PTC 5757 - Exercício Aula 2 (1)

3.1-4

(a)

$$G(f) = \int_0^1 4 e^{-j2\pi f t} dt + \int_1^2 2 e^{-j2\pi f t} dt =$$

$$= \cancel{4}^{2} \cdot \frac{e^{-j2\pi f t}}{-j\cancel{2}\pi f}\Bigg|_0^1 + \frac{\cancel{2}}{-j\cancel{2}\pi f} e^{-j2\pi f t}\Bigg|_1^2 =$$

$$= \frac{j}{\pi f}\left[2\left(e^{-j2\pi f} - 1\right) + e^{-j4\pi f} - e^{-j2\pi f}\right] =$$

$$= \frac{j}{\pi f}\left(e^{-j4\pi f} + e^{-j2\pi f} - 2\right)$$

$$\int \underset{f}{x}\, \underset{g'}{e^{-ax}} = \frac{x e^{-ax}}{-a} + \int \frac{e^{-ax}}{a}$$

$$= -\frac{1}{a}\left(x e^{-ax} + \frac{1}{a} e^{-ax}\right)$$

$$= \frac{x e^{-ax}}{a} + \frac{1}{a^2}$$

$$= -\frac{e^{-ax}}{a^2}(ax + 1)$$

(b)

$$G(f) = \int_{-\tau}^{0} -\frac{t}{\tau} e^{-j\omega t} dt + \int_0^{\tau} \frac{t}{\tau} e^{-j\omega t} dt$$

$$= -\frac{1}{\tau}\left[+\frac{e^{-j\omega t}}{+\omega^2}(+j\omega t + 1)\Bigg|_{-\tau}^{0}\right] + \frac{1}{\tau}\left[\frac{e^{-j\omega t}}{\omega^2}(j\omega t + 1)\right]_0^{\tau} =$$

$$= \frac{-1}{\tau\omega^2}\left[1 - e^{j\omega\tau}(-j\omega\tau + 1)\right] + \frac{1}{\omega^2\tau}\left[e^{-j\omega\tau}(j\omega\tau + 1) - 1\right]$$

$$= \frac{1}{\omega^2\tau}\left[j\omega\tau e^{-j\omega\tau} + e^{-j\omega\tau} - 1 - 1 - j\omega\tau e^{j\omega\tau} + e^{j\omega\tau}\right]$$

$$= \frac{1}{\omega^2 \tau} \left[2\cos\omega\tau - j\omega\tau\,(j\,2\,\mathrm{sen}\,\omega\tau) - 2 \right]$$

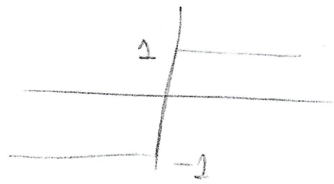
$$= \frac{1}{\omega^2 \tau} \left[2\cos\omega\tau + 2\omega\tau\,\mathrm{sen}\,\omega\tau - 2 \right]$$

$$= \frac{2}{\omega^2 \tau} \left[\cos\omega\tau + \omega\tau\,\mathrm{sen}\,\omega\tau - 1 \right]$$

3.2-2

$$1 \overset{TF}{\longleftrightarrow} \delta(f)$$

$$\mathrm{sgn}\, t \overset{TF}{\longleftrightarrow} \frac{2}{j2\pi f}$$



$$u(t) = \frac{1}{2}\mathrm{sgn}\, t + \frac{1}{2} \Leftrightarrow$$

$$\Rightarrow U(f) = \frac{1}{2} \cdot \frac{2}{j2\pi f} + \frac{1}{2}\delta(f) \Rightarrow$$

$$u(t) \overset{TF}{\longleftrightarrow} \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

3.2-3

$$\mathrm{sen}(2\pi f_0 t + \theta) = \frac{e^{j2\pi f_0 t} \cdot e^{j\theta} - e^{-j2\pi f_0 t} \cdot e^{-j\theta}}{j2}$$

$x(t) =$

$$\sin(2\pi f_0 t + \theta) = \frac{e^{j\theta}}{j2} e^{j2\pi f_0 t} - \frac{e^{-j\theta}}{j2} e^{-j2\pi f_0 t}$$

$$\Updownarrow TF$$

$$X(f) = \frac{e^{j\theta}}{j2} \cdot \delta(f - f_0) - \frac{e^{-j\theta}}{j2} \delta(f + f_0) =$$

$$= \frac{1}{2} e^{-j(\pi/2 - \theta)} \delta(f - f_0) + \frac{1}{2} e^{j(\pi/2 - \theta)} \delta(f + f_0)$$

