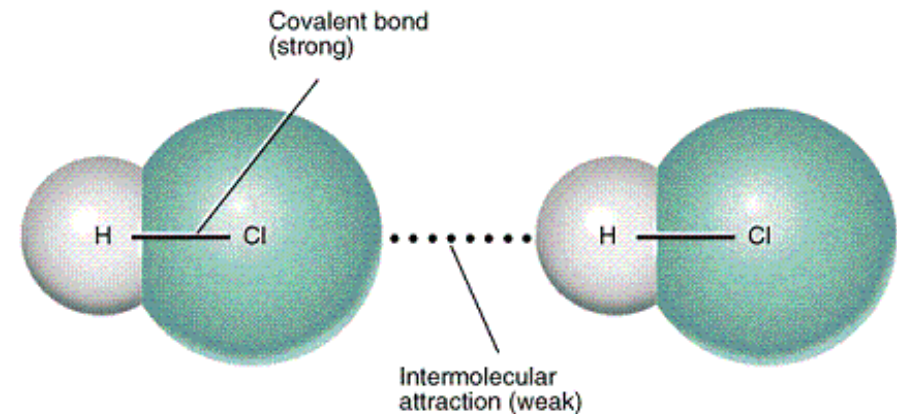


# Física do Calor (4300159)



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**A07**



**Teoria Cinética dos  
Gases**

Data	Programa do curso
August 9	Temperatura e escalas
August 12	Expansão Térmica
August 16	Calorimetria
August 19	Condução, convecção Radiação (Corpo Humano)
August 23	Equação de Estado
August 26	Propriedades moleculares da Matéria
August 30	<b>(Aula de Exercícios e Revisão)</b>
September 2	<b>Aula Modelo do Gas Ideal</b>
September 6	Feriado
September 9	Feriado
September 13	<u>Prova 3 1/4 - Temperatura e Calor</u> - Capacidade Térmica
September 16	Velocidade molecular (Corpo Humano)
September 20	<b>(Aula de Exercícios e Revisão)</b>
September 23	<u>Prova 3 2/4 - Propriedades da Matéria</u> - Aula Fases da matéria
September 27	Prova 1: Temperatura, Calor e Propriedades da Matéria
September 30	Calor e trabalho
October 4	A primeira lei da Termodinâmica
October 7	Processos termodinâmicos
October 11	Semana de Ensino (IFUSP)
October 14	Semana de Ensino (IFUSP)
October 18	Termodinâmica do Gas Ideal
October 21	<b>(Aula de Exercícios e Revisão)</b>
October 25	<u>Prova 3 3/4 - Primeira Lei da Termodinâmica</u> - Aula Processos adiabaticos
October 28	Processos reversíveis e irreversíveis (Corpo Humano)
November 1	Maquinas térmicas, Ciclo de Otto e Refrigerador (Corpo Humano)
November 4	Segunda Lei da Termodinâmica
November 8	Ciclo de Carnot
November 11	<b>(Aula de Exercícios e Revisão)</b>
November 15	Feriado
November 18	Entropia Micro estados
November 22	<u>Prova 3 4/4 - Segunda Lei da Termodinâmica</u> - Aula Micro estados
November 25	Prova 2: Primeira e Segunda Lei da Termodinâmica
November 29	Prova Sub

# Teoria Cinética dos Gases

Ultima Aula, saímos das leis de Newton e chegamos aqui:

$$pV = \frac{2}{3}N \left[ \frac{1}{2}m\langle v^2 \rangle \right]$$

Energia cinética molecular

$$pV = \frac{2}{3}K_{\text{tr}}$$

$$pV = nRT$$

$$K_{\text{tr}} = \frac{3}{2}nRT$$

Energia cinética  
transacional de  $n$   
moles de gas ideal

# Teoria Cinética dos Gases

$$K_{\text{tr}} = \frac{3}{2}nRT$$

Energia cinética translacional de  $n$  moles de gas ideal

$$N = nN_A$$

$$\frac{K_{\text{tr}}}{N} = \frac{1}{2}m\langle v^2 \rangle = \frac{3}{2N}nRT$$

$$\langle v^2 \rangle = \frac{3k_B T}{m}$$
$$v_{\text{rmq}} = \sqrt{\frac{3k_B T}{m}}$$

$$\frac{K_{\text{tr}}}{N} = \frac{1}{2}m\langle v^2 \rangle = \frac{3}{2} \frac{R}{N_A} T = \frac{3}{2} k_B T$$

