CHAPTER IX *The Analytical Representation of Process and the Economics of Production*

1. The Partial Process and Its Boundary. Occasionally, the use of a term spreads through the scientific literature with amazing swiftness but without a valid birth certificate, that is, without having been defined in some precise manner. Actually, the swifter the spreading, the greater is everyone's confidence that the meaning of the term is perfectly clear and well understood by all. One of the most glaring examples of this state of affairs is supplied by "process." It must be admitted, though, that process is a particularly baffling concept, for process is Change or is nothing at all. And as we have seen in Chapter III the intricate issues surrounding the idea of Change have divided philosophers into opposing schools of thought, one holding that there is only Being, the other that there is only Becoming. Science, however, can follow neither of these teachings. Nor can it follow the dialectical synthesis of the two into Hegel's tenet that "Being is Becoming." Science can embrace only the so-called vulgar philosophy according to which there is both Being and Becoming, for by its very nature it must distinguish between object and event. In other words, science must try to remain analytical throughout, even though, as I have argued earlier, it cannot succeed in this forever. The upshot is that science must have a clear idea of how to represent a process *analytically*. Failure to do so before the game starts is apt to become a source of important errors. In physics, we may remember, the opposition between particle and wave in quantum phenomena compelled the physicists to become more careful in interpreting observed processes. In social sciences—especially in economics where the paper-and-pencil arguments generally have only a remote contact with actual data—"process" is an abused term: it is used to denote almost anything one pleases. Witness the variety of mathematical formulae by which such a basic element of economic theory as the production process is represented. Witness, too, the practically total lack of concern for what the symbol-letters of these formulae stand for in actual terms.[1]

In approaching the problem of how to describe a process analytically, we should note that we must go along with the dialectics of Change on at least one point: Change cannot be conceived otherwise than as a relation between one thing and "its other" (to use Hegel's convenient terminology). To explain: in viewing a tree as a process we oppose it in our thought to everything that is not that tree even though we may not be fully conscious of this opposition all the time. Only for the absolute totality—the entire universe in its eternity—Change has no meaning; nothing corresponds to "its other." There certainly is Change within such a totality, but in order to discover it we must get inside, so to speak. More exactly, we must divide the totality into parts, into partial processes. The notion of a partial process necessarily implies some slits cut into the seamless Whole with which Anaxagoras identified actuality.[2] It is at this point that the dialectical thorns of the idea of partial process come to be appreciated even if we do not wish to go too deep into dialectics. Hardly anyone would deny that a living organism is a partial process; most would leave it at that. Yet, as Bohr reminds us, it is nigh impossible to say in every case whether a particular atom of the totality belongs to the organism in question or to "its other."[3] Economists, too, should be aware of the difficulty in deciding whether a truck hired by company A from company B and riding on some highway loaded with goods for company C is part of the activity of A, of B, or of C. Or to cite a still more intricate case: is the hired worker in a capitalist system in essence owned by the capitalist, as Marx argued? We are here confronted with the same issue that opposes dialectical (in my own meaning) to arithmomorphic notions. Analysis cannot accept a penumbra between one individual process and "its other." For if it does, it must set it as another partial process ana then it ends with three partial processes instead of two. We would thus be drawn into an infinite regress.

One obvious conclusion of the foregoing observations is that analysis must, in this case as in all others, proceed by some heroic simplifications

[1] This imbroglio is exposed together with its symbiotic fallacies in my paper "Chamberlin's New Economics and the Unit of Production," chap. ii in *Monopolistic Competition Theory: Studies in Impact*, ed. R. E. Kuenne (New York, 1967), pp. 38–44.

[2] See Chapter III, note 28.

[3] Niels Bohr, *Atomic Physics and Human Knowledge* (New York, 1958), p. 10.

and totally ignore their consequences. The first such step is to assume that actuality can be divided into two slices—one representing the partial process determined by the topical interest, the second, its environment (as we may say)—separated by an analytical boundary consisting of an arithmomorphic void. In this way, everything that goes on in actuality at any time is a part either of the process in point or of its environment. The first element, therefore, that the analytical picture of a process must necessarily include is the analytical boundary. *No analytical boundary, no analytical process.* The point deserves emphasis because often we may catch ourselves in the act of speaking about a process without having the faintest idea where its boundary should be drawn. On such occasions we are simply abusing the term "process."

Precisely because the Whole has no seams, where to draw the *analytical* boundary of a partial process—briefly, of a process—is not a simple problem. Plato to the contrary, there are not even joints in actuality to guide our carving.[4] One may slice actuality anywhere one pleases. This does not mean that any boundary cut by mere whim determines a process that has some significance for science. Analysis has already compartmented the study of actuality into special fields, each one with its own *purpose*. So, every special science draws process boundaries where it suits its special purpose. Without an intimate knowledge of the phenomenal domain of chemistry, for instance, one would not know where to draw a compatible boundary. In other words, a relevant analytical process cannot be divorced from purpose and, consequently, is itself a primary notion—that is, a notion that may be clarified by discussion and examples but never reduced to other notions by a formal definition.

If we consider further the nature of the boundary of a process, one point should arrest our attention: such a boundary must necessarily consist of two distinct analytical components. One component sets the process against its "environment" at any point of time. For lack of a better term, we may refer to this component as the *frontier* of the process. We should be careful, however, not to let this term mislead us into believing that the frontier of a process is geographical, i.e., spatial. Thought itself is a partial process; yet one can hardly say that it is enclosed within a definite space. The same is true of numerous sociological or political processes. Nor should we lose sight of another difficulty: the process may be such that it alters its own frontier. But this difficulty is not insuperable provided that we grant the analyst the faculty of perceiving that an oak and the acorn from which it grew belong to the *same* process. And we could not possibly deny him this faculty without denying all articulation to knowledge in general.

[4] See Chapter III, note 29.

The boundary must also contain a temporal component, the *duration* of the process. We must specify the time moments at which the analytical process we have in mind begins and ends. In view of the fact that it is for the sake of science that nature is sliced into partial processes, the temporal component of any such process must necessarily be a finite time interval. It must begin at some $t_0 > -\infty$ and end at some $t_1 < +\infty$. For if $t_0 = -\infty$ we would not know all that has gone into the process and if $t_1 = +\infty$ all that it does. Extrapolation may be in order in some special cases, but to walk on firm ground we must start with a finite duration. For the same reason, the case of $t_0 = t_1$ should also be excluded from the category of analytical processes proper. To recall Whitehead's dictum, a durationless process, an event at an instant of time as a primary fact of nature, is nonsense. Like the everlasting process, the point-process is an analytical abstraction of the second order and, like it, can be reached only by approximation.

A process involves, above all, some happening. How to represent this happening analytically is our next problem. But two observations are necessary before we tackle this new task.

The first is that by deciding to identify a process by its boundary we have implicitly given up any thought of describing what happens within that boundary, that is, inside the process. Should we wish to learn something about what happens inside, we must draw *another* boundary across the process and thus divide it into *two* processes to be studied separately. These processes could not have been part of our analytical picture before the new boundary was drawn because of the simple principle "no boundary, no process." Conversely, if for some reason or another we need to focus our attention only on the process obtained by subsuming two processes into one, we must remove from the analytical picture the boundary separating them and *also everything connected with it*. Should we aim at a complete description of everything that happens inside a process, we shall be drawn into an infinite regress whose resolution uncovers the inherent vice of any plan to represent actuality by an analytical framework. Indeed, there is no end to the division of nature by one analytical boundary after another. The limit of this algorithm is an abstract matrix in which every process is reduced to a point-instant of the space-time. All partial processes will thus vanish from our ambitious portrait of actuality. In other words, analysis, after starting from the position that there is both Being and Becoming, is in the end saddled with a matrix in which neither Being nor Becoming exists any more. It is because of this paradox of analysis that we may rest assured that physics, whose aim is to get further and further inside matter, will always cling to the idea that matter is made of atomic, i.e., indivisible yet sizable, particles.

The second observation is that in saying that the duration of the process *begins* at t_0 and *ends* at t_1 we must take the underscored words in their strictest sense. At $t < t_0$ or $t > t_1$ the *analytical* process is out of existence. By this I do not mean that outside the duration we have chosen for an analytical process the corresponding part of actuality is inexistent. What I mean is that we must abstract from what may have happened in actuality before t_0 and from what will happen after t_1. The corresponding mental operation should be clear: an analytical process should be viewed in itself as a hyphen between one *tabula rasa* and another.

2. The Analytical Coordinates of a Partial Process. Because analysis must renounce the idea of including in the description of a process what happens either inside or outside it, the problem of describing the happening associated with a process reduces to recording only what crosses the boundary. For convenience, we may refer to any element crossing the boundary from the environment into the process as an *input* and to any element crossing it in the opposite direction as an *output*.[5]

At this juncture, analysis must make some additional heroic steps all aimed at assuming away dialectical quality. Discretely distinct qualities are still admitted into the picture as long as their number is finite and each one is cardinally measurable. If we denote the elements that may cross the boundary of a given process by $C_1, C_2, \ldots, C_m$, the analytical description is *complete* if for *every* C_i we have determined two nondecreasing functions $F_i(t)$ and $G_i(t)$, the first showing the cumulative input, the second, the cumulative output of C_i up to the time t. Naturally, these functions must be defined over the entire duration of the process which may be always represented by a closed time interval such as $[0, T]$.

The question of whether this analytical model is operational outside paper-and-pencil operations cannot be decided without an examination of the nature of the elements usually found in actual processes. Such an examination reveals that there always exists numerous elements for which either $F_i(t)$ or $G_i(t)$ is identically null for the entire duration of the process. Solar energy is a typical example of an element which is only an input for any terrestrial process. The various materials ordinarily covered by the term "waste" are clear examples of elements which are only outputs. In all these cases, we may simplify the analytical picture by representing each element by one coordinate only, namely, by

$$E_i(t) = G_i(t) - F_i(t). \quad (1)$$

For an output element, $E_i(t) = G_i(t) \geq 0$; for an input element, $E_i(t) =$

[5] In the above context the terms have a precise meaning, a fact that contrasts with the current practice in economics where they are used so loosely that we see them applied to services of capital and labor as well.

$-F_i(t) \leq 0$. The sign of $E_i(t)$ suffices to tell which is actually the case.

A second category of elements is typified by the Ricardian land, i.e., by land viewed only in its "original and indestructible powers." If we refer to the simple case of a process consisting of growing corn from seed on an acre of land, the coordinates of the Ricardian land are

$$(2) \qquad \begin{aligned} &F_\alpha(t) = 1 \quad \text{for } 0 \leq t \leq T; \\ &G_\alpha(t) = 0 \quad \text{for } 0 \leq t < T, \qquad G_\alpha(T) = 1. \end{aligned}$$

In the same example, we find that corn, too, belongs to this ambivalent category. As seed, corn is an input; as crop, it is an output. Thus, assuming that one bag of corn is used as seed and the crop is ten bags, we have

$$(3) \qquad \begin{aligned} &F_\beta(t) = 0 \quad \text{for } 0 \leq t < t', \qquad F_\beta(t) = 1 \quad \text{for } t' \leq t \leq T; \\ &G_\beta(t) = 0 \quad \text{for } 0 \leq t < T, \qquad G_\beta(T) = 10; \end{aligned}$$

where t' is the time of seeding.[6] One may cite numerous cases of the same nature. One particular example, which I shall use often later on, is supplied by the hammers used in hammering additional hammers. With the aid of some analytical refinements we may represent each element of this category, too, by one single coordinate, $E(t)$. However, for reasons to become apparent in Section 4, below, it is preferable to abide by the more direct representation such as (2) and (3).