3.3-3

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$\boxed{3.3\text{-}3}$ a

$$x(t) = \Pi\left(\frac{t+T/2}{T}\right) - \Pi\left(\frac{t-T/2}{T}\right)$$

$$X(f) = T\,\mathrm{sinc}(\pi f T)\,e^{+j2\pi f T/2} -$$

$$- T\,\mathrm{sinc}(\pi f T)\,e^{-j2\pi f T/2}$$

$$\Rightarrow X(f) = T\,\mathrm{sinc}(\pi f T)\, j2\,\mathrm{sen}(\pi f T)$$

$$\boxed{X(f) = j2T\,\mathrm{sinc}(\pi f T)\,\mathrm{sen}(\pi f T)}$$

$\mathrm{sen}(t-\pi) = \mathrm{sen}\,t\cos\pi - \cancel{\cos t\,\mathrm{sen}\,\pi}$

b

$$x(t) = \mathrm{sen}\,t\,(u(t) - u(t-\pi))$$

$$= \mathrm{sen}\,t\,u(t) + \mathrm{sen}(t-\pi)\,u(t-\pi) \qquad (f_0 = 1/2\pi)$$

$$= \cancel{\frac{1}{4j}\left[\delta(f - 1/2\pi) - \delta(f + 1/2\pi)\right]} + \frac{1}{1-(2\pi f)^2}$$

$$+ \cancel{\frac{1}{4j}\left[\delta(f - 1/2\pi) - \delta(f + 1/2\pi)\right] e^{-j2\pi^2 f}} + \frac{1}{1-(2\pi f)^2}\, e^{-j2\pi^2}$$

$$= \frac{1}{1-(2\pi f)^2}\left[1 + e^{-j2\pi^2 f}\right]$$

(c) $x(t) = \cos t\, u(t) - \cos t\, u(t - \pi/2) =$

$= \cos t\, u(t) + \operatorname{sen}(t - \pi/2)\, u(t - \pi/2) =$ | $f_0 = \frac{1}{2\pi}$ | $t_0 = \pi/2$

$X(f)$

$= \frac{1}{4}\left[\delta(f - 1/2\pi) + \delta(f + 1/2\pi)\right] + \frac{j}{1 - (2\pi f)^2} +$

$\frac{1}{4j}\left[\delta(f - 1/2\pi) - \delta(f + 1/2\pi)\right] e^{-\pi^2 j f} + \frac{1}{1 - (2\pi f)^2}\, e^{-j\pi^2 f} =$

$= \cancel{\frac{1}{4}\delta(f - \frac{1}{2\pi})} + \cancel{\frac{1}{4}\delta(f + 1/2\pi)} + \cancel{\frac{1}{4j}\delta(f - 1/2\pi)} \cdot e^{-j\frac{\pi}{2}} \cancel{-\frac{1}{4j}\delta(f + 1/2\pi)}$ $-j$

$+ \frac{1}{1 - (2\pi f)^2}\left[j + e^{-j\pi^2 f}\right] =$

$= \frac{1}{1 - (2\pi f)^2} \cdot \left[j + e^{-j\pi^2 f}\right]$

(d) $x(t) = e^{-at}\left[u(t) - u(t - T)\right] = e^{-at} u(t) - e^{-aT} e^{-a(t-T)} u(t-T)$

$X(f) = \frac{1}{a + j2\pi f} - e^{-aT} \cdot \left[\frac{1}{a + j2\pi f}\right] \cdot e^{-j2\pi f T} =$

$$= \left[\frac{1}{a+j2\pi f}\right]\left(1 - e^{-aT} e^{-j2\pi f T}\right) \Rightarrow$$

$$\boxed{X(f) = \frac{1}{a+j2\pi f}\left(1 - e^{-(a+j2\pi f)T}\right)}$$

3.3-9

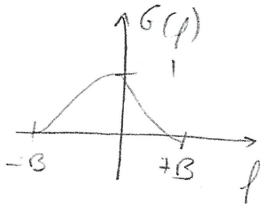
$$y(t) = [g(t)\cos 2\pi f_0 t]\, 2\cos 2\pi f_0 t =$$

