PTC 5757 Aula 01

(1

2.1-3

$g(t) = C\cos(\omega_0 t + \theta)$

$$P_g = \frac{1}{T}\int_0^T g^2(t)\,dt \qquad \left(T = \frac{2\pi}{\omega_0}\right)$$

$$P_g = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} C^2\cos^2(\omega_0 t + \theta)\,dt = C^2\frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} \frac{1}{2} + \frac{1}{2}\cos(2\omega_0 t + 2\theta)\,dt$$

$$= \frac{\omega_0}{2\pi}C^2\cdot\left[\frac{\pi}{\omega_0} + \frac{1}{2}\cancel{\frac{\operatorname{sen}(2\omega_0 t + 2\theta)}{2\omega_0}}\Bigg|_0^{2\pi/\omega_0}\right] = \frac{C^2}{2}$$

2.1-5

$T = 4$

$$P_g = \frac{1}{4}\int_{-2}^{2} t^6\,dt = \frac{1}{4}\left[\frac{t^7}{7}\right]_{-2}^{2} = \frac{1}{28}\,2^7\cdot 2 =$$

$$= \frac{1}{14}\cdot 128 = \frac{64}{7}$$

a) $-g(t) \rightarrow P_g = \dfrac{64}{7}, \quad V_{rms} = \dfrac{8}{\sqrt{7}} = \dfrac{8\sqrt{7}}{7}$

(b) $2g(t)$

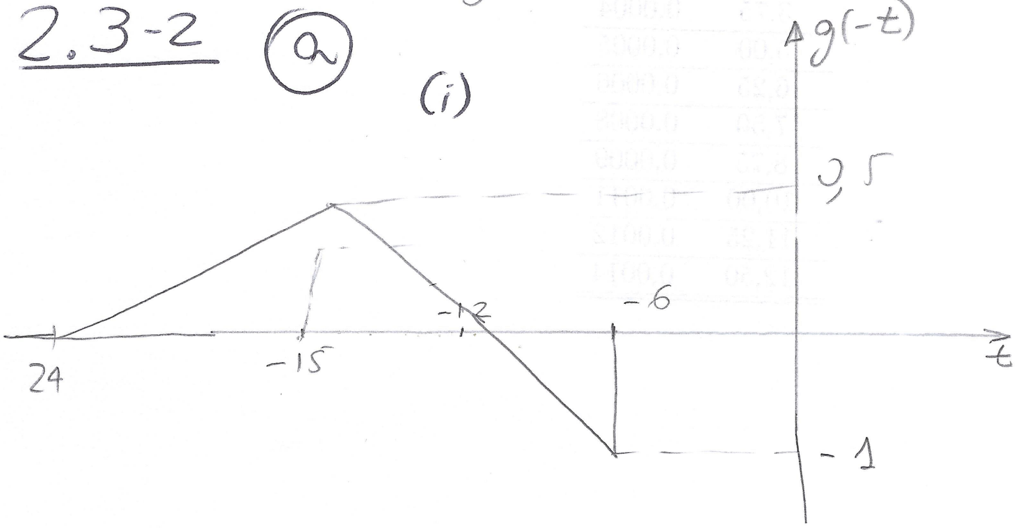
$$P_g = \frac{64 \cdot 4}{7} = \frac{256}{7}, \quad V_{rms} = \frac{16}{\sqrt{7}} = \frac{16\sqrt{7}}{7}$$

(c) $cg(t)$

$$\boxed{P_g = c^2 \cdot \frac{64}{7}}, \quad \boxed{V_{rms} = c \cdot \frac{8\sqrt{7}}{7}}$$

2.3-2 (a) $x(t) = g(-t)$

(i)

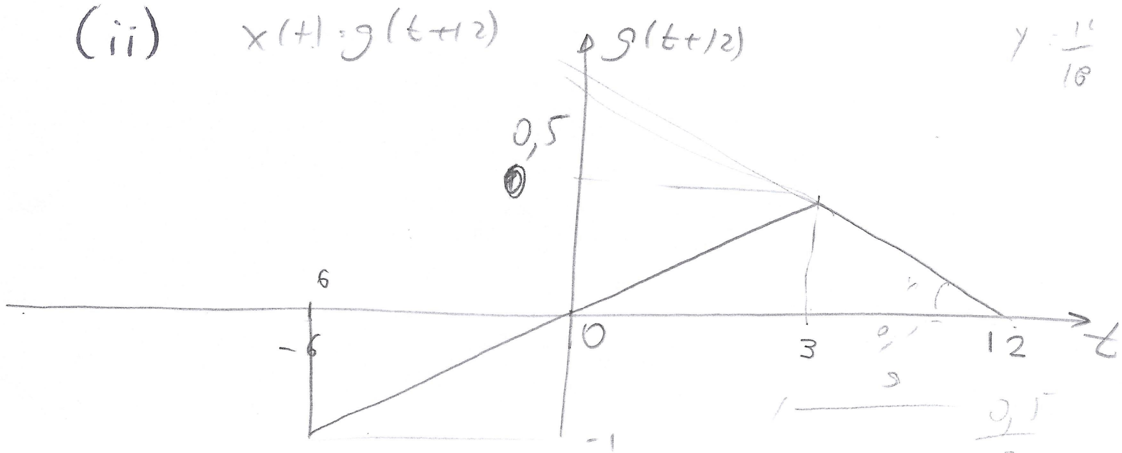


(b)