Depende apenas de  $T$

$$k_B = \frac{R}{N_A} = \frac{8.314 \text{ Joules}}{6.022 \times 10^{23} \text{ Molecula K}}$$

### Example 18.6 Calculating molecular kinetic energy and $v_{\text{rms}}$

(a) What is the average translational kinetic energy of a molecule of an ideal gas at a temperature of  $27^\circ\text{C}$ ? (b) What is the total random translational kinetic energy of the molecules in 1 mole of this gas? (c) What is the root-mean-square speed of oxygen molecules at this temperature?

#### SOLUTION

**IDENTIFY:** This problem involves the translational kinetic energy of an ideal gas on a per-molecule basis and a per-mole basis, as well as the rms speed of molecules in the gas.

**SET UP:** We are given temperature  $T = 27^\circ\text{C}$  and number of moles  $n = 1$  mol, and the molecular mass  $m$  is that for oxygen. We use Eq. (18.16) to determine the average kinetic energy of a molecule, Eq. (18.14) to find the total molecular kinetic energy, and Eq. (18.19) to find the rms speed of a molecule.

**EXECUTE:** (a) To use Eq. (18.16), we first convert the temperature to the Kelvin scale:  $27^\circ\text{C} = 300$  K. Then

$$\begin{aligned}\frac{1}{2}m(v^2)_{\text{av}} &= \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K}) \\ &= 6.21 \times 10^{-21} \text{ J}\end{aligned}$$

This answer does not depend on the mass of the molecule.

(b) From Eq. (18.14), the total translational kinetic energy of a mole of molecules is

$$\begin{aligned}K_{\text{tr}} &= \frac{3}{2}nRT = \frac{3}{2}(1 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K}) \\ &= 3740 \text{ J}\end{aligned}$$

This is about the same kinetic energy as that of a sprinter in a 100-m dash.

(c) From Example 18.5 (Section 18.2), the mass of an oxygen molecule is

$$m_{\text{O}_2} = (53.1 \times 10^{-24} \text{ g})(1 \text{ kg}/10^3 \text{ g}) = 5.31 \times 10^{-26} \text{ kg}$$

From Eq. (18.19),

$$\begin{aligned}v_{\text{rms}} &= \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})}{5.31 \times 10^{-26} \text{ kg}}} \\ &= 484 \text{ m/s}\end{aligned}$$

This is 1740 km/h, or 1080 mi/h! Alternatively,

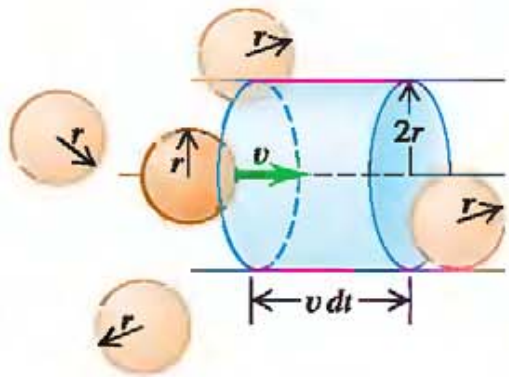
$$\begin{aligned}v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{32.0 \times 10^{-3} \text{ kg/mol}}} \\ &= 484 \text{ m/s}\end{aligned}$$

**EVALUATE:** We can check our result in part (b) by noting that the translational kinetic energy per mole must be equal to the average translational kinetic energy per molecule from part (a) multiplied by Avogadro's number  $N_A$ :  $K_{\text{tr}} = (6.022 \times 10^{23} \text{ molecules})(6.21 \times 10^{-21} \text{ J/molecule}) = 3740 \text{ J}$ .

In part (c), note that when we use Eq. (18.19) with  $R$  in SI units, we must express  $M$  in *kilograms* per mole, not grams per mole. In this example we use  $M = 32.0 \times 10^{-3} \text{ kg/mol}$ , *not*  $32.0 \text{ g/mol}$ .