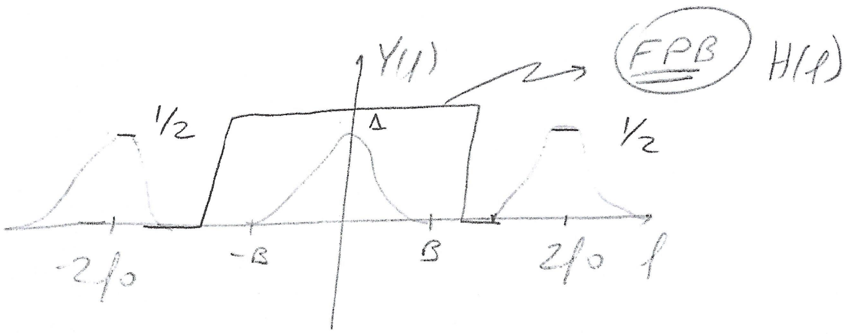
$$= g(t)\cdot 2\left[\frac{1}{2} + \frac{1}{2}\cos 4\pi f_0 t\right] =$$

$$= g(t) + g(t)\cos 4\pi f_0 t$$

$$\updownarrow$$

$$Y(f) = G(f) + \frac{1}{2}\left[G(f-2f_0) + G(f+2f_0)\right]$$





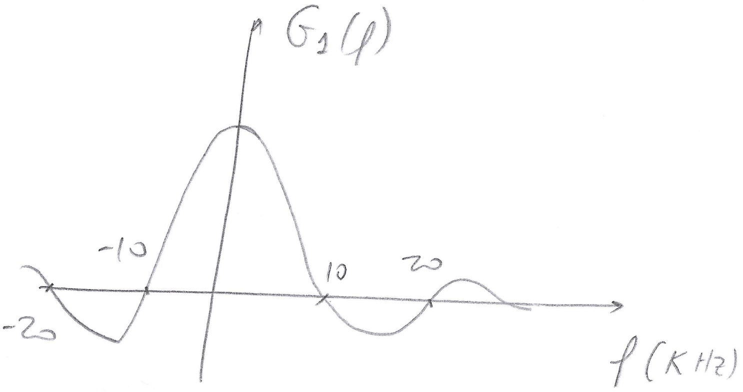
$$Y_f(f) = G(f) \Rightarrow y_f(t) = \underline{\underline{g(t)}}$$

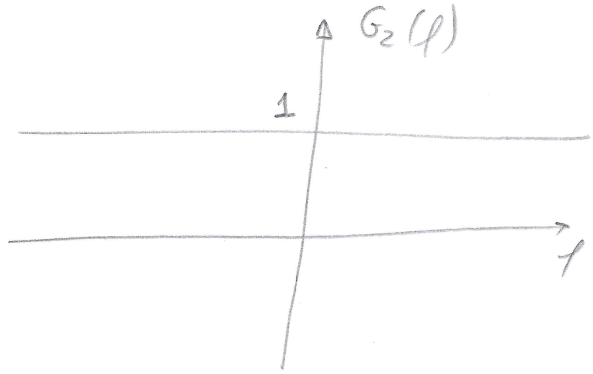
3.4.2

a) $G_1(f) = 10^4 \cdot 10^{-4}\,\mathrm{sinc}\left(\frac{\pi f}{10000}\right) \Rightarrow \boxed{G_1(f) = \mathrm{sinc}\left(\frac{\pi f}{10000}\right)}$

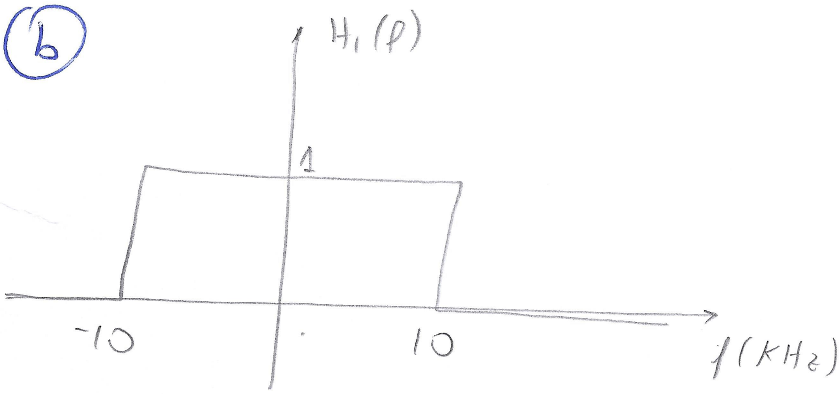
$$\boxed{\Pi\left(\frac{t}{\tau}\right) \xleftrightarrow{TF} \tau\,\mathrm{sinc}(\pi f \tau)}$$