A third (and last) category of elements, which is illustrated by workers and tools, poses a special problem. A worker is a *rested* man when he goes into the process but comes out a *tired* man. A tool may be *new* when it enters the process but it is *used* when it comes out. In view of the analytical condition of discrete distinctness between the elements C_i, the "same" worker must be split into two distinct entities, one representing the worker when rested, the other, when tired. On the surface, this point may seem to be of a practical order only. In fact, it is a palpable symptom of the difficulty of separating the proper notion of process from that of qualitative change, a difficulty on which I have insisted in Chapter III. The elimination of qualitative change, we see, forces us to bar such a basic notion as that of sameness from our analytical picture of a process. Needless to add, from the formal viewpoint nothing pleads against representing a rested worker (or a new tool) by one C_k and the same worker when tired (or the same tool when used) by a different C_j. That is, the "same" worker may be represented by one input and one output coordinate:

$$(4) \qquad \begin{aligned} &E_k(t) = 0 \quad \text{for } 0 \leq t < t', \qquad E_k(t) = -1 \quad \text{for } t' \leq t \leq T; \\ &E_j(t) = 0 \quad \text{for } 0 \leq t < t'', \qquad E_j(t) = 1 \quad \text{for } t'' \leq t \leq T; \end{aligned}$$

[6] Seeding and harvesting, being processes, have durations. But the simplification involved in (3) has no effect on the point discussed here.

where t' and t'', $t' < t''$, are the times when the worker enters and leaves the process, respectively. Rested and tired workers and new and used tools may thus be included in the same category as the other ordinary inputs and outputs such as solar energy, waste, raw materials, etc.

The analytical description of a process is thus complete. We may associate it with a point in an abstract space of functions and write it symbolically as follows:

$$(5) \qquad [\overset{T}{\underset{0}{E_i(t)}};\ \overset{T}{\underset{0}{F_\alpha(t)}},\ \overset{T}{\underset{0}{G_\alpha(t)}}].$$

In this expression the subscript i covers all elements that are only inputs or only outputs, the subscript α, those that are both inputs and outputs. And a point we should not fail to note: the representation (5) keeps in permanent focus the fact that every process has a duration T.[7] Alternatively, the same representation can be laid out in a less abstract form as a series of graphs, each graph representing one of the functions involved in (5).[8]

An analytical picture in which the same worker (or the same tool) is split into two elements would undoubtedly complicate matters beyond description. The reason why these complications have not upset the various other analytical models currently used in natural or social sciences is that the issue of qualitative change has been written off *ab initio* by various artifices.[9] For example, the chemist usually draws the boundary of a chemical process in such a manner that the material structure—say, the test tube—inside which a reaction takes place is not listed as an element of the process. Perhaps he is justified in abstracting the test tube—chemical reactions may also occur in *open* space. However, even a chemist would mention the use of a catalyst (when necessary) even though a catalyst, like the Ricardian land, is not transformed by the process. On the other hand, a chemical engineer must, under heavy penalty, not lose sight of the fact that a dyeing vat deteriorates with use. All the more then we should expect an economist to make room in his analytical representation of a production process for this important economic factor—

[7] In the case of a production process we may use instead Marx's convenient term "time of production." Karl Marx, *Capital* (3 vols., Chicago, 1932–33), II, 272 f.

[8] For which see Fig. 1 of my article "Process in Farming vs. Process in Manufacturing: A Problem of Balanced Development," in *Economic Problems of Agriculture in Industrial Societies*, Proceedings of a Conference held by the International Economic Association at Rome (1965), eds. Ugo Papi and Charles Nunn (New York, 1969), pp. 497–528.

[9] One notable exception, to which I shall refer more than once, is Marx's analysis of the worker's participation in the productive process, an analysis which occupies a prominent place in the first volume of *Capital* and which, its shortcomings notwithstanding, is distinctly superior to everything else I have been able to come across in the literature.

the wear and tear. This he does, at times, explicitly. But in doing so he resorts to evaluating depreciation in money terms according to one of the conventional rules set up by bookkeepers. The solution is not only arbitrary, but also logically circuitous: it presupposes that prices and the interest rate, which in fact are influenced by production, are independent of it.

An inspection of the basic models of production (in real terms) reveals however, that none includes the tired worker or the used tool among their coordinates. In addition to the formal complications already mentioned, there are other reasons which command the economist to avoid the inclusion of these elements in his analytical representations of a process. The economist is interested first and last in *commodities*. To wit, no economist would nowadays draw the boundary of a process so that melted glass, for instance, should be an output or an input element. Melted glass, no doubt, is an indispensable factor in the production of glass wares; it is not something one would throw away—in a sense, it has economic value. But it is not a commodity under the present technology. The notion of commodity reflects not only the dialectical individuality of human wants but also (and especially) the fact that production under any state of the arts is carried out by fairly well individualized processes. At any one time, therefore, the spectrum of commodities is determined by the prevailing technology. Until recently, half-baked bread or ready-mixed cement were not commodities any more than melted glass is today. At any one time, however, the boundaries of the processes in which the economist is interested are drawn where the circulation of commodities can be observed, i.e., where they pass from one production unit to another or from one production unit to a consumption unit.

Even though there is no fast and general rule for determining what is and what is not a commodity, by no stretch of the imagination could we say that tired workers and used tools are commodities. They certainly are *outputs* in every process, yet the aim of economic production is not to produce tired workers and worn-out equipment. Also, with a few exceptions—used automobiles and used dwellings are the most conspicuous ones—no used equipment has a market in the proper sense of the word and, hence, no "market price." Moreover, to include tired workers and used tools among the products of industry would invite us to attribute a cost of production to such peculiar commodities. Of course, the suggestion is nonsense. Economics cannot abandon its commodity fetishism any more than physics can renounce its fetishism of elementary particle or chemistry can renounce that of molecule.

The conclusion is that at least for the purpose of microanalysis the representation of an economic process in the form to which the considerations of this section have led us is highly cumbersome, to say the least.

The question before us is whether there is some other mode of describing analytically a process, a mode that is both manageable and adequate in the sense that it does not leave out any essential factor. And the wear and tear, this work of the Entropy Law, is such a factor.

3. Stocks and Flows. The analytical models currently used in economics for representing a production process fall into two main categories, each category being related to an entirely different viewpoint. Although opposite to each other, the two views fared side by side in economics long before the era of mathematical models. One view, which began its great vogue with the advent of the Leontief static input-output system, is that a process is completely described by its flow coordinates, explicitly, by "the rate of flow per unit of time of each of the *N commodities* involved."[10] Assumingly, the flow rates are determined on the boundary identifying the process (although the idea of a boundary is never mentioned in the related works). The complex that characterizes this approach—and which is apparent from the arguments and applications of the flow models—is that the process is viewed as a continuously going affair which is approached by the observer at any time he may please *but only from the outside*. That is, during his tour of duty the observer is supposed to record only the flows that cross the frontier of such a going on process. What was already inside that process when he arrived on the scene and what remained inside it when he left are no concern of his. In its strict form, a flow model does not start with a *tabula rasa* nor ends with one.

The other type of analytical representation of a process reflects the diametrically opposite view: a complete representation of a process consists of two snapshots, as it were, one at the time when the observer comes on the scene, the other when he leaves. Or to put it differently, the observer takes two censuses, one at the beginning of his period of observation and one at the end. He pays no attention whatsoever to *what crosses the frontier* at any time. According to this viewpoint, a process is represented analytically by a two-row matrix

$$\begin{bmatrix} A'_1, A'_2, \cdots, A'_n \\ A''_1, A''_2, \cdots, A''_n \end{bmatrix}, \tag{6}$$

where the vectors (A') and (A'') represent the stocks of *commodities* inside

[10] T. C. Koopmans, "Analysis of Production as an Efficient Combination of Activities," in *Activity Analysis of Production and Allocation*, ed. T. C. Koopmans (New York, 1951), p. 36 (my italics). As hinted above, this conception of a process had already been advocated in nonmathematical quarters; e.g., G. Stigler, *The Theory of Competitive Price* (New York, 1942), p. 109. That the same conception is the analytical cornerstone of Leontief's input–output system is obvious from the statements which stud his major contribution, W. W. Leontief, *The Structure of the American Economy: 1919–1939* (2nd edn., New York, 1951), especially pp. 12 f, 27.

the boundary at two time instants $t' < t''$, respectively.[11]

One point, which I made some years ago, should be abundantly clear from the preceding remarks: each of the two types of models tells only one different part of the whole story.[12] For an incisive example, let us consider the case in which $(A') = (A'')$. Unless the frontier of the process includes the entire universe—alternatively, unless we know that we are dealing with an isolated system—it is impossible for us to say whether (6) represents a stationary state (in which something does happen) or a frozen conglomerate (in which nothing happens). Turning to the flow models, let us take the case of two processes having exactly the same flow coordinates. In this situation, we have no way of knowing whether they are identical or one is more efficient (in some particular sense) than the other. The observer being supposed to approach the process from the outside as the process is going on, the flow representation of an agricultural process should not include the Ricardian land.[13] Nor could the tools already in use be included in such a model if its rationale is strictly followed. Should the observer be by chance meticulous indeed, he may, at most, record the output rate of scrap. But no model builder yet seems to have been meticulous to that extent.

The opposition between the two types of models brings to mind the famous antinomy between flow and stock. For if both the flow and the stock models offer an adequate representation of a process—as each model claims for itself—the antinomy between flow and stock should be fictitious. As it happens, the two models are neither equivalent nor contradictory. We may be thus tempted to conclude that, after all, the concepts of flow and stock are not strictly antinomic. The antinomy is nonetheless as irreducible as antinomy can be.

No alert economist would nowadays make the same kind of statement as that by which Adam Smith opened his magnum opus: "The *annual* labor of every nation is the *fund* which originally supplies it with all the necessaries and conveniences of life."[14] That much is certain after we have

[11] John von Neumann, "A Model of General Economic Equilibrium," *Review of Economic Studies*, XIII (1945), 2. For a sample of the numerous works in which the stock-process conception has been advocated, see A. L. Bowley, *The Mathematical Groundwork of Economics* (Oxford, 1924), pp. 28 f; J. R. Hicks, *The Theory of Wages* (London, 1932), p. 237; Paul A. Samuelson, *Foundations of Economic Analysis* (Cambridge, Mass., 1948), p. 57.

[12] See my article "The Aggregate Linear Production Function and Its Applications to von Neumann's Economic Model," in *Activity Analysis of Production and Allocation*, ed. Koopmans, pp. 100 f.

[13] But see note 30 below.

[14] Adam Smith, *The Wealth of Nations*, ed. E. Cannan (2 vols., 5th edn., London, 1930), I, 1 (my italics). But the acme of surprise is that Léon Walras—a mathematics aspirant in his youth—associates income with stock. See his *Elements of Pure Economics* (Homewood, Ill., 1954), pp. 212 f. The same thought echoes in J. A. Schumpeter, *The Theory of Economic Development* (Cambridge, Mass., 1934), p. 46: "the reservoirs which we call income."

been repeatedly instructed not to confuse what flows with what stands still.[15] The oft-quoted dictum on this issue is Irving Fisher's: "Stock relates to a *point* of time, flow to a *stretch* of time."[16]

Like all tersely formulated principles, this rule has walked the rounds so swiftly that practically everyone's mind felt satisfied without looking into the thoughts behind it. It was thus very easy for the modern tide of formalism to bury the antinomy under a trite formula which now walks the rounds with still greater ease. The formula could not possibly refurbish such a fundamental concept as that of stock. Stock continued to be conceived as a qualityless entity—we would say—which exists as a quantum in a definite "place" and has a cardinal measure at any instant during the time interval in focus. Flow, however, came to be defined simply as the difference between two instances of a stock at two different instants of time. The idea is crystallized in the tautological formula

$$\Delta S = S(t_1) - S(t_0), \tag{7}$$

where $S(t_0)$ and $S(t_1)$ are the measures of the correlative stock at the instants $t_0 < t_1$. That this approach hides away the antinomy is beyond question. The difference between two quanta of, say, wheat is also a quantum of wheat whether the two quanta refer to the same storehouse at two different instants or to two storehouses at the same instant. It is because of this truism that we are apt to commit the error of confusing stock with flow. According to formula (7) both an income over any period and a bank balance consist of dollars indistinguishable from each other. Why should we then treat income and wealth as two different essences?