# Colisão/Caminho livre

**18.15** In a time  $dt$  a molecule with radius  $r$  will collide with any other molecule within a cylindrical volume of radius  $2r$  and length  $v dt$ .



$$\text{Volume do cilindro} = 4\pi r^2 v dt$$

$$\frac{N}{V} \quad \text{Densidade de Moléculas}$$

Apenas uma molécula se movendo  
 $dt$  é o tempo necessário para uma  
colisão,  $dN=1$

$$dN = 4\pi r^2 v dt \frac{N}{V}$$

$$\frac{dN}{dt} = \frac{4\pi r^2 v N}{V}$$

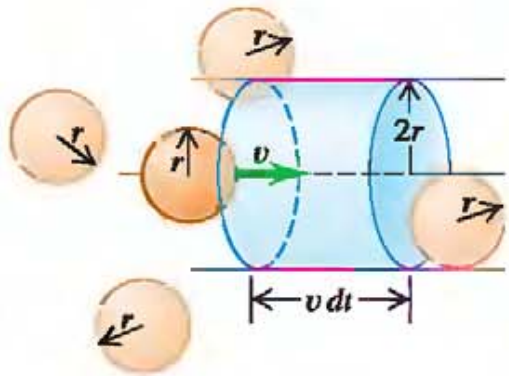
Colisões por unidade de tempo

Todas as molécula se movendo,  
**conta complexa**, resultado:

$$\frac{dN}{dt} = \frac{4\pi r^2 \sqrt{2} v N}{V}$$

# Colisão/Caminho livre

**18.15** In a time  $dt$  a molecule with radius  $r$  will collide with any other molecule within a cylindrical volume of radius  $2r$  and length  $v dt$ .



Todas as molécula se movendo,  
**conta complexa**, resultado:

$$\frac{dN}{dt} = \frac{4\pi r^2 \sqrt{2} v N}{V}$$

$$dN = 1, \quad t_{\text{medio}} = \frac{V}{4\pi r^2 \sqrt{2} v N}$$

$$\lambda = v t_{\text{medio}} = \frac{V}{4\pi r^2 \sqrt{2} N}$$

$$pV = nRT = Nk_B T$$

$$\lambda = \frac{k_B T}{4\pi r^2 \sqrt{2} p}$$



### Example 18.8 Calculating mean free path

(a) Estimate the mean free path of a molecule of air at  $27^\circ\text{C}$  and 1 atm. Model the molecules as spheres with radius  $r = 2.0 \times 10^{-10}$  m. (b) Estimate the mean free time of an oxygen molecule with  $v = v_{\text{rms}}$ .

#### SOLUTION

**IDENTIFY:** This problem uses the concepts of mean free path and mean free time (which are our target variables).

**SET UP:** We use Eq. (18.21) to determine the mean free path  $\lambda$ . To find the mean free time  $t_{\text{mean}}$  we could use Eq. (18.20), but it's more convenient to use the basic relationship  $\lambda = vt_{\text{mean}}$  in Eq. (18.21). For the speed  $v$  we use the root-mean-square speed for oxygen calculated in Example 18.6.

**EXECUTE:** (a) From Eq. (18.22),

$$\begin{aligned}\lambda &= \frac{kT}{4\pi\sqrt{2}r^2p} \\ &= \frac{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})}{4\pi\sqrt{2}(2.0 \times 10^{-10} \text{ m})^2(1.01 \times 10^5 \text{ Pa})} \\ &= 5.8 \times 10^{-8} \text{ m}\end{aligned}$$

The molecule doesn't get very far between collisions, but the distance is still several hundred times the radius of the molecule. To get a mean free path of 1 meter, the pressure must be about  $5.8 \times 10^{-8}$  atm. Pressures this low are found 100 km or so above the earth's surface, at the outer fringe of our atmosphere.

(b) From Example 18.6, for oxygen at  $27^\circ\text{C}$  the root-mean-square speed is  $v_{\text{rms}} = 484$  m/s, so the mean free time for a molecule with this speed is

$$t_{\text{mean}} = \frac{\lambda}{v} = \frac{5.8 \times 10^{-8} \text{ m}}{484 \text{ m/s}} = 1.2 \times 10^{-10} \text{ s}$$

This molecule undergoes about  $10^{10}$  collisions per second!