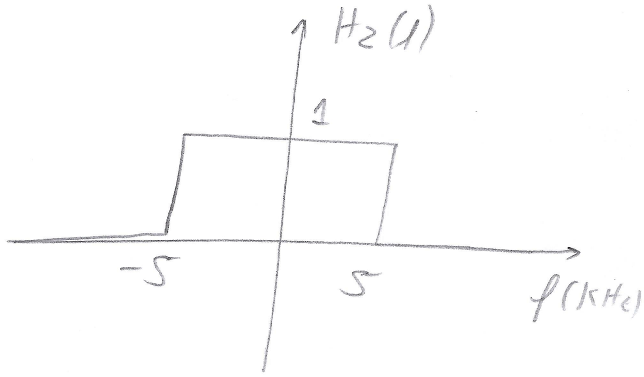
$$\boxed{G_2(f) = 1}$$





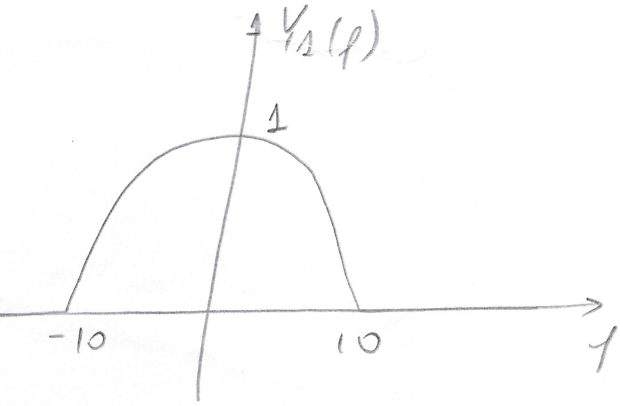
(b)

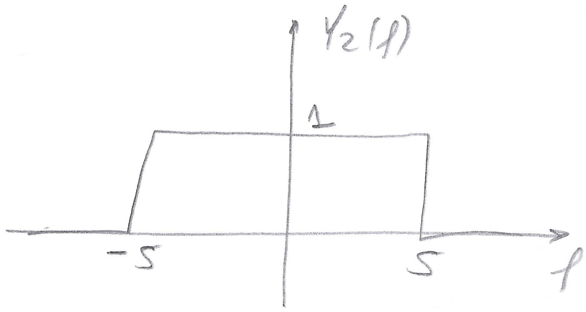




(c) $Y_1(f) = G_1(f) \cdot H_1(f)$

$Y_2(f) = G_2(f) \cdot H_2(f)$

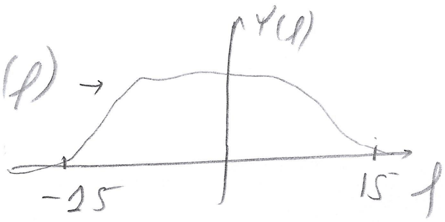




(d) largura de banda de $y_1(t)$ → 10 KHz

largura de banda de $y_2(t)$ → 5 KHz

$y(t) = y_1(t) \cdot y_2(t) \rightarrow Y(f) = Y_1(f) * Y_2(f) \rightarrow$



↳ largura de banda de $y(t) \Rightarrow$ 15 KHz

3.7.4

$$g(t) = \frac{2a}{t^2+a^2} \qquad G(t) \Leftrightarrow g(-f)$$

$$\boxed{e^{-a|t|} \longleftrightarrow \frac{2a}{a^2+(2\pi f)^2}} \rightarrow 2\pi e^{-2\pi a|t|}$$

$$G(t) = \frac{4\pi^2 \cdot 2a}{(2\pi t)^2 + (2\pi a)^2} = \frac{2\pi \cdot 2 \cdot 2\pi a}{(2\pi t)^2 + (2\pi a)^2}$$

$$\boxed{G(f) = 2\pi e^{-2\pi a|f|}}$$

$$E_g = \int_{-\infty}^{\infty} |G(f)|^2 df = 2\int_0^{\infty} 4\pi^2 e^{-4\pi a f} df = 8\pi^2 \left[\frac{e^{-4\pi a f}}{-4\pi a}\right]_0^{\infty}$$

$$= \cancel{8}^{2} \pi^{\cancel{2}} \left[\frac{1}{4\pi a}\right] = \frac{2\pi}{a}$$