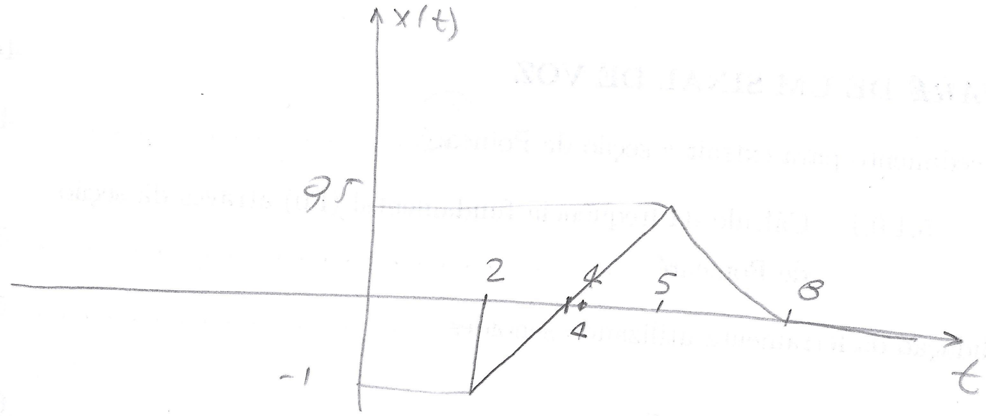
(ii) $x(t) = g(t+12)$

$$\frac{y}{12} = \frac{1}{18}$$

$$y = \frac{12}{18}$$

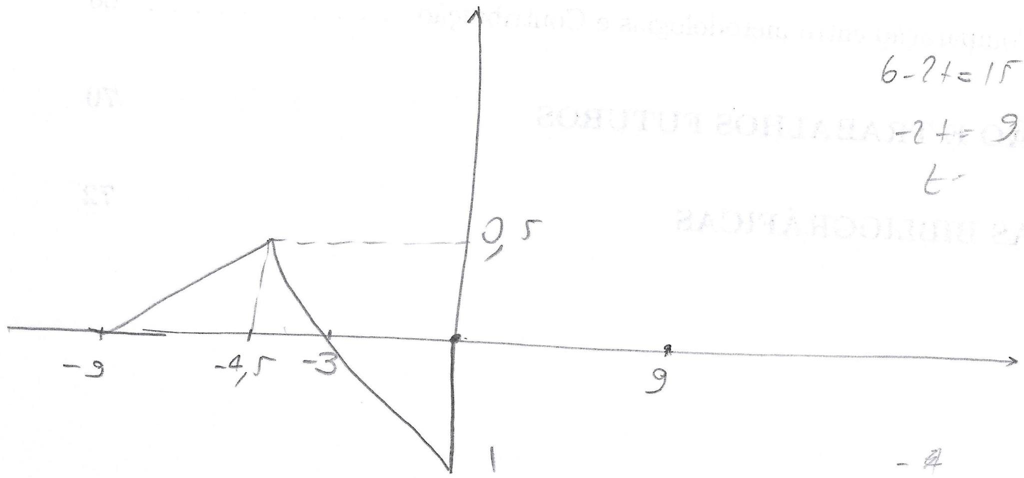


(iii) $x(t) = g(3t)$



iv) $x(t) = g(6-2t)$ $-2t = 6$ $t = -3$ $-2t = -18$ $t = ?$

$6-2t = 15$
$-2t = 9$
$t =$



$\frac{12}{18}$ $\frac{2}{3}$

(b) (i)

$$g(t+12) = \begin{cases} t/6, & -6 \le t \le 3 \\ -\frac{t}{18} + \frac{2}{3}, & 3 < t < 12 \end{cases}$$

$-\frac{3}{18} + \frac{2}{3}$

$-\frac{1}{6} + \frac{2}{3} = \frac{-1+4}{6} = \frac{5}{3}$

$$E_g = \int_{-6}^{3} \frac{t^2}{36}\,dt + \int_{3}^{12} \left(-\frac{t}{18} + \frac{2}{3}\right)^2 dt =$$

$\frac{3}{36}$ $\frac{6}{6}$

$$= \left[\frac{t^3}{36 \cdot 3}\right]_{-6}^{3} + \left[-\frac{t^2}{36} + \frac{2t}{3}\right]_{3}^{12} = \frac{1}{36 \cdot 3}(27+216) +$$

$$+\left[-\frac{\cancel{144}^{\,4}}{\cancel{36}}+8+\frac{\cancel{9}}{\cancel{36}}_{\,1/4}-2\right]=$$

$$=2{,}25+(2{,}25)=4{,}5$$

i) $E_g = 4{,}5$

ii) $E_g = 4{,}5$

iii) Se $g(t) = x(at)$

$$E_g = \int_{-\infty}^{\infty} x^2(at)\,dt = \int_{-\infty}^{\infty} x^2(\mu)\,\frac{1}{a}\,d\mu \rightarrow \frac{1}{a}E_x$$

$at \rightarrow \mu$

$dt = \frac{1}{a}\,d\mu$

Então $E_g = \frac{1}{3}\,4{,}5 = 1{,}5$

iv) $E_g = \frac{1}{2}\,4{,}5 = 2{,}25$

2.3.5

Para $\boxed{x(t) = -g(t)}$

$$E_x = \int_{-\infty}^{\infty} (-g(t))^2 dt = \int_{-\infty}^{\infty} g^2(t)\, dt = E_g.$$

Para $\boxed{x(t) = g(-t)}$

$$E_x = \int_{-\infty}^{\infty} [g(-t)]^2 dt = -\int_{\infty}^{-\infty} g^2(u)\, du = \int_{-\infty}^{\infty} g^2(u)\, du = E_g$$

$-t \to u$

$dt \to -du$

Para $\boxed{x(t) = g(t-T)}$

$$E_x = \int_{-\infty}^{\infty} [g(t-T)]^2 dt = \int_{-\infty}^{\infty} g^2(u)\, du = E_g$$

$t - T \to u$

$dt \to du$

Para $\boxed{x(t) = g(at)}$

$$E_x = \int_{-\infty}^{\infty} g(at)\, dt = \int_{-\infty}^{\infty} g(u) \frac{1}{a}\, du = \frac{1}{a} E_g$$

$at \to u$

$dt \to \frac{1}{a} du$

Para $x(t) = g(at-b)$

$$E_x = \int_{-\infty}^{\infty} g(at-b)\,dt = \int_{-\infty}^{\infty} g(\mu)\cdot\frac{1}{a}\,d\mu = \frac{1}{a}E_g$$

$\mu \to at-b$

$d\mu \to a\,dt$

a) $x(t) = g(t/a) \to \boxed{E_x = a\,E_g}$

b) $x(t) = a\,g(t)$

$$E_x = \int_{-\infty}^{\infty} a^2 g^2(t)\,dt = a^2 E_g \Rightarrow \boxed{E_x = a^2 E_g}$$

2.4.3

a) $$\int_{-\infty}^{\infty} g(\tau+a)\,\delta(t-\tau)\,d\tau = g(t+a)$$

b) $$\int_{-\infty}^{\infty} \delta(\tau)\,g(t-\tau)\,d\tau = g(t)$$

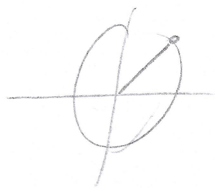
c) $$\int_{-\infty}^{\infty} \delta(t)\,e^{-j\omega t}\,dt = 1$$

(d) $\int_{-\infty}^{\infty} \delta(t-2) \sin \pi t \, dt = \sin 2\pi = 0$

(e) $\int_{-2}^{\infty} \delta(t+3)\, e^{-t} dt = e^{-3}$

(f) $\int_{-2}^{2} (t^2+4)\, \delta(1-t)\, dt = 5$

(g) $\int_{-\infty}^{\infty} g(2-t)\, \delta(3-t)\, dt = g(-1)$



$x = 3/2$

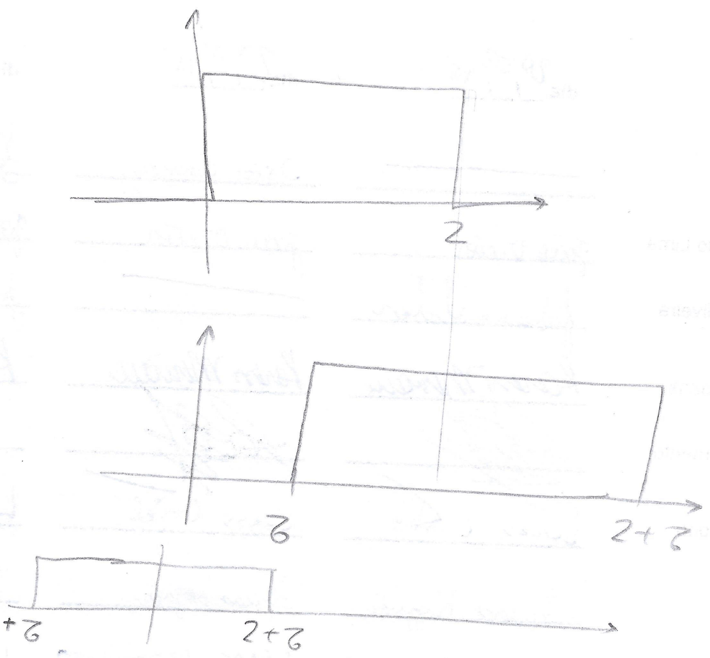
(h) $\int_{-\infty}^{\infty} \cos \frac{\pi}{2}(x-5)\, \delta(2x-3)\, dx = \cos \frac{\pi}{2}\left[\frac{3}{2} - 5\right] =$

$= \cos \frac{\pi}{2}\left(\frac{3-10}{2}\right) =$

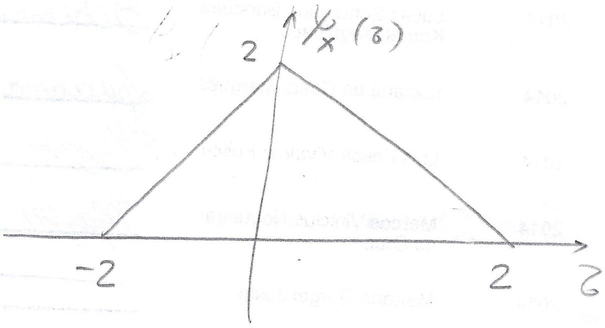
$\cos -\frac{7\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

2.6-2 $x(t) = u(t) - u(t-2)$

$$\psi_x(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t-\tau)\, dt$$



$$\psi_x(\tau) = \begin{cases} (2-\tau), & 0 \leq \tau \leq 2 \\ 2+\tau, & -2 \leq \tau < 0 \\ 0, & \text{c.c} \end{cases}$$



2.7.1 a) i) (2, −1) iv) (1, 2)

ii) (−1, 2) v) (2, 1)

iii) (0, −2) vi) (3, 0)

b) i) e iv)
$$\langle g_1, g_4 \rangle = \int_{-\infty}^{\infty} (2x_1(t) - x_2(t)) \cdot (x_1^*(t) + 2x_2^*(t))\, dt$$

$$= \int_{-\infty}^{\infty} \cancel{2|x_2(t)|^2} + 4\cancel{\int_{-\infty}^{\infty} x_2(t)\, x_2^*(t)\, dt} - \cancel{\int_{-\infty}^{\infty} x_2(t)\, x_1^*(t)\, dt}$$

2 ($E_{x_1}=1$) ortogonais 0 0

$$- 2 \cancel{\int_{-\infty}^{\infty} |x_2(t)|^2 dt} = 0$$

-2 ($E_{x_2}=1$)

ii e V

$$\langle g_2, g_5 \rangle = \int (-x_2(t) + 2x_2(t)) \cdot (2x_1^*(t) + x_2^*(t))\, dt$$

$$= -2 + 0 + 0 + 2 = 0$$

iii e vi

$$\langle g_3, g_6 \rangle = \int_{-\infty}^{\infty} -x_2(t) \cdot 3x_2(t)\, dt = 0$$

2.B.3

$$g(t) = \underbrace{\frac{1}{2}[g(t) + g(-t)]}_{g_e(t)} + \underbrace{\frac{1}{2}[g(t) - g(-t)]}_{g_o(t)}$$

$g_e(t) = g_e(-t) \rightarrow$ par

$g_o(t) = -g_o(-t) \rightarrow$ ímpar

(i) $g(t) = u(t)$

$$g_e(t) = \frac{u(t) + u(-t)}{2} = \frac{1}{2}$$

$$g_o(t) = \frac{1}{2}[u(t) - u(-t)] = \frac{1}{2}\operatorname{sgn}(t)$$

(ii) $g(t) = e^{-at} u(t)$

$$g_e(t) = \frac{1}{2}[e^{-at} u(t) + e^{at} u(-t)]$$

$$g_o(t) = \frac{1}{2}[e^{-at} u(t) - e^{+at} u(-t)]$$

(iii) $g(t) = e^{jt}$

$$g_e(t) = \frac{1}{2}[e^{jt} + e^{-jt}] = \cos t$$

$$g_o(t) = \frac{1}{2}[e^{jt} - e^{-jt}] = j \operatorname{sen} t$$