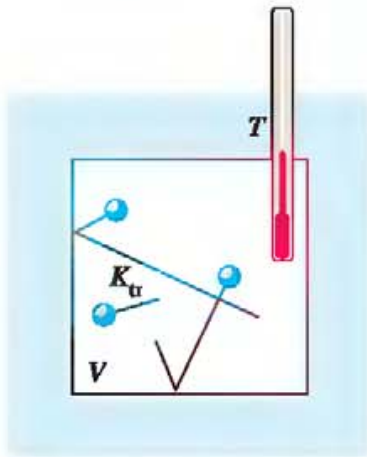
**EVALUATE:** Note that the mean free *path* calculated in part (a) doesn't depend on the molecule's speed, but the mean free *time* does. Slower molecules have a longer average time interval  $t_{\text{mean}}$  between collisions than do fast ones, but the average *distance*  $\lambda$  between collisions is the same no matter what the molecule's speed.



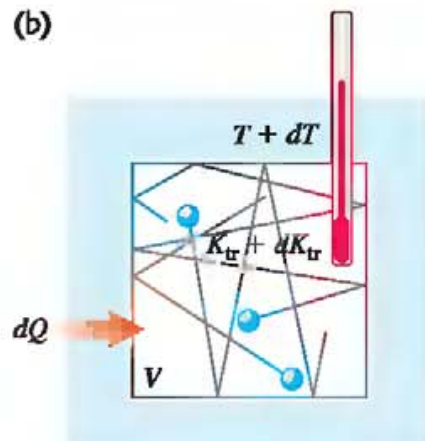
# Capacidade Térmica

**18.17** (a) A fixed volume  $V$  of a monatomic ideal gas. (b) When an amount of heat  $dQ$  is added to the gas, the total translational kinetic energy increases by  $dK_{tr} = dQ$  and the temperature increases by  $dT = dQ/nC_v$ .

(a)



(b)



Quando a temperatura muda, a energia cinética muda

$$dK_{tr} = \frac{3}{2}nRdT$$

$$dQ = nC_v dT$$

$$dQ = dK_{tr}$$

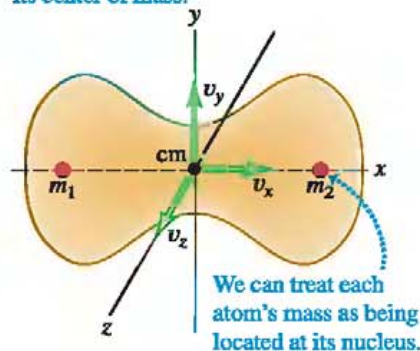
$$nC_v dT = \frac{3}{2}nRdT$$

$$C_v = \frac{3}{2}R$$

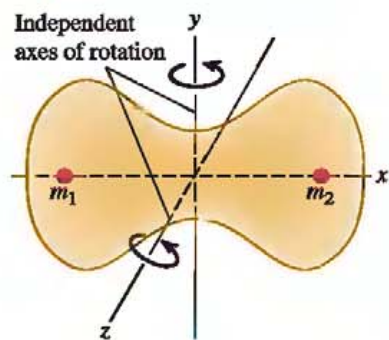
$$C_v = \frac{3}{2}8.314 = 12.47 \text{ J/mol K}$$

Type of Gas	Gas	$C_v$ (J/mol · K)
Monatomic	He	12.47
	Ar	12.47
Diatomic	H <sub>2</sub>	20.42
	N <sub>2</sub>	20.76
	O <sub>2</sub>	21.10
	CO	20.85
Polyatomic	CO <sub>2</sub>	28.46
	SO <sub>2</sub>	31.39
	H <sub>2</sub> S	25.95