(3.7.5)

Preciso encontrar B tal que $2\int_0^B |G(f)|^2 df = \frac{99}{100} \cdot \frac{2\pi}{a}$

$$\int_0^B |G(f)|^2 df = \int_0^B 4\pi^2 e^{-4\pi a|f|} df = 4\pi^2 \left[\frac{e^{-4\pi a f}}{-4\pi a}\right]_0^B =$$

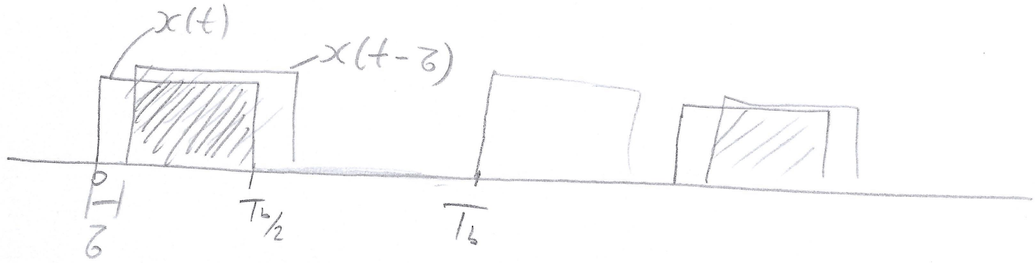
$$= \pi \left[\frac{e^{-4\pi a B}}{-a} + \frac{1}{a} \right] = \frac{\pi}{a}\left(1 - e^{-4\pi a B}\right)$$

$$\frac{2\pi}{a}\left(1 - e^{-4\pi a B}\right) = \frac{2\pi}{a} \cdot \frac{99}{100} \Rightarrow$$

$$\Rightarrow e^{-4\pi a B} = \frac{1}{100} \Rightarrow \quad 4\pi a B = +\ln 100 \Rightarrow \boxed{B = \frac{\ln 100}{4\pi a}}$$

3.8-2

A figura a seguir mostra as forma de onda $x(t)$ e $x(t-\tau)$ para $\tau < T_b/2$.