One answer—on which Fisher himself fell back—is that what is after all opposed to stock is not ΔS but $\Delta S/(t_1 - t_0)$, that is, the flow *rate*.[17] A flow rate, certainly, is not of the same essence as a stock. But the relation between this difference and the old antinomy is only superficial. To wit, an instantaneous flow rate also refers to a point of time. When driving, I can read on the panel instruments both the speed of the car and the mileage driven from home at any chosen instant. But if I do not know what speed actually means, I am apt to tell the policeman who stops me for driving sixty miles per hour in the center of the town,

[15] To recall, S. Newcomb, in his *Principles of Political Economy* (New York, 1886), p. 316 and *passim*, was first to draw the attention of economists to the error of the crude Wage Fund doctrine which confused—as Adam Smith did—an annual flow with a fund.

[16] Irving Fisher, "What Is Capital?" *Economic Journal*, VI (1896), 514. The caution comes up repeatedly in most of Fisher's later writings down to his *The Nature of Capital and Income* (New York, 1919), chap. iv.

[17] Fisher, "What Is Capital?" pp. 514 f.

"Officer, it cannot possibly be so, I have not driven sixty miles since I left home." True, I may avoid committing such an error if my attention is drawn to the fact that ΔS and $\Delta S/\Delta t$ do not have the same dimensionality. According to this line of thought, the only reason against confusing the monthly rate of income with a monthly bank balance is the general principle that concepts of different dimensionalities must be kept separate in our minds and in our operations. The upshot is that in the particular case under discussion the role of time becomes accidental. Fisher's dictum would convey nothing more than countless other rules of the same form, say, the rule that "height relates to a *point* in space, slope to a *stretch* of space." The answer mentioned at the beginning of the paragraph misses the point that the antinomy between flow and stock does not involve only the difference between the dimensionality of flow rate and stock.

Even though most economic models nowadays use formula (7) by rote in order to pass from stock to flow coordinates and vice versa, they offer no occasion for the reader to sense the antinomy that has intrigued Fisher and many other careful analysts. Actually, this antinomy is implicitly but unmistakably denied by the argument that the stock model is the more comprehensive of the two because the flow coordinates can be derived by (7) from stock data but the stocks can be determined from flows only beyond an arbitrary constant (or only if the stocks are known at some instant). One should, the argument concludes, prefer the stock model to the other.[18]

The advice harbors the fallacy, manifest in one model after another, that a census taker must come out with exactly the same list of elements as the custom official who records only what crosses the frontier. In other words, the list of the elements C_i must be identically the same in the stock and in the flow representations of the same process.[19] This is the natural consequence of settling the issue of flow by (7). For if that equation

[18] Among the authors that I could cite in support of the judgment expressed in this paragraph are authorities such as John Hicks, for instance. In his recent *Capital and Growth* (New York, 1965), p. 85, he tells us explicitly that "We do not need to distinguish between stocks and flows; for stocks and flows enter into the determination of equilibrium in exactly the same way." In this volume, just as in the earlier *Value and Capital* (Oxford, 1939), Hicks adopts the idea expressed by (7) to derive the flows from the stock coordinates by which he prefers to represent a process.

[19] We may cite here the case of the dynamic Leontief system where the lists of current and capital input–outputs are identical. Cf. W. Leontief *et al.*, *Studies in the Structure of the American Economy* (New York, 1953), pp. 55–58. True, in a formal model one may use the same list for the flow and the stock items and let some of the coordinates be set to zero in the concrete applications of the model. But such a procedure is likely to conceal from view a very important feature of process. See Section 9, below.

is accepted as the only definition of a flow we cannot avoid the conclusion that whenever there is a flow there must be a stock, and conversely. Clearly, if one side of a definitional formula has a meaning, so must the other. A few simple counter-examples suffice to show that between the lists of flow and the stock elements of the same process there does not exist even a relation of inclusion: normally, the lists overlap. A census must include the land of a country, its roads, its river dams, its factories, etc., etc.—items never found in any import-export statistics. On the other hand, most private homes use a flow of electricity; yet a census taker may find no stock of electricity in it. But even if we take an item such as "raw rubber"—which is both a stock and a flow coordinate of the United States viewed as one partial process—we shall find that the stocks and the flow coordinates do not as a rule satisfy (7).

The crux of the issue under discussion is that *a flow does not necessarily represent either a decrease or an increase in a stock of the same substance.* The melted glass that flows into the rolling machines does not decrease the stock of melted glass in the furnace. In the ultimate analysis, it decreases the stocks of sand, coal, etc.—that is, the stocks of other substances in nature. The flow of food consumed by mankind since its origin has not come out from a stock in existence at the time of Creation. But, for an analogy that should make the point crystal clear, there is the fact that Time always flows but never exists as a stock.

The position that formula (7) takes perfect care of the notion of flow because every flow comes from one stock and goes into another stock can be traced back to the epistemological fallacy which I have endeavored to confute in some of the preceding chapters. The fallacy is that Change consists of locomotion and nothing else. As a result, the intricate notion of flow, which is intimately connected with qualitative change, is reduced to motion from one slice of actuality to another. There are, no doubt, cases in which formula (7) expresses directly the connection between *two* stocks and *one* flow. Still, for the overwhelming number of the relevant cases the true connection is between *one* stock and *one* flow. For a simple illustration, let us consider the flow of melted glass that pours from the furnace into the rolling machines. We may simply visualize the stock of melted glass that would have accumulated during some interval if it had not been almost instantaneously transformed into glass plate. Or we may visualize the stock of wheat that would be accumulated by now if, say, all the wheat produced since 1900 had not been consumed in step with every harvest.

The moral of these illustrations is plain: *a flow is a stock spread out over a time interval.* The stock to which this definition refers may have an analytical existence only—as in the case of the last examples—or an actual existence, in which case it corresponds to ΔS of formula (7). The definition, I believe, is far more incisive than Fisher's dictum.

Whether the flow comes out of a stock or goes into one, or whether it is of the nature of an event, it can be represented analytically by a coordinate such as $E(t)$ of relation (1) in Section 2 above, *defined over an appropriate time interval*.[20] Often, we may be satisfied with a less sharp description and simply say that a flow of ten tons of melted glass *occurred* during five hours. In this case, the analytical representation is the *pair* (S, T), where S is a stock and T is a stretch of time. The explicit mention of the corresponding stock *and* of the duration is indispensable. And in fact, this is done in every statistical table of production data, for example. Only, the time component is separated from S and included in the title of the table, which may read "The *Yearly* Production of the Steel Industry." But the data in the body of the table are stocks, as said above. To say *only* that the rate of flow was on the average two tons per hour—i.e., to replace the pair (S, T) by a single coordinate S/T—does not constitute a complete description of the flow even in the simplified form.

4. Funds and Services. The main point of the preceding section—that a flow does not necessarily come out or go into an actual stock—is connected with the plain fact that products are *created*. If the boundary of a process that produces automobiles, for instance, is appropriately drawn, we will find no stock out of which the product flows. Conversely, a boundary of a process may be drawn in such a way that many input flows are *annihilated* the instant they enter the process. In economic jargon, they are consumed. The inputs falling in this category are characterized by one interesting feature. Although when we settle the final accounts we see that the completion of a process requires a definite amount of such an input, this amount is not needed all at once, but only as a flow spread over time in some specific manner. Think, for instance, of the amount of solar energy or the amount of rainfall necessary to bring to completion a process of raising corn. A painter, also, does not need to buy all the paint for a job at once. If material constraints arising from discontinuous units were not present, we could visualize him buying a continuous flow of paint.

What we have just said about solar energy, rainfall, and paint does not apply to all inputs. These other inputs are characterized by two correlated

[20] Obviously, the essence of $E(t)$ is that of stock in all cases. At times, $E(t)$ cannot be determined otherwise than by an instrumental measure of the instantaneous flow rate, $e(t) = E'(t)$. In this case, we must not lose sight of the fact that a flow always consists of some substance in the broad sense of the term. Otherwise, we may find ourselves speaking of a stock of voltage, if we read the wrong instrument.

features. First, they are not consumed in the process; instead they come out of the process, albeit with some scars. The ladder of a painter is a good illustration. But the most stringent example of this category is the Ricardian land which comes out in exactly the same amount and quality. Most of the inputs of this category exist only in some indivisible physical units. They are typified by any tool that outlasts the process in which it participates as well as by a worker. But in all these cases, we speak of land, of tools, and workers as being *used* in, not *consumed* by, the process. And we are right in making this distinction.

The point is not new. The way it has often been presented is that the distinction arises from the fact that some things can be consumed at once but others are durable because their consumption requires duration.[21] As expected, positivist arguments have assailed this position on the ground that no event is durationless and no fast line can be drawn to separate durable from nondurable factors of production.[22] The fault of the position, however, is that it claims—as is apparent from the literature—that any object can be classified as durable or nondurable independently of the process in which it is an input. The sin is similar to that of the general dichotomy of commodities into consumer and producer goods. Analysis may abstract from a dialectical penumbra but not if the penumbra happens to cover almost the entire spectrum of discourse.

Inputs can be classified into nondurable and durable in a manner that meets the requisites of analysis if we adopt a relative criterion. In relation to a *given* process an input is only used (but not consumed) if it can be connected with an output element by reason of identity of substance—like the clover seed in growing clover seed—or sameness of object—like the painter's ladder. If this is not the case, the input is consumed in the process. The classification is, of course, dialectical because we find no tool in the positivist paraphernalia for recognizing sameness. A few extreme illustrations may be in order for additional clarification. A space rocket would at present be classified as a consumable input; yet in the technology of tomorrow it may become a durable input used successively in several space flights. Also, we may conceive processes with no durable input besides mere space and some raw form of matter-energy—the evolution of the universe from the Big Bang to the present, for instance. A completely tragic expedition in the Sahara is another example. Finally, let us note that the economic process of mankind from its inception to this

[21] E.g., Léon Walras, *Elements of Pure Economics*, p. 212.

[22] Time and again, the oily inconsistency of the positivist dogma comes up to the surface. We may recall that on other occasions the same dogma finds nothing wrong with the idea of an event at an instant of time. Cf. Chapter III, Sections 4 and 5, above.

day has no durable input of human or technical nature and only a few of other nature. For such processes the above classification of inputs into consumable and durable, although still workable, may not be relevant. They raise some issues that must be handled differently. So, let us make abstraction of them for the time being and concentrate on the overwhelming number of processes for which the distinction is highly enlightening.

About a durable input—a machine, for instance—economists say not only that it can be *used* in a production process, but also that it can be *decumulated*. They also speak of capital *accumulation* when a new factory is built. We should note, however, that in these expressions the meanings of "accumulation" and "decumulation" differ profoundly from those in saying that a flow accumulates into a stock or a stock decumulates into a flow. In the last cases "accumulation" and "decumulation" represent some mechanical operations akin to locomotion. Because the difference thus screened is of paramount analytical importance, the ambiguous usage has served as a hotbed of idle controversy and a source of grave errors—one of which will presently have our attention.