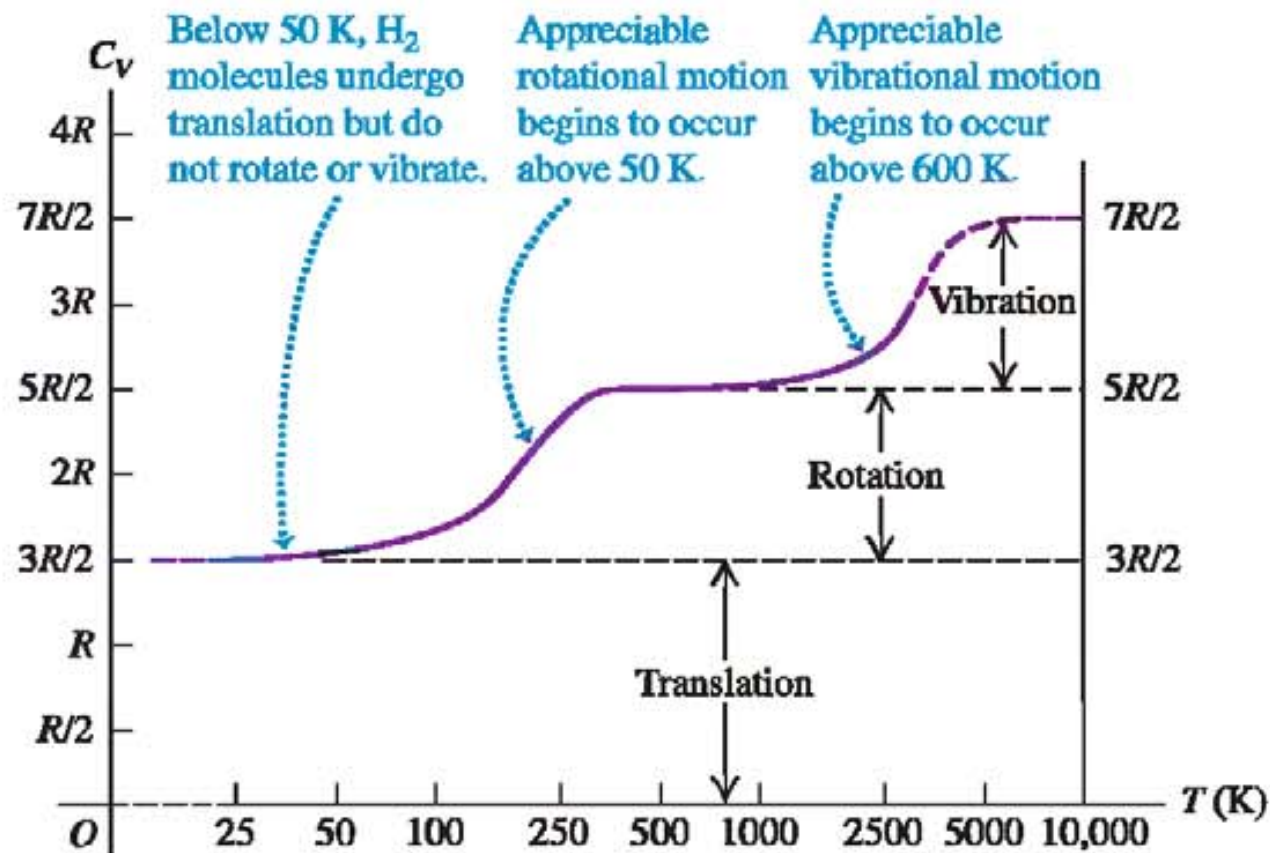
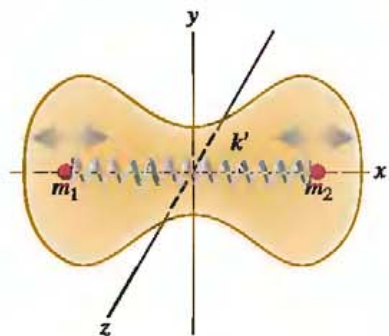
**(a) Translational motion.** The molecule moves as a whole; its velocity may be described as the  $x$ -,  $y$ -, and  $z$ -velocity components of its center of mass.