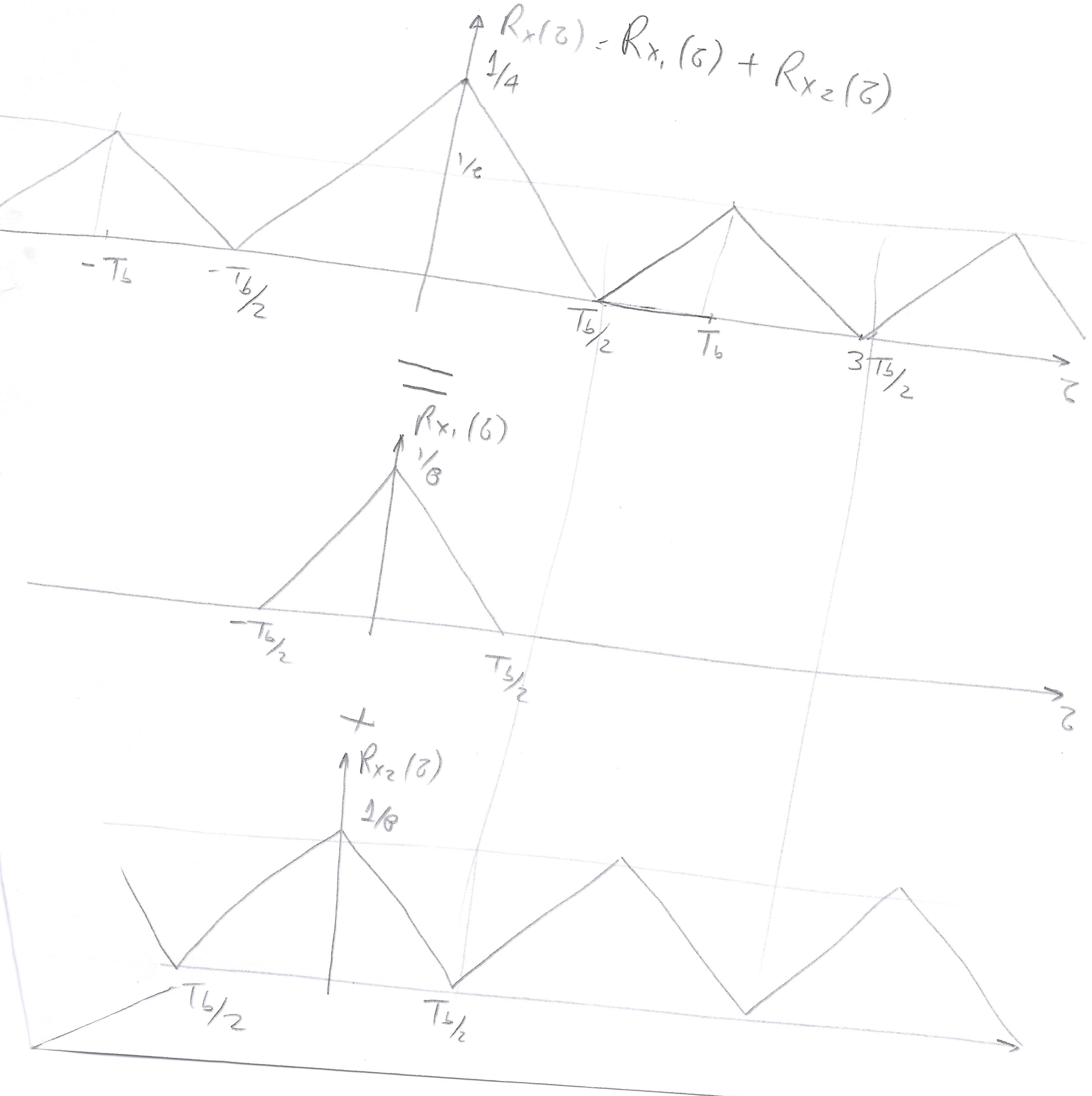
Seja $T = N T_b$. Na média existem $N/2$ pulsos na forma de onda com duração T. Assim, a área sob o produto $x(t)x(t-\tau)$ é $\frac{N}{2} \cdot \left(\frac{T_b}{2} - \tau\right)$. Portanto,

$$R_x(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, x(t-\tau)\, dt =$$

$$= \lim_{N \to \infty} \frac{1}{\cancel{N} T_b} \frac{\cancel{N}}{2} \left(\frac{T_b}{2} - \tau\right) = \frac{1}{2}\left(\frac{1}{2} - \frac{\tau}{T_b}\right), \quad \tau < \frac{T_b}{2} \text{ ou}$$

$$R_X(\tau) = \frac{1}{2}\left(\frac{1}{2} - \frac{|\tau|}{T_b}\right) \quad , \boxed{|\tau| < \frac{T_b}{2}}$$

Para $\tau = T_b/2$, não há sobreposição entre pulsos. Assim, $R_X(\tau) = 0$. Para $\frac{T_b}{2} \leq |\tau| \leq \frac{3T_b}{2}$, pulsos novamente se sobrepõem. Mas em média, apenas metade dos pulsos se sobrepõem. Assim, $R_X(\tau)$ se repete a cada T_b segundos, mas apenas com metade da amplitude, ou seja,



Assim, $$S_{X1}(f) = \frac{T_b}{16}\,\mathrm{sinc}^2\left(\frac{2\pi f T_b}{4}\right)$$

$\delta(at) = \frac{1}{a}\,\delta(t)$

$\delta(at)$

$$S_{X2}(f) = \frac{1}{16}\sum_{n=-\infty}^{\infty} \mathrm{sinc}^2\left(n\frac{\pi}{2}\right)\,\delta\left(f - \frac{n}{T_b}\right)$$

e

$\frac{\pi f T_b}{2} = \pi$

$$\boxed{S_X(f) = \frac{T_b}{16}\,\mathrm{sinc}^2\left(\frac{2\pi f T_b}{4}\right) + \frac{1}{16}\sum_{n=-\infty}^{\infty} \mathrm{sinc}^2\left(n\frac{\pi}{2}\right)\,\delta\left(f - \frac{n}{T_b}\right)}$$