \gamma^{-2} B_1 B_1^T X + -(I - \gamma^{-2} YX)^{-1} Y C_2^T C_2 \\
B_K &= (I - \gamma^{-2} YX)^{-1} Y C_2^T \\
C_K &= -B_2^T X \\
D_K &= 0
\end{aligned}
$$

sendo X e Y as soluções das equações algébricas de Ricatti relacionadas com as Hamiltonianas

$$
H_\infty = \begin{bmatrix} A & \gamma^{-2} B_1 B_1^T - B_2 B_2^T \\ -C_1^T C_1 & -A^T \end{bmatrix}
$$

e

$$
J_\infty = \begin{bmatrix} A^T & \gamma^{-2} C_1^T C_1 - C_2^T C_2 \\ -B_1^T B_1 & -A \end{bmatrix}
$$

- Idéia

$$\tau_i = \begin{cases} \tau_i^+ & se \;\; r_i(\mathbf{e}_i, \dot{\mathbf{q}}_i) > 0 \\ \tau_i^- & se \;\; r_i(\mathbf{e}_i, \dot{\mathbf{q}}_i) < 0 \end{cases}$$

par $i = 1, \cdots, n$ e r_i são as superfícies de deslizamento

$$r_i(\mathbf{e}_i, \dot{\mathbf{q}}_i) = \lambda_i \mathbf{e}_i + \dot{\mathbf{q}}_i$$

- Sejam

$$\begin{aligned}\mathbf{r} &= \Lambda\mathbf{e} + \dot{\mathbf{e}}\\ \dot{\mathbf{q}}_r &= \Lambda\mathbf{e} + \dot{\mathbf{q}}^d\\ \Lambda &= \mathrm{diag}[\lambda_1,\ \lambda_2,\ \cdots,\lambda_n],\quad \lambda_i > 0\\ \mathrm{sgn}(\mathbf{r}) &= [\mathrm{sgn}(\mathbf{r}_1), \mathrm{sgn}(\mathbf{r}_2), \cdots, \mathrm{sgn}(\mathbf{r}_n)]^T\end{aligned}$$

sendo

$$\mathrm{sgn}(\mathbf{r}_i) = +1 \ \ se\ \mathbf{r}_i > 0, \qquad \mathrm{sgn}(\mathbf{r}_i) = -1 \ \ se\ \mathbf{r}_i < 0$$

- Controlador

$$\tau = \hat{M}(\mathbf{q})\ddot{\mathbf{q}}_r + \hat{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_r + \hat{G}(\mathbf{q}) + K\mathrm{sgn}(\mathbf{r})$$

sendo

$$K = \mathrm{diag}[k_1,\ k_2,\ \cdots,k_n],\quad k_i > 0$$