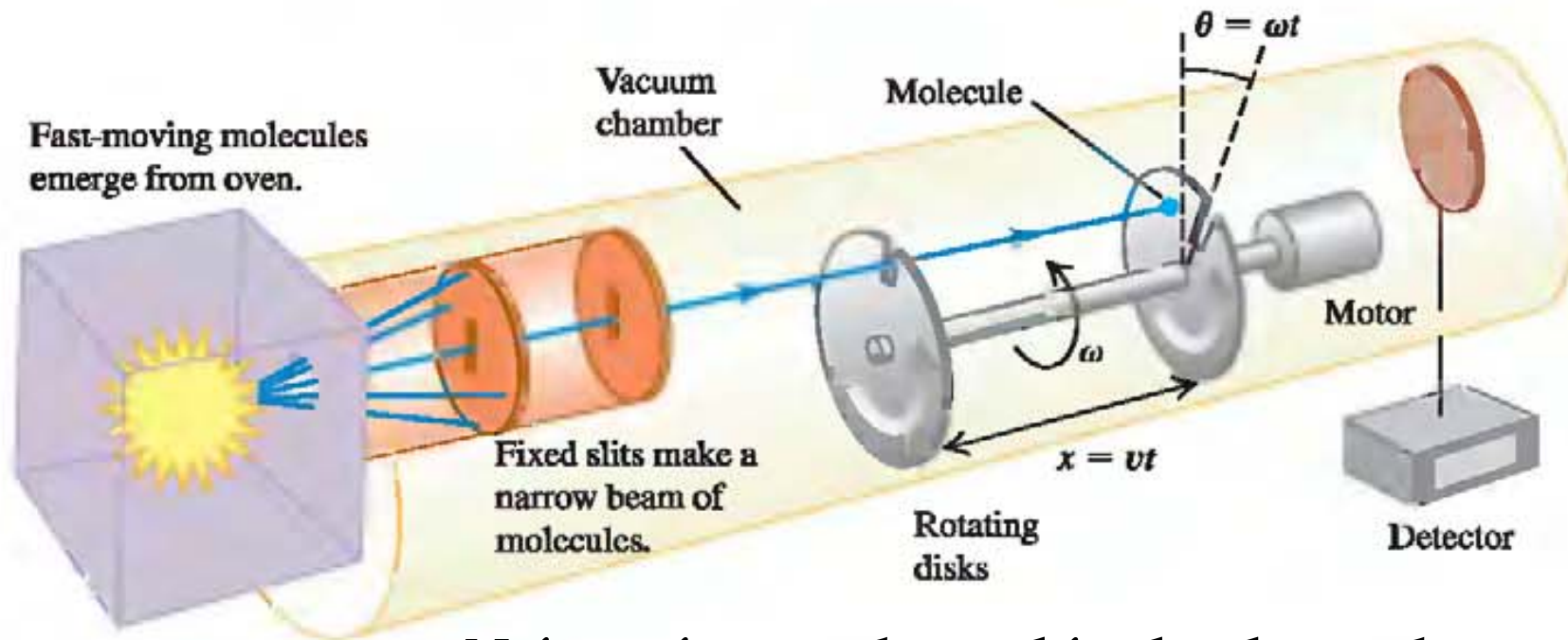
**(b) Rotational motion.** The molecule rotates about its center of mass. This molecule has two independent axes of rotation.



**(c) Vibrational motion.** The molecule oscillates as though the nuclei were connected by a spring.



# Velocidade Molecular



$N$  é o número de moléculas lançadas

$dN$  o numero que atingiu o detector

as moléculas detectadas possuem velocidade entre  $v$  e  $v+dv$

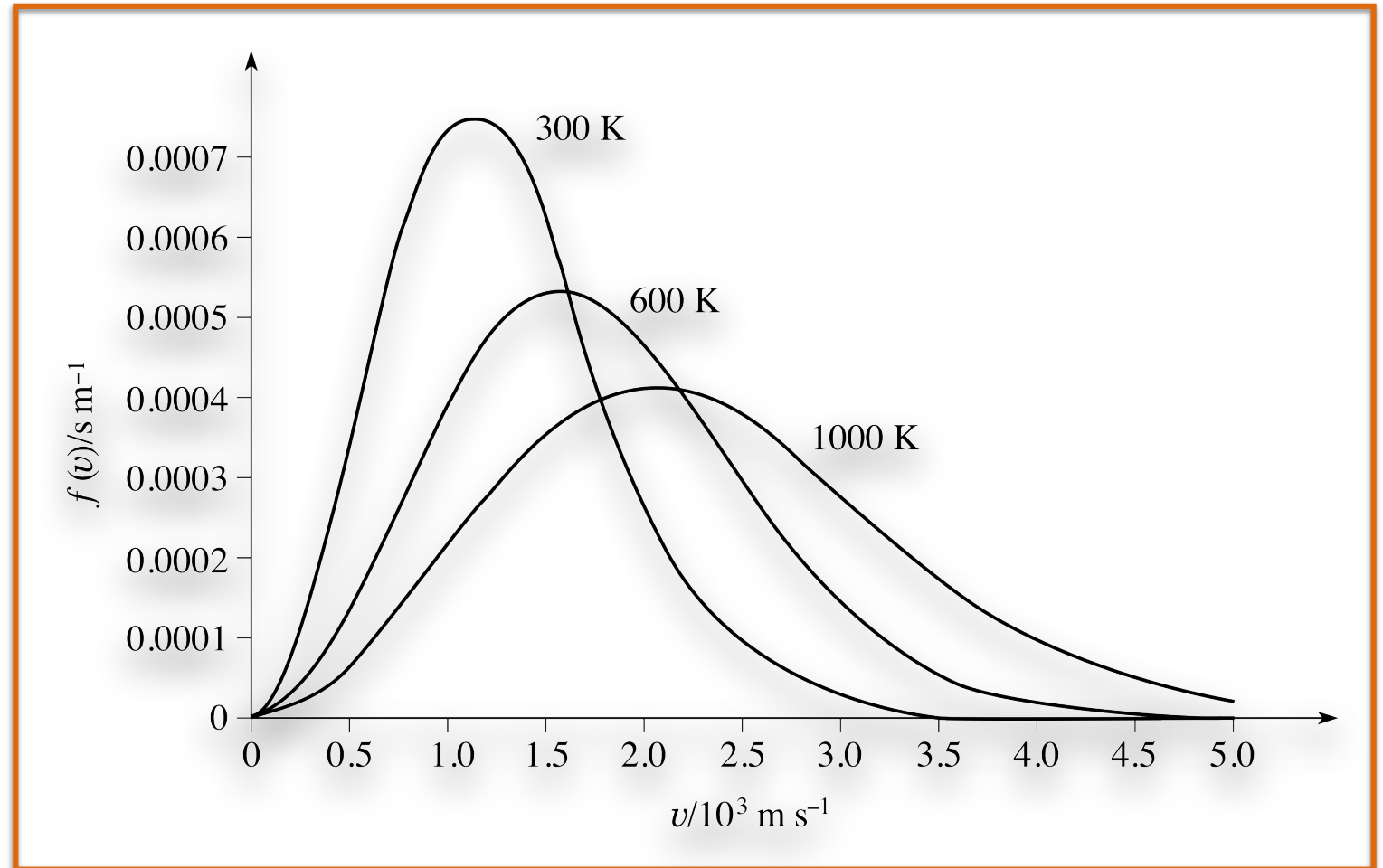
$$\text{Fração} = \frac{dN}{N} \quad f(v)dv = \frac{dN}{N}$$

$f(v)$  é a função distribuição das velocidades moleculares

# Velocidade Molecular

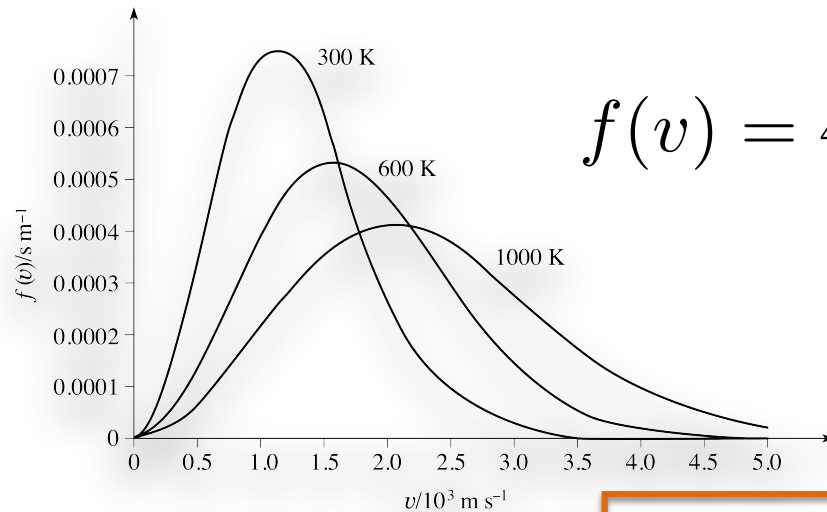
$$f(v)dv = \frac{dN}{N}$$

$$\int_0^{\infty} f(v)dv = 1$$



A função de distribuição de Maxwell-Boltzmann (velocidade) para uma amostra de gás a três temperaturas diferentes. A temperaturas mais elevadas, o pico da função de distribuição torna-se menor, mais amplo, e ocorre a uma velocidade maior

# Distribuição de Maxwell-Boltzmann



$$f(v) = 4\pi \left( \frac{m}{2\pi k_B T} \right)^{3/2} v^2 e^{-mv^2/2k_B T}$$

$$v_{\text{prob}} = \sqrt{\frac{2k_B T}{m}}$$

$$\langle v \rangle = \sqrt{\frac{8k_B T}{\pi m}}$$

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$$