There can be no doubt that the decumulation of a machine is not a mechanical spreading in time of the machine as is the case with the stock of provisions of an explorer, for instance. When we "decumulate" a machine we do not separate it into pieces and use the pieces one after another as inputs until all parts are consumed. Instead, the machine is used over and over again in a temporal sequence of tasks until it becomes waste and has to be thrown away. A machine is a material stock, to be sure, but not in the sense the word has in "a stock of coal." If we insist on retaining the word, we may say that a machine is a stock of services (uses). But a more discriminating (and hence safer) way of describing a machine is to say that it is a *fund of services*.

The difference between the concept of stock and that of fund should be carefully marked, lest the hard facts of economic life be distorted at everyone's expense. If the count shows that a box contains twenty candies, we can make twenty youngsters happy now or tomorrow, or some today and others tomorrow, and so on. But if an engineer tells us that one hotel room will probably last one thousand days more, we cannot make one thousand roomless tourists happy *now*. We can only make one happy today, a second tomorrow, and so on, until the room collapses. Take also the case of an electric bulb which lasts five hundred hours. We cannot use it to light five hundred rooms for an hour *now*. The use of a fund (i.e., its "decumulation") requires a duration. Moreover, this duration is determined within very narrow limits by the physical structure of the fund. We can vary it only little, if at all. If one wishes to "decumulate" a pair of shoes, there is only one way open to him: to walk until they

become waste.[23] In contrast with this, the decumulation of a stock may, conceivably, take place in one single instant, if we wish so. And to put the dots on all significant i's, let us also observe that the "accumulation" of a fund, too, differs from the accumulation of a stock. A machine does not come into existence by the accumulation of the services it provides as a fund: it is not obtained by storing these services one after another as one stores winter provisions in the cellar. Services cannot be accumulated as the dollars in a saving account or the stamps in a collection can. They can only be used or wasted.

Nothing more need be said to prove that also the use of the term "flow" in connection with the services of a fund is improper if "flow" is defined as a stock spread over time. In fact, the generally used expression "the flow of services" tends to blur—at times, it has blurred—the important differences between two mechanisms, that by which the prices of services and that by which the prices of material objects are determined. The inevitable trap of this ambiguous use of "flow" is that, because a flow can be stored up, we find it perfectly normal to reason that services are "embodied" in the product.[24] Only the materials that flow into a production process can be embodied in the product. The services of the tailor's needle, for example, cannot possibly be embodied in the coat—and if one finds the needle itself embodied there it is certainly a regrettable accident. The fact that in certain circumstances the value of services passes into the value of the product is to be explained otherwise than by simply regarding a machine as a stock of services that are shifted one after another into the product.

The difference between flow and service is so fundamental that it separates even the dimensionalities of the two concepts. For this reason alone, physicists would not have tolerated the confusion for long. The amount of a flow is expressed in units appropriate to substances (in the broad sense)—say pounds, quarts, feet, etc. The rate of flow, on the other hand, has a mixed dimensionality, (substance)/(time). The situation is entirely reversed in the case of services. The amount of services has a mixed dimensionality in which time enters as a factor, (substance) × (time). If a plant uses one hundred workers during a working day (eight

[23] Of course, one may sell the shoes. But this would mean decumulation of the shoes as a stock, not decumulation of the shoes as a fund of services. Besides, selling the shoes implies a buyer who presumably is interested in using them himself. The elementary fact that funds cannot be decumulated except by use over a fairly determined duration accounts not only for the economic ills of recession but also for the structural locks of many Latin American economies. Cf. my article "O Estrangulamento: Inflação Estrutural e o Crescimento Econômico," *Revista Brasileira de Economia*, XXII (March 1968), 5–14.

[24] E.g., A. C. Pigou, *The Economics of Stationary States* (London, 1935), pp. 20, 117.

hours), the total of the services employed is eight hundred *man* × *hours*. If by analogy with the rate of flow we would like to determine the rate of service for the same situation, by simple algebra the answer is that this rate is one hundred *men*, period. The rate of service is simply the size of the fund that provides the service and consequently is expressed in elemental units in which the time factor does not intervene. A rate with respect to time that is independent of time is, no doubt, a curiosity. It was all the more necessary to point out that it exists and to show the reason why it exists.

5. A Flow–Fund Model. As manifested by the standard of numbers, the present temper in economics is to jump directly to tackling only the "big" problems, of growth or of development. But, especially among the rank and file, not all economists who write on development or who are engaged in planning seem to heed one elementary object lesson of mechanics, which is that one cannot speak of accelerated motion otherwise than as a passage from one *uniform* motion to another such motion. For, just like the accelerated motion, growth cannot be conceived otherwise than as a passage from one stationary state to another. The study of growth must begin with the study of the stationary state and develop up from this basis if it is to be a well-planned scientific enterprise.[25] The view—expressed quite often, albeit sotto voce rather than solemnly—that the concept of a stationary state constitutes only a textbook cumber is therefore inept. Actually, the reverse is true: ordinarily, writers do not pay enough attention to clarifying the concept.[26] A complement of the same mistaken view is that the concept of a stationary state is in addition factually irrelevant. This reflects both a superficial knowledge of facts and a misunderstanding of what "factually relevant" means in science. Even a practicing mechanical engineer, who is interested only in facts as they are, would not say that uniform motion is factually irrelevant for him. And just as there are actual motions that are *almost* uniform and, hence, can be treated as being uniform, so in the history of mankind we do find cases after cases of almost stationary economic states. From the dawn of man's economic evolution to this day only the present interlude constitutes an exception to the rule that human society has advanced at such a slow speed that the change becomes visible only in the perspective of centuries or even millennia. On a lower level, what is a normally functioning factory if not a quasi stationary state or a steady-going concern, if you wish?

[25] For an example of this procedure at its best the reader is invited to look up Part III of Leontief, *Structure of the American Economy*.

[26] One notable exception standing in a class by itself is the masterly analysis by Pigou in his *Economics of Stationary States*, now almost completely buried by oblivion.

The quality of being stationary may be defined in several equivalent ways. The direction from which Karl Marx approached the problem appears to suit best the scope of this chapter.[27] A system is stationary if whatever it does can be repeated identically over and over again. "Stationary state" and Marx's "simple reproduction" are therefore perfectly synonymous terms. But in order that a partial process be capable of being repeated after its conclusion, it is imperative that the fund factors involved in it should not come out degraded. From what we have seen already in this essay, this condition leads to an impasse.

However, the impasse can be resolved and the solution comes straight out of the economic literature of older vintage. It is the idea of capital equipment *being kept as a constant fund by the very process in which it participates*.[28] Strictly interpreted, this idea is a fiction. A process by which something would remain indefinitely outside the influence of the Entropy Law is factually absurd. But the merits of the fiction are beyond question. Like the notion of uniform motion (i.e., a motion without entropic friction), that of a process which maintains its equipment constant is not as remote from actuality as it may at first seem. We need only look around in almost any factory or home to convince ourselves that normally efforts are constantly directed toward keeping every piece of equipment in good working condition. For let us not fail to note that "maintaining capital constant" does not imply that a piece of capital is an indestructible monolith. All it means is that the *specific* efficiency of every piece of capital is kept constant. It matters not that a machine looks old, is scratched, dented, out of fashion, etc., as long as it is as efficient as when it was new. In places that the jet planes cannot yet reach we see hundreds of DC-3 planes, some twenty years old, doing now as good a job as when they were new and flying between the metropolises of the world. There is, though, a snag in the idea of capital's being maintained constant, but the snag pertains to analytical, not factual, considerations.

To keep a spade in good working condition, a farming process needs, among other things, a file. The file, being now a necessary element of the process, must also be kept in good order and, hence, it calls in turn for another tool (say, a wire brush); this tool calls for another, and so forth. We are thus drawn into a regress which might not end until we have included in the process in question a very large part of the entire produc-

[27] See Marx, *Capital*, I, 619 f.

[28] The reason given by Marx (*Capital*, I, 221 f) for his choice of the term "constant capital" to denote the material means of production is that the value of these means passes unaltered into the value of the product. To emphasize in this manner the main tenet of the labor doctrine of value was natural for him. But his analysis of the diagram of simple reproduction (*ibid.*, II, 459 f) clearly suggests that he had in mind mainly the idea mentioned in the text.

tion sector of the economy. And that is not all. Since workers are funds, they must be kept "in good working condition," too. The initial process has then to be expanded until the household process of almost every worker and practically every production line in the world are included in it. This conclusion is a glaring example of the ways in which the seamless actuality resists being divided into arithmomorphic parts. If we insist on connecting a process with some enduring entity, some form of Being, we are forced to go back to the Whole. Such a broad viewpoint may have its merits in other respects—as we shall see in time—but it forbids us from dealing with microprocesses, a plant or even an industry, for instance. Some sort of compromise is necessary in order to circumvent the difficulty. It consists of admitting that maintenance may be achieved in part also through services brought in from outside and ignoring the daily wear and tear of the worker (which in fact is always restored outside, in the household). The dividends of this compromise are paid by a clearer picture of the practical implications of a production process.

The factors of production can now be divided into two categories: the fund elements, which represent the *agents* of the process, and the flow elements, which are *used* or *acted upon* by the agents. Each flow element continues to be represented by one coordinate $E_i(t)$ as defined in Section 2 above. But in view of the fact that a fund element enters and leaves the process with its efficiency intact, its analytical representation can be greatly simplified. Specifically, we can represent the participation of a fund C_α by a single function $S_\alpha(t)$ showing the amount of services of C_α up to the time t, $0 \leq t \leq T$.[29] We still need to refer to a point in an abstract space for the analytical representation of the process, but this representation is much simpler than (5) of Section 2:

$$[\overset{T}{\underset{0}{E_i(t)}};\ \overset{T}{\underset{0}{S_\alpha(t)}}]. \tag{8}$$

Formal results such as the one just reached tend to screen issues and points that must nonetheless be continuously borne in mind lest we turn into symbol spinners. Most important of all is to remember that the question whether a factor is classified as a fund or as a flow element in the

[29] Alternatively, C_α can be represented by a function $U_\alpha(t)$ showing how much of the fund is participating in the process at t, with the convention that a fund is *in* at the instant it enters and *out* at the instant it leaves the process. This convention is the symmetrical aspect of the fact that an input in a partial process is an output of the environment, and vice versa. The graph of $U_\alpha(t)$ looks like a skyline of a city and has the advantage of bringing into focus the periods when C_α is idle, i.e., when it is not needed by the process. See Fig. 1 in my paper "Process in Farming vs. Process in Manufacturing" (cited in note 8, above).

analytical representation of an actual process depends upon the duration of that process. If the process in which, say, an automobile is used is relatively short, the efficiency of the automobile may be maintained by replacing now and then the spark plugs or the tires, for instance. If it is longer, we may have to replace the motor, the chassis, or the body parts. The flow elements will not be the same in the two cases. Conceivably, any automobile may be maintained forever through flows of all its constituent parts. In the long run, the automobile will have only the name in common with the initial one. But this fact need not bother the analyst: the process needs the services of an automobile of a definite type, not those of a particular automobile identified by the serial number. Cost is one reason why in actuality automobiles and other pieces of equipment are not maintained forever but are discarded from time to time. But even if cost would not be an impediment, obsolescence would ultimately bring about the same result. Novelty, therefore, is the main cause why an automobile, a machine, a bridge, or a highway are discarded and replaced in the long run. Of course, in the very, very long run it is the work of the Entropy Law that prevents anything from lasting forever. The limitations of the flow–fund model as an analytical representation of actual processes should not, therefore, be ignored. But neither should the merits of the model in casting a great deal of light on many analytical points be belittled.

Another important point to be borne in mind is that the division of factors into flow and fund elements does not mean that the same item cannot appear both as a flow and as a fund. Let us recall one illustration of Section 2, namely, the process in which hammers are used to hammer hammers. It should be obvious that in this case the item "hammer" is an output flow—and as such must be represented in (8) by one $E_i(t)$—and also a fund—to be represented by one $S_\alpha(t)$. Yet the point has often been missed. And it is not a point of minor importance. One notable illustration is provided by the analytical difficulties into which Marx got himself by failing to distinguish in his diagram of simple reproduction between the fund-hammers and the flow-hammers. The problem has many instructive facets and I shall come back to it (Section 14, below).

6. Further Reflections on the Flow–Fund Model. The general expression (8) can be made more telling by bringing into it the broad categories into which the fund and the flow elements may be classified according to their specific nature or role in the process. For funds we may take our cue from the Classical division of production factors and distinguish them into Ricardian land (L), capital proper (K), and labor power (H). Among flow elements we may distinguish first the inputs of the so-called natural resources (R)—the solar energy, the rainfall, the "natural" chemicals in the air and the soil, the coal-in-the-ground, etc. Second, there are the current input flows (I) of the materials which are normally transformed into products and which come from other production processes—the lumber in a furniture factory, the coke in a foundry, etc. Third, there are the input flows needed for maintaining capital equipment intact, (M)—lubricating oil, paint, parts, etc. Fourth, there is the output flow of products (Q). And, finally, there is the output flow of waste (W). With the correlative notations, expression (8) may be replaced by

$$[\underset{0}{\overset{T}{R}}(t),\ \underset{0}{\overset{T}{I}}(t),\ \underset{0}{\overset{T}{M}}(t),\ \underset{0}{\overset{T}{Q}}(t),\ \underset{0}{\overset{T}{W}}(t);\ \underset{0}{\overset{T}{L}}(t),\ \underset{0}{\overset{T}{K}}(t),\ \underset{0}{\overset{T}{H}}(t)]. \tag{9}$$

The natural factors of production have always offered a matter of disagreement to economists. The reason why my classification differs from that of the Classical school, which includes all these factors under "land," should by now be obvious. One may, though, object that, in view of the fact that I have associated the concept of fund with that of agent, it is inconsistent to place the Ricardian land—an inert element—in the fund category. However, I submit that the Ricardian land *is* an agent in the true sense of the term. Just as a net catches fish even if left by itself in the sea, the Ricardian land catches rainfall and, above all, solar radiation. Moreover, it is the only net that can do this. Had our planet a radius twice as great, we could catch four times as much of this most vital energy for man's existence. By and large, the scarcity of land derives from this role; the role of providing mere space is secondary. Conceivably, we could get more land-space by building acres on top of acres, but only those on the very top would be green acres.

One may also count on the objection that the inclusion of both the Ricardian land and the solar energy among the factors of production constitutes a double-counting. The point that the rain and the solar energy flow by themselves and hence are "free gifts of nature" is a familiar leitmotiv of all major doctrines of economic value. This is no reason, however, for omitting the natural factors from our scientific report of a process. The issue of what has and what has not value must not be prejudiced—as has generally been—by a trimmed-off representation of a process in *real* terms.[30] A glaring illustration of the danger of simplifying

[30] Among the authors of the main models of production, Koopmans ("Analysis of Production," pp. 37–39) is perhaps the only exception in that he lists the flow of resources from nature and the flow of land services among the coordinates of a productive process. The general trend follows the pattern of both the static and the dynamic input–output systems of Leontief in ignoring all natural factors. This is all the more curious since these systems are intended as instruments of *material* planning of production rather than abstract foundations for an analysis of value.

this representation is, again, provided by Marx. It is from Marx, I believe, that we have inherited the heresy that if the maintenance flow of, say, a bridge is included as a factor in the representation of a process using the bridge, then the inclusion of the bridge itself constitutes a double-counting that serves the interest of the capitalist exploiters. So eager was Marx to avoid the slightest suggestion of the idea that the services of capital proper may contribute to the value of the product something more than the value of the maintenance flow, that he painstakingly avoided any reference to services even in the case of the laborer. Instead, he used such veiling expressions as the work performed by a machine or the life activity of the worker.[31]

In the light of our flow–fund model Marx's tour de force lets itself be seen in detail and admired as well. It is beyond question that Marx started by viewing the worker as a fund.[32] Labor power—one of the many useful terms introduced by Marx—means "the aggregate of those mental and physical capabilities existing in a human being, which he *exercises* whenever he produces a use-value of any description."[33] In a plain incontrovertible manner the same idea is expressed by Engels: "Labor power exists in the form of the living worker who *requires* a definite amount of means of subsistence for his existence as well as for the maintenance of his family."[34] Marx, we should note, did not say in the cited phrase that the worker *consumes* his capabilities in production. Nor did Engels say that the services of labor power are in some precise sense *equivalent* to the maintenance flow required by the worker. Yet Marx, as he comes to the crux of his argument, suddenly introduces the equivalence by reducing the participation of the worker in a production process to a "definite quantity of human muscle, nerve, brain, etc., [which] is wasted" during work.[35] By this equivalence Marx simply covered up the plain fact that the worker participates in the production process with his entire stock of muscle, nerve, brain, etc. Nature is such that no instructor can discharge his duty by sending to class only that part of his nervous or muscular energy he usually spends during a lecture. And the reason for the im-

[31] E.g., Marx, *Capital*, I, 589, and his "Wage, Labor and Capital" in K. Marx and F. Engels, *Selected Works* (2 vols., Moscow, 1958), I, 82. In one place, *A Contribution to the Critique of Political Economy* (Chicago, 1904), p. 34, Marx did use the term "service" in arguing that the question in the exchange value "is not as to the service which it renders, but as to the service which it has been rendered in its production," only to follow this remark by a sneer at J. B. Say and F. Bastiat. The remark, obviously, gives him away.

[32] Cf. Marx, *Capital*, I, 189 f and, especially, 622.

[33] *Ibid.*, 186. My italics.

[34] Engels, "Marx's Capital," in *Selected Works*, I, 464. My italics. See also Marx, *Capital*, I, 189.

[35] Marx, *Capital*, I, 190.

possibility is elementary. To teach, an instructor must be present in class with all his labor power, i.e., with the aggregate of all his "mental and physical capabilities." A service must not be confused with a partial decumulation of one's stock of energy even if one insists on considering only the material factors of an economic process. If it were true that one can cross a river on the maintenance flow of a bridge or drive a maintenance flow of an automobile on the maintenance flow of a highway, there would be practically no financial problem in saturating the world with all the river crossings and automotive facilities. Economic development itself could be brought about everywhere almost instantaneously. These are the well-concealed implications of Marx's doctrine in which the main agent of the economic process—the human being—is degraded to a mere stock of energy for the sole purpose that the material means of production may also be denied the quality of agent.

7. *The Production Function.* During the foregoing discussion of how a process may be represented analytically, one question should have brewed up gradually. It is this: why is a production process represented in Neoclassical economics by an ordinary vector (in which every coordinate is a number) if, as I have argued, each coordinate in the analytical representation of a process is a function of time? The opening remarks of this chapter contain the only explanation of the discrepancy: economists, more so than other scientists, have treated the concept of process in a cavalier manner. The tone was set by P. H. Wicksteed as he sought to improve on Walras' treatment of production by introducing the general concept of production function: "*The product being a function of the factors of production we have* $P = f(a, b, c, \cdots)$."[36] Numberless others after him have made and still make the same swift passage from "function" understood in the broadest sense to the "point function" of mathematics. In addition, Wicksteed's presentation leaves us completely in the dark on what process means and why a process is represented by an ordinary vector $(P, a, b, c, \cdots)$. The situation even worsened after the vapid terms of input and output spread throughout the economic literature. At their best, the modern works liken the description of a process to a recipe from a cookbook, which in itself is a good starting point. But the sequel is rather a regress. According to his cookbook—we read—an iron manufacturer knows that if he "mixes so much ore, so much lime, so much coke, and so much heat for so many hours, [he will get so] much iron."[37] One is thus invited to read only the list of ingredients, which in cookbooks is usually

[36] Philip H. Wicksteed, *The Co-ordination of the Laws of Distribution*, (London, 1894), p. 4. Reprinted as No. 12 of the Scarce Tracts in Economic and Political Science.

[37] Kenneth E. Boulding, *Economic Analysis* (3rd edn., New York, 1955), pp. 585 f.

printed above the recipe proper, and ignore the rest. Obviously, the recipe being reduced to "that much of this" and "that much of that," the description of the process, too, is reduced to a list of quantities.

Once this result is reached, albeit surreptitiously, the concept of production function encounters no difficulties. As Samuelson views it,[38] the production function is a catalog of all recipes found in the cookbook of the prevailing state of arts for obtaining a *given* product out of *given* factors. And since each recipe now tells us only that we can obtain the quantity z of product by using the quantities $x, y, \cdots$ of this and that factor, the catalog itself is reduced to a point function.

$$z = f(x, y, \cdots). \tag{10}$$

To quote Boulding again, a "basic transformation function of an enterprise is its *production function*, which shows what quantities of inputs (factors) can be transformed into what quantities of output (product)."[39] In this short sentence, there is packed almost every misleading notion that surrounds the conception of process in the economic literature.

Yet Boulding's idea that the description of a process is a recipe is, as I have already said, a very fortunate one. Let us start again from it. First, we should clarify our thoughts on one point. One may speak vaguely of a recipe for making, say, tables. But there is a host of such recipes. Tables are made in the shops of cabinet makers; they are also made in small-scale or large-scale industrial plants. At times, they are made out of dressed lumber and wood panels, at others out of raw lumber, and at others out of living trees. Whatever the case, I propose to consider that recipe which describes the partial process by which one table considered by itself is produced in each particular system of production.[40] I shall refer to such a partial process as an *elementary process*, on the ground that every production system of any type whatsoever is a system of elementary processes. In the shop of a cabinet maker the elementary process by which a piece of furniture is produced develops unclouded in front of our eyes. But even in a more complex system, it can be isolated if one follows the rules outlined earlier for drawing the boundary and recording the analytical coordinates of a partial process. The point is that the concept of elementary process is well defined in every system of production. In fact, it should not

[38] Samuelson, *Foundations*, p. 57.

[39] Boulding, *Economic Analysis*, p. 585.

[40] For products such as "gasoline" or "steel," the elementary process may be associated with a molecule or, better, with a "batch" appropriately chosen to fit the concrete conditions in each case. Even for bread, we may associate the elementary process with a batch of loaves, the number of loaves being determined by the capacity of the oven, for example.

be difficult to reconstruct it by an attentive examination of all the orders issued by the production manager.

Nothing need be added now to what has been said in some previous sections in order to see that precisely the process described by a cookbook recipe cannot be completely represented by an ordinary vector. Only an expression such as (9) can represent it completely. A catalog of all *feasible* and *nonwasteful* recipes then consists of a set of points in an abstract space, as opposed to Euclidean space. The set may be regarded as a variety within the abstract space and, hence, represented by a relation of the form

$$(11)\qquad \overset{T}{\underset{0}{Q(t)}} = \mathscr{F}[\overset{T}{\underset{0}{R(t)}}, \overset{T}{\underset{0}{I(t)}}, \overset{T}{\underset{0}{M(t)}}, \overset{T}{\underset{0}{W(t)}}; \overset{T}{\underset{0}{L(t)}}, \overset{T}{\underset{0}{K(t)}}, \overset{T}{\underset{0}{H(t)}}],$$

which in mathematical jargon is called a *functional*.[41] This is a relation from *a set of functions to one function*. Consequently, (11) is a far cry from the Neoclassical production function (10), which is a point function, i.e., a relation from *a set of numbers to one number*. Yet there is a connection between (11) and (10). To unravel it and to make it explicit is our next task.

8. The Economics of Production. All elementary processes have one important feature in common. In relation to any given elementary process most of the fund factors involved in it must remain idle during a great part of the production time. This idleness, it should be emphasized, is not the result of our own fault or wish. It is an unavoidable consequence of the material conditions of the process itself. A superficial observation of a cabinet maker at work should suffice to convince us of the general validity of this truth. The saw, the plane, the sander, etc., are never used simultaneously in the production of a table considered by itself. Every tool is used by turns; in the meantime it lies idle. Should there be specialized workers—say, one specialized in operating the saw, another in applying varnish, etc.—they, too, would be idle by turns in relation to every elementary process. Moreover, all tools and all workers are idle (in the same sense) during the time when the varnished table is set out to dry. During this phase, nature is the silent partner of man, its forces operating through some flow elements included under (R). A flow input of oxygen from the air oxidizes the varnish while the varnish solvent evaporates as an output flow. All these facts are even more conspicuous in a farming

[41] The fact that the functional does not exist for every point $R, I, \ldots, H$ may well be ignored at this juncture. But we should note that, since the functional represents an elementary process, we have $Q(t) = 0$ for $0 \leq t < T$ and $Q(T) = 1$ with unity standing for the unit of product associated with the elementary process.

process, but they are part and parcel of any elementary process, be it in manufacturing, mining, construction, or transportation.

Another important observation is that if the flow of demand is such that only one table is demanded during an interval equal to or greater than the time of production, the production of tables has to be carried out by partial processes arranged *in series*, i.e., in such a way that no process overlaps with another in time. This was the case of every craft shop in older times and is now the case for canals, bridges, large maritime ships, and so forth. New plants also are ordinarily produced in series. The point to be retained is that a low intensity of demand imposes on most fund elements long periods of idleness. The human factor can find employment only by shifting periodically to other lines of production—as thousands of peasants do by seeking employment in the cities during the idle periods on the farm. But this seasonal employment also falls back on the existence of some demand. Besides, not all partial processes include sufficiently long periods of idleness to make the shift operative. We can understand now the reason why, as long as the demand for most manufactured goods remained at a very low level, specialization of tools and especially of labor was uneconomical. The craftsman of the Middle Ages, for instance, had to know how to perform all the tasks required by the elementary process of his trade. Otherwise, he would have had to remain idle part of the time and share with others the revenue accruing to labor. Under such conditions, specialization was *uneconomical*.

The case in which more than one table is demanded during an interval equal to the duration of the elementary process leads to two alternatives.

First, production may be carried out by the appropriate number, n, of elementary processes set *in parallel*, i.e., started at the same time and repeated after they are completed. In many cases the resulting system is a typical case of processes that are added externally.[42] To describe it we need only multiply every coordinate of the elementary process in point by n. The corresponding production function is then easily derived from (11):

$$(12)\qquad [n\overset{T}{\underset{0}{Q(t)}}] = \mathscr{F}\{[n\overset{T}{\underset{0}{R(t)}}], \cdots, [n\overset{T}{\underset{0}{W(t)}}]; (n\overset{T}{\underset{0}{L(t)}}], \cdots\}.$$

The point that deserves to be stressed is that the arrangement in parallel offers little or no economic gain. Most kinds of fund factors are now needed in an amount n times as great as in the elementary process. In addition, the idleness of each such fund factor is *ipso facto* amplified by n. The only exceptions are the fund factors that—like a large bread oven, for

[42] For which see Chapter IV, Section 5.

instance—may accommodate several elementary processes simultaneously. But even though the capacity of such a fund factor would be more fully utilized, its idleness period would remain the same.

The second alternative is to arrange the appropriate number of processes *in line*. In this system, the time of production is divided into equal intervals and one elementary process (or a batch of such processes) is started at each division point. In more familiar language, the elementary processes are uniformly staggered in time. There is no need to go here over the mathematical proof—which, in fact, is quite simple—of the following proposition:

If the number of the elementary processes is sufficiently large and all periods during which each fund factor renders service are commensurable with the time of production, then there is a minimum number of elementary processes that can be arranged in line so that every fund factor is continuously employed.[43]

In plain words, the proposition says that if the demand for a product is sufficiently large, then production may be arranged so that no fund factor employed in it is ever idle. Obviously, this arrangement represents the factory system, where every tool and every worker shifts from one elementary process to the next as soon as they have performed their services in the first. No tool and no worker is thus idle during the time when the process of the whole factory goes on.

9. The Factory System and Its Production Function. To arrive at the analytical representation of the process consisting of a factory system we have simply to follow our basic rule of starting with one *tabula rasa* and ending with another and observe the distinction between flow and fund elements. The duration of the process to be described may be chosen arbitrarily: a factory system once set in order is a steady state in which all funds are kept in good working conditions at all times. However, to simplify the notational scaffold I shall make the perennial assumption of continuity, i.e., I shall assume that a batch of elementary processes is started at each instant of time and, by necessity, all flow elements are continuous entities. In this case, it is straightforward that the flow coordinates are homogeneous linear functions of t:

[43] For a diagrammatical illustration of this proposition, see Fig. 2 in my article "Process in Farming vs. Process in Manufacturing," cited in note 8, above. But because the point is related to some aspects of size, it deserves to be made more explicit. The number of elementary processes to be started at each division point is the smallest common multiple of the numbers of such processes that can be accommodated at the same time by each unit of the various funds. The intervals between two consecutive batches is T/d, where d is the greatest common divisor of T and of the intervals during which the various kinds of funds are needed in an elementary process.

$$R(t) = rt, \quad I(t) = it, \quad M(t) = mt, \quad Q(t) = qt, \quad W(t) = wt. \tag{13}$$

The same is true of the fund coordinates. But to render our representation more discriminating we need to set apart two new categories of capital funds. The first includes the stores of commodities, the inventories in the narrow sense, which are related to some (ordinarily, all) flow elements included under I, M, Q, and W. The real role of these stores is to dampen the irregular fluctuations in the number of accidents in the process of production and in the rhythm of sales. One must have on hand a certain number of fuses so that even if many fuses happen to blow at the same time they can be replaced without delay. A certain quantity of each product must also be stored in order to take care of any fortuitous concentration of orders.[44] Let us denote this category, generically, by S.

The second new category of capital funds corresponds to the familiar term of "goods in process." But "goods" is here a patent misnomer: melted glass, half-tanned hides, half-wired radio sets, for example, can hardly fit the term. It is nevertheless true that at any time there exists inside the factory system a *process-fund*, $\mathscr{C}$, in which is mirrored the entire transformation of the material inputs into the final products. The time of production of, say, an aircraft may be several months or even a couple of years. But in the process-fund of a factory producing such an aircraft there must exist at any time at least one "aircraft" in each phase of its transformation. If we take one photograph of each successive phase on the same film roll, and then project the film as if it were a movie, we will indeed see a movie—a movie showing how one aircraft is made out of metal sheets, motor parts, cables, etc. The whole qualitative change—a Becoming—is thus frozen, as it were, into a time-less Being—the process-fund. This fact explains my choice of the term.

Let us also note that without the *process-fund* no factory is complete. The role of the process-fund is fundamental. It can be likened to the water in the vertical pipe of a hand pump. Unless the water fills that pipe, the pump is not primed; we must work the pump for some time before we can get any water. If, on the contrary, the pump is primed, water starts to flow the minute we move the pump's handle. In a factory, too, the outputs included under Q and W begin to flow out the moment the factory opens in the morning and the inputs R, I, M begin to flow in. This is possible only because a factory at closing time is left in a primed state, with its $\mathscr{C}$ intact. The continuous maintenance of tools and buildings requires, we remember, some special assumptions. By contrast, the process-

[44] It is clear then that speculative inventories are left out of account—as they should be in a description of the purely material process of production.

fund is maintained by the very manner in which the elementary processes are arranged in a factory system.[45]

Looking at a factory from the outside, as the flow complex does, one would certainly see only the flow coordinates (13). Moreover, one may very well say that production is instantaneous, i.e., that a batch of input materials is instantaneously transformed into a batch of products. What happens in fact is rather similar to what happens when we push the end of a perfectly inelastic rod: the other end moves at the very same instant. In the case of a factory, the process-fund plays the role of such a rod. In all probability, it is this peculiar property of a factory system that has led some economists to argue that, since production is instantaneous, wages are always paid out of the product, never out of capital. The waiting doctrine of capital would thus be baseless. Of course, once a factory is built and primed, there is no longer any waiting. But both to build and prime a factory require duration. Only to prime an aircraft factory, for instance, we may have to wait several months.

The fund coordinates of a factory system being

(14) $$L(t) = Lt, \quad K(t) = Kt, \quad S(t) = St, \quad \mathscr{C}(t) = \mathscr{C}t, \quad H(t) = Ht,$$

its analytical representation is now complete. And according to what has been said earlier, the production function of a factory process—the catalog of all factory processes by which the *same* products can be obtained from the *same* factors—is a functional involving all functions listed in (13) and (14):

(15) $$\overset{T}{\underset{0}{(qt)}} = \mathscr{G}[\overset{T}{\underset{0}{(rt)}}, \cdots, \overset{T}{\underset{0}{(wt)}}; \overset{T}{\underset{0}{(Lt)}}, \cdots, \overset{T}{\underset{0}{(Ht)}}].$$

In contrast with the functionals (11) and (12), where T is a physical coordinate determined by the nature of the elementary process, in this last functional T is a freely varying parameter. The consequence of this fact is that the relation expressed by (15) boils down to a relation only between the coefficients of t in the functions (13) and (14). In other words, the functional in this case *degenerates* into an ordinary point function, namely, into

(16) $$q = F(r, i, m, w; L, K, S, \mathscr{C}, H).$$

Alternatively, the same functional may be replaced by a point function

[45] A factory system is like a music box, which starts to play the moment it is opened and stops playing the moment it is closed. Of course, if laid idle for a long period of time, any factory would need some additional work to remove the damage done by the Entropy Law.

between the amounts of flows and services over an arbitrary time interval t. *But in this function t must appear as an explicit variable*:

(17) $$qt = \Phi(rt, \cdots, wt; Lt, \cdots, Ht; t).$$

We should not fail to note that, in contrast with the function F of (16), Φ is a homogeneous function of the first degree with respect to *all* its variables, that is, including t.[46] Obviously, we have the identity

$$\Phi \equiv tF.$$

We can see now the thought which, possibly, may have guided the Neoclassical economists who, in the past as well as now, represent any production process by the jejune formula (10) about which we are told only that the dependent variable stands for "output" and all other symbols stand for "inputs". No wonder then that economists took liberties with the interpretation of these terms—some defining the production function as a relation between the quantity of product and the quantities of inputs, others as a relation between the output of products per unit of time and the input of factors per unit of time. Some have even adopted the two definitions within the same work. As the analysis developed in the foregoing sections clearly shows, once the production function is defined as a catalog of recipes, its formula cannot be decreed by our whims—reasonable though they may seem on the surface.[47] The production function is always a functional, either (11), or (12), or (15), according to the system in question.

That in the case of a factory we should prefer (16) to the pseudo functional (15) is perfectly natural. Yet (16) looks like a black sheep amidst the flock of other functionals: in contrast with them, it does not involve the time element. In the process of passing from the degenerate functional to the point function (16), we have let the time element slip through our analytical fingers. As a result, the production function (16) does not tell us what the corresponding system *does*, but only what it *may do*. The variables involved in it consist only of the rates of the flow factors and the sizes of the fund factors. They describe the process in the same manner in which the inscription "40 watts, 110 volts" on an electric bulb or "B.S. in Chemical Engineering" on a diploma describe the bulb or the engineer. Neither description informs us how long the bulb burnt

[46] This homogeneity expresses the trivial truth that the flows and the services of any factory during, say, eight hours are eight times as great as during one hour. Clearly, it has absolutely no bearing on whether there is an optimum size of the unit of production or not. See also note 48, below.

[47] For further details, some apparently so surprising that they were denounced as false on the spot when I first presented them, see my articles cited in notes 1 and 8 above.

yesterday or how many hours the engineer worked last week. Similarly, (16) may tell us that a man with a 100 hp tractor which uses three gallons of gasoline and one quart of oil per hour can plow two acres per hour.

It stands to reason then that what a factory is capable of doing is a function of its purely technical structure alone. The point is that a competent person should be able to determine from the blueprint of a factory what the factory can do and also what it needs for this. Consequently, the production function (16) may be decomposed into several elements which together constitute a more faithful picture of the factory process. The first two elements are

$$q^* = G(L, K), \qquad H^* = H(L, K), \tag{18}$$

where q^* represents the maximum rate of product flow of which the factory is capable if properly manned with H^*. However, the human element being as variable as it is in actuality, q^* is rather an unattainable limit. To take into account that the actual rate of product flow depends on the quality as well as the size of the personnel employed, we have to replace (18) by

$$q = f(L, K, H) \leq q^*. \tag{19}$$

This relation should not, however, be confused with the form currently used in theoretical and applied works. As defined by (19), if $q < q^*$, q need not (and usually does not) decrease if either L and K are decreased while H is kept constant.

The other fund factors, S and $\mathscr{C}$, are also determined by the same basic funds, L, K, H. Hence, we have

$$S = S(L, K, H), \qquad \mathscr{C} = C(L, K, H). \tag{20}$$

There remains to examine the relations binding the other flow elements. The case of the maintenance flow is easily settled. Its rate must be a function of the capital to be maintained and of the labor fund. Moreover, by virtue of the Conservation Law of matter and energy, it must be equal to the flow rate of wear-and-tear waste, w_1. We are thus led to put

$$m = m(K, H), \qquad w_1 = m. \tag{21}$$

These relations take care also of the fact that the capital proper may be more intensively or less intensively used according to the size of the manpower employed.

According to the same Conservation Law of matter and energy, there must exist in each case some relation between the other input and output flows:

$$qt = g(rt, it, w_2t), \tag{22}$$

where w_2 denotes the flow rate of that waste which arises only from transformation. Since (22) must be true for any positive value of t, the function g must be homogeneous of the first degree. This result may be reached also by a familiar argument: double the amounts of timber, of impregnating material, *and of waste*, and the amount of railroad ties will necessarily double, too.[48] However, to double q, we need another factory, i.e., another fund combination (L, K, H). With this new combination, the amount of waste may not be (and usually is not) doubled. This is precisely one meaning of the statement that one technical recipe is more efficient than another: the value of w_2 is smaller for the more efficient recipe. We are thus led to put

$$w_2 = w_2(L, K, H). \tag{23}$$

Relation (22) then becomes

$$q = g[r, i, w_2(L, K, H)]. \tag{24}$$

To sum up, the catalog of all factory systems that produce the same products with the same factors (flow or fund) consists not of one, but of seven basic functions, listed as (19), (20), (21), (23), and (24) in this section. There are therefore some definite limitationalities inherent to the structure of production by the factory system. Technical features peculiar to each process may introduce additional binding relations between factors. We may recall the customary examples of gold in the production of wedding rings and of a tractor needing only one driver. But these special cases apart, we must not jump to the conclusion that the factors included in any of the point functions representing the catalog of the factory recipes for a given product are substitutable in the sense assumed by the current theory of production. To recall, in these point functions K represents generically capital equipments of various qualities, K_i meaning a certain amount of capital of "quality i." The same applies to L and H. Moreover, there is not necessarily a process corresponding exactly to every possible combination (L, K, H). A more capital intensive process normally requires a different type of capital. Therefore, if we consider a given process, there may be no process corresponding to the substitution of more of K_i for a decrease in H_j. Substitution means rather that K_c and L_d are used instead of K_a and L_c. And if this is the case, substitution

[48] It may be well to point out that this argument does not imply that there is no optimum size of a factory, although those who have argued against the existence of the optimum size may have been influenced by it. The absence of the optimum size requires that the functions in (18) be homogeneous of the first degree. Cf. Chapter IV, Section 4, above, and my article "Chamberlin's New Economics and the Unit of Production," cited in note 1, above.

cannot be represented in terms of two coordinates—one representing "capital," the other "labor"—as is done in the familiar map of isoquants. Neoclassical economists, after censuring Marx for his idea that every concrete labor is a congealed form of general abstract labor, returned to their own shop to outdo him in this very respect by assuming that concrete capital, too, is congealed abstract capital.

As a highly abstract simile, the standard form of the Neoclassical production function—as a function of K, the cardinal measure of homogenous "capital," and H, the cardinal measure of homogeneous "labor"—is not completely useless. But, in sharp contrast with the ophelimity function (where substitutability is a result of the individual's subjective weighing), the value of the standard form of the production function as a blueprint of reality is nil. It is absurd therefore to hold on to it in practical applications—as is the case with the numberless attempts at deriving it from cross-section statistical data. The K_i in these data are not all qualitatively identical and, hence, have no common measure. For the same reason, there is no sense of speaking of the elasticity of substitution between homogeneous capital and homogeneous labor. Marginal productivity, too, comes out as an empty word. True, capital and labor may be rendered homogeneous but only if they are measured in money. All this shows that the theorems which adorn the theory of marginal pricing are in essence misleading analytical ornaments. In fact, to explain the adaptation of production to prices, whether in the case of a factory or any other arrangement of elementary processes, we do not require the existence of either Neoclassical substitutability or marginal physical productivity. Such an adaptation is secured regardless of the number and the nature of the limitationalities a production function may contain.[49] Cost is the only element that counts in this problem. And in cost, all qualitative differences between factors vanish into one homogeneous entity, money. The only role the production function (as developed above) has in this particular case is to enable us to know what factors, and in what amounts, enter into the cost of every possible factory process. As I have argued elsewhere, E. H. Chamberlin's "idea of analyzing the problem of optimum scale with the aid of a diagram of a family of average cost curves seems far more promising than using the production function and its isoquants—however more respectable the latter approach may be."[50]

10. Production and the Time Factor. I have already underscored the

[49] For which see my article reprinted as Essay 7 in AE.

[50] See my article "Measure, Quality, and Optimum Scale" in *Essays on Econometrics and Planning Presented to Professor P. C. Mahalanobis* (Oxford, 1964), p. 255.

fact that the basic relation (19) does not tell us what a factory does. To describe what a factory did yesterday or what it does every day, we need an additional coordinate, not included in (19). This coordinate is the time during which the factory works each day. We may refer to this time interval as the working day of the factory and denote it by δ. If the ordinary day is taken as time unit, then $\delta \leq 1$. The daily output of a factory, $Q = \delta q$, follows straightforward from (19):

$$Q = \delta f(L, K, H). \tag{25}$$

This formula is again a far cry from the Neoclassical production function (10), which does not contain the time factor as an explicit variable. I can foresee that this statement may be questioned on the ground that in the Neoclassical formula the symbols stand for *quantities* of flows and services, and thus the time factor is not ignored. Many economists have indeed proclaimed on intuitive grounds that the production function is a relation between quantities.[51] But their intuition has failed to perceive one point which is made so obvious by the analysis in the foregoing section. The relation between the quantity of product and the quantities of flows and services must include time as an explicit variable—as in (17). The conclusion is that no matter what position we consider—whether the symbols in (10) represent rates of flows and services or represent amounts of flows and services—the Neoclassical mode of representing the production function ignores the time factor.

This is a regrettable, albeit understandable, regress from Marx's analysis of the production process in which the time factor—whether as the time of production of what I have called an elementary process or as the working day of the worker—occupies a quite prominent place.[52] Marx looked for every analytical element that may evolve historically. The Neoclassical school, on the contrary, planned to ignore the march of history. Indeed, the most favorable excuse for the omission of the working day from the formula intended to describe what factories do is that the Neoclassical economists regard δ as a given social coordinate. Being a given coefficient, δ does not have to appear explicitly in a general formula any more than any other physical coefficients.

This excuse does not alter the fact that the consequences of the omission of the factory's working time from the standard analytical apparatus are more complicated than one would like to think. Some are aggravated by

[51] See note 39, above.

[52] Cf. Marx, *Capital*, vol. I, ch. x. Incidentally, formula (25) lends support to one dearest tenet of Marx's, namely, that labor time, though it has no value itself, is a measure of value. *Ibid.*, pp. 45, 588.

another fault of the same apparatus—the confusion of (19) with (18). This confusion is tantamount to another omission, that of neglecting the intensity of capacity utilization, which is measured either by q/q^* or by H/H^*. Both omissions seriously vitiate the studies in which the argument involves the capital-output or the capital-labor ratio and which have been rendered highly popular by some of the highest economic authorities as well as by such respectable institutions as the National Bureau of Economic Research.

In the light of the preceding analysis, it is clear that an objective definition of capital intensiveness in a factory process must be grounded in relations (18). Hence, K/q^*—alternatively, K/Q^* where Q^* corresponds to q^* and $\delta = 1$—and K/H^* constitute the only objective measures of capital intensiveness. The point deserves unparsimonious emphasis: capital intensiveness is essentially a coordinate of the factory's blueprint, not of what a factory happens to do. It would be a gross mistake to measure capital intensiveness by the ratio $K/Q = K/(\delta q)$: the daily (or the annual) production, Q, varies both with δ and the intensity of capacity utilization. The same applies to the capital-labor ratio measured by K/N, where N is the *average* number of employees (or of the production workers) over the year: N varies with both the intensity of capacity utilization and the number of shifts. Clearly, if *ceteris paribus* a factory passes from using one shift to using two shifts of the same size, K/N would be halved, even though the capital intensiveness of the process has not changed. The ratios K/Q and K/N, therefore, are affected by the working day of the factory, the number of shifts, and the intensity of capacity utilization. These coordinates, in turn, fluctuate according to the momentary business outlook in the corresponding line of activity. The moral is that any comparisons of the ratios K/Q or K/N, either between one year and another for the same industry or between two industries for the same year, do not necessarily reflect a change in capital intensiveness. This is especially true of interindustrial comparisons.

Yet, to my knowledge, all studies concerned with capital intensiveness use these last measures of capital-output and capital-labor ratios. And even though one finds occasional mention of some possible reason for the noncomparability of these measures, I know of no author to insist on all the implications of the problem of measuring capital intensiveness. The curious thing is that, had anybody seen that the correct measures are K/q^* and K/H^*, he would have not been able to arrive at a statistical estimation of these ratios. The reason is that even the most sophisticated statistical agencies do not include in their censuses of manufactures the data required for deriving these last ratios from the ordinary data on production, capital, and employment. Nothing more normal for a statis-

tical bureau than to orient its data collection according to the inventory of the tool box of the analytical social scientist.[53]

Now, the consequences of the fact that the elements mentioned in the preceding paragraphs have been omitted from the analytical tools of the Neoclassical economist are not confined to purely academic matters. The omission of the length of the working day, δ, is responsible, I believe, for the strange fact that no Neoclassical planning expert seems to realize that, as correctly assessed by Marx and confirmed by (25), one of the "secrets" by which the advanced economies have achieved their spectacular economic development is a long working day.[54] The length of the working day, although an economic lever that can be used directly and without delay, is not a coordinate in any Neoclassical model of economic development found in the general literature and, probably, in any other. In view of our loudly proclaimed aims, to help the underdeveloped economies not only to make progress but to make *rapid* progress, the legal regimen of the eight-hour day in such economies (even in those where overpopulation brings about unwanted leisure) is a patent incongruity, if not a planned anachronism as well.

Were we in the situation in which there were enough manpower to keep all factories working around the clock by four, six, or even twelve shifts, there would be no economic objection (besides the cost of changing shifts) to have a six-hour, a four-hour, or a two-hour day. But what underdeveloped economy, nay, what economy is in this position? The basic shortage in underdeveloped economies—as we have finally come to realize recently[55]—is capital in both its forms: *machines and skilled labor*. The two go together simply because skilled labor is a package of labor *and* skill and because skill is akin to capital: it takes time to acquire it.[56]

[53] An excellent example is supplied by the epochal impact the Leontief system had on the collection of statistical data pertaining to interindustrial transactions.

[54] We need not rely only on the relation by F. Engels in his *The Condition of the Working Class in England in 1844* (London, 1892). According to W. S. Woytinsky and Associates, *Employment and Wages in the United States* (New York, 1953), p. 98, in the United States as late as 1850 the average working week was seventy hours. The first attempt to limit the work of children under twelve to a ten-hour day was made only in 1842 by the Commonwealth of Massachusetts. The ten-hour day did not become a widespread rule for the other workers until 1860. See Philip S. Foner, *History of the Labor Movement in the United States* (4 vols., New York, 1947), I, 218, and G. Gunton, *Wealth and Progress* (New York, 1887), pp. 250 f.

[55] E.g., Theodore W. Schultz, *The Economic Value of Education* (New York, 1963).

[56] Strangely, this last point has been long ignored by those who opposed the idea that in many countries overpopulation is a reality which requires an economic handling different from that prescribed by Neoclassical economics. Cf. my article "Economic Theory and Agrarian Economics" (1960), reprinted in AE, pp. 372–374.

Ordinarily, the shortage of skilled labor in underdeveloped economies is so acute that many a factory cannot be worked around the clock. In this case, the idleness of the inert factors would not be inherent to the physical nature of the process itself—as in the case of an elementary process—but to the shortage of their human companions in work. To set the same legal limit to the working day as in the advanced economies—where, thanks to the abundance of capital, leisure has economic value—is tantamount to decreeing an unnecessarily high amount of idleness and a cut in the potential income of the country. For the same reason, any factory built in an underdeveloped country in addition to any other producing the same product and operating with only one or two shifts of eight hours each is a waste of resources. If there is already, say, a shoe factory which works with only one shift, it makes no economic sense to build another shoe factory also operated by one shift. The two shifts can produce the same output (practically) with the old factory, and the additional capital can be invested in another line to support further growth.

"Economic Development Takes Time" would make a very appropriate inscription above the entrance of every economic planning agency, so that the passers-by be continuously reminded of the bare truth, however disappointing. But inside every office the inscription should read "Do not make this time longer by unnecessary idleness." For unnecessary idleness results in a waste of time. I am convinced that all economic plans harbor, in a larger or smaller measure, idleness unconsciously planned. No wonder we feel or even recognize occasionally that most plans of economic development have not been speedy enough. Perhaps all this could be avoided if in planning economic development we would bear in mind the economic object lesson of the factory system.

11. The Factory: Its Advantages and Limitations. A factory is such a familiar object in the industrialized world in which most economists have been reared that we seem to have lost sight of two important facts.

The first fact is that the factory system is one of the greatest *economic* inventions in the history of mankind—comparable only to the invention of money but just as anonymous in origin. The word "economic" should indeed be underscored, because the advantages of the factory system are independent of technology and also above it. We may be told that the factory system was a creation of the industrial revolution, that is, of the mass of technological innovations of the eighteenth century and thereafter. In my opinion, the causal relationship is the reverse: the factory system, which had already begun to be practiced in the old craft shops because of an increased demand, was one of the main factors that spurred the technological innovations.

The factory system, as the preceding sections amply attest, is superior

to all other arrangements of the elementary processes, not because it increases the power of a tool or the command of man over natural forces, but because it does away with the idleness of the fund factors which is inherent to any recipe. And the gain is availing whatever the technology may be: cloth could be produced by a factory system using the technique of the Egyptians in the time of the Pharaohs. Whether we can take advantage of this gain depends, not on the technology available, but on the level of the demand for the product under consideration. To wit, if transoceanic Queens are not produced under a factory system, i.e., in line, it is only because they are not demanded fast enough in relation to the time of production. Strange though it may seem, if the technology in shipbuilding were still that of a hundred years ago, we might be building Queens in line provided the demand for them would still be what it is today. In some cases, therefore, technological progress may work against the factory system if the demand does not increase in step with it.

The upshot is that the intimate connection which undoubtedly exists between the factory system and technological progress involves mainly the work of demand. Just as a low intensity of demand renders uneconomical any specialization, so an increase in demand paves the way for further specialization. The point is easily proved by observing that if a particular task of an agent in an elementary process is divided into several distinct tasks, the number of elementary processes needed for an arrangement in line without any idleness generally increases (and rather sharply). The output flow, therefore, must also increase. If the demand flow does not increase in the same proportion, specialization would only result in costly idleness.[57]

The role of demand as a stimulant of technological innovation is seen even in those cases in which, for some reason or other, the elementary processes have to be set in parallel. To wit, as the demand for bread in a small community increases, the baking industry may find it economical to replace the ovens used daily in parallel by a larger oven instead of adding more ovens of the same size. Actually, technological progress has always consisted of a blend of specialization *and* concentration of several tools into one unit of a larger but more efficient capacity. In both cases, the result has been an increase in the size of the unit of production. The limits beyond which this size cannot go are set by the laws of matter, as

[57] As we all know, it was Adam Smith who first argued that "the division of labor is limited by the extent of the market." *The Wealth of Nations* (ed. Cannan), I, 19. But the analysis of the factory system in Section 9 and especially the theorem of note 43, above, set this proposition on a clear foundation and also extend it to the specialization of capital equipment as well.

we have seen in Chapter IV, Section 4, but what stimulates the increase is a growing demand alone.[58]

The second fact of which we often lose sight is that the factory system cannot be applied to everything man needs or wants to produce. We have already seen that one obstacle is a low demand for some commodities. Another, more subtle, reason is related to the fact that normally *we produce not only commodities but also processes*. Only in a stationary economy is production confined to commodities. Because in such an economy every extant process maintains itself, none needs to be produced. But in a changing world we must also produce new processes, in addition to those that exist or in place of those that have become obsolete. And it stands to reason that it is impossible to produce all these processes by factory systems. At least the factory producing a new type of factory must be produced anew, that is, not by an existing factory. A third reason, the most relevant of all for the actual world, has its roots in the conditions of human life on this planet.

12. The Factory and the Farm. In order to arrange the elementary processes in line uninterruptedly, it is necessary that we should be able to start an elementary process at any moment in Time we may please. In a great number of cases we can do so. A hobbyist, for instance, is free to start his project of making a desk at three o'clock in the morning, on a Monday or a Friday, in December or August. Without this freedom the production of furniture, automobiles, coke, etc., could not go uninterruptedly around the clock throughout the year in factory systems. By contrast, unless one uses a well-equipped greenhouse one cannot start an elementary process of growing corn whenever he may please. Outside a few spots around the equator, for every region on the globe there is only a relatively short period of the year when corn can be sown in the fields if one wants a corn crop. This period is determined in each place by the local climatic conditions. These, in turn, are determined by the position of our planet and its rotation in relation to the sun as well as by the geographical distribution of land and water on the surface of the globe. So vital is the dependence of terrestrial life on the energy received from the sun that the cyclic rhythm in which this energy reaches each region on the earth has gradually built itself through natural selection into the reproductive pattern of almost every species, vegetal or animal. Thus, lambs are born in the spring, chickens hatch in early spring, calves are

[58] Because of this connection between demand and the technological recipes, I take exception to the view, shared by many of my fellow economists, that for the economic theorist the production functions are given data "taken from disciplines such as engineering and industrial chemistry." Stigler, *Theory of Competitive Price*, pp. 109 f. See also Pigou, *Economics of Stationary States*, p. 142; Samuelson, *Foundations*, p. 57; J. R. Hicks, *Theory of Wages*, p. 237.

born in the fall, and even a fish such as the turbot is not worth eating unless caught in April or May. So, in husbandry too an elementary process cannot be started except during one specific period dictated by nature.

These facts are commonplace. Yet the general tenor among economists has been to deny any substantial difference between the structures of agricultural and industrial productive activities. In the socialist literature of the past this fact was unmistakably reflected in the claim that under socialism the backward farms will be replaced by "open-air factories." In the Neoclassical literature the production function (10) is used regardless of whether the problem at hand refers to agricultural or industrial activity.[59] The elementary processes in agricultural production, however, cannot be arranged in line without interruption. True, if we view a corn plant as a unit of product, the elementary processes are arranged in line as the plowing and the sowing go on. The rub is that this line cannot go on forever: there is a point in time after which no seed sown will mature properly into a plant. In order that all the corn fields in a climatic region be cultivated in time, farmers have to work their fields *in parallel*. In view of the short length of time during which the field of a single farm is plowed, seeded, weeded, or harvested, it is quite safe to describe the production system of each individual farm by assuming that all elementary processes are started at the same time. With this convenient simplification, the production function of a farm system is the *nondegenerate* functional (12) of Section 8, above.[60]

Again, the difference between this production function and that of the factory—the point function (16) or (17)—is not a mere academic nicety. On the contrary, it teaches us some important economic questions. Long ago, Adam Smith argued that "the improvement of the productive powers of labor [in agriculture] does not always keep pace with their improvement in manufactures."[61] The proposition led to the controversy over the difference of returns in agriculture and industry and thus failed to be retained in modern economic thought. However, the foregoing analysis

[59] Actually, in no other economic field are so many studies confined to merely fitting a production function—usually, the perennial Cobb-Douglas type—to some particular product in a particular region as in agricultural economics.

[60] For completion, I may add that there are other activities besides agriculture which are subject to the rhythm of the climate: hostelry in tourist resorts comes immediately to mind, and so does construction. Most of what can be said about cost of production in agriculture applies *mutatis mutandis* to such activities, too. Thus, if you happen to arrive in Oslo and find no room to your liking, do not blame the Norwegians for not building more or bigger hotels for tourists. Such hotels would be idle during ten months each year, so short is the tourist season there. Only a millionaire can afford the waste of a villa on the French Riviera which he occupies only a few days each year, if at all.

[61] *The Wealth of Nations*, I, 8.

reveals one of the deep-seated reasons why the proposition is true even in a stronger form. One reason why technological progress has, by and large, proceeded at a slower rate in agriculture is that agricultural elementary processes cannot be arranged in line.[62] Curiously, the association of agricultural activity with an appreciable amount of labor unemployment is a fact accepted even by those who challenge Adam Smith's proposition. But our analysis not only shows why this association is inevitable, but also brings to the surface some interesting aspects of it.

There is one important difference between industrial and agricultural unemployment. An idle industrial worker is free to take a job and stay with it. A farmer even when idle is still tied to his job. If he accepts a regular job elsewhere, he creates a vacancy on the farm. Only in the case of overpopulation are there villagers unemployed in the strict sense of the term. But the inherent *idleness* is