Resolução teste nº 4

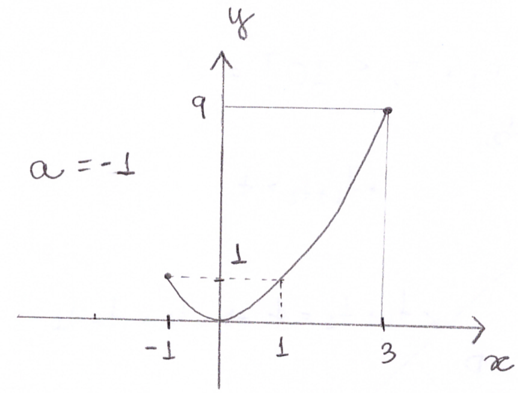
$$f_X(x) = \begin{cases} 1/4 & ; \quad a \le x \le a+4 \\ 0 & ; \quad c.c. \end{cases}$$

I) $Y = X^2$, $E[Y] = ?$

$$Y = g(X) \Rightarrow E[Y] = E[g(X)] = \int_{-\infty}^{+\infty} g(x) \cdot f_X(x)\, dx = \int_a^{a+4} x^2 \cdot \frac{1}{4}\, dx =$$

$$= \frac{1}{4} \cdot \frac{x^3}{3} \Big|_a^{a+4} = \frac{1}{12}\left((a+4)^3 - a^3\right) = \frac{1}{12}\left(\cancel{a^3} + 12a^2 + 48a + 64 - \cancel{a^3}\right) =$$

$$= a^2 + 4a + \frac{16}{3} .$$



$X \sim$ uniforme$(-1, 3)$

II) $f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy}\left(P[Y \le y]\right)$

$$P[Y \le y] = P[-\sqrt{y} \le X \le \sqrt{y}]$$
$$= F_X(\sqrt{y}) - F_X(-\sqrt{y})$$

Logo, $f_Y(y) = \frac{d}{dy} F_X(\sqrt{y}) - \frac{d}{dy} F_X(-\sqrt{y})$

Utilizando a regra da cadeia, temos:

y no denominador

$$f_Y(y) = f_X(\sqrt{y}) \cdot \frac{1}{2\sqrt{y}} - f_X(-\sqrt{y}) \cdot \left(\frac{-1}{2\sqrt{y}}\right) , \quad 0 < y \le 9$$

$$\left.\begin{array}{l} \text{Para } 0 < y \le 1 : \quad f_X(\sqrt{y}) = f_X(-\sqrt{y}) = \frac{1}{4} \\ \text{Para } 1 < y \le 9 : \quad f_X(\sqrt{y}) = \frac{1}{4} \text{ e } f_X(-\sqrt{y}) = 0 \end{array}\right\} \text{gráfico y por x!}$$

Portanto: $f_Y(y) = \begin{cases} \frac{1}{4\sqrt{y}} , & 0 < y \le 1 \\ \frac{1}{8\sqrt{y}} , & 1 < y \le 9 \end{cases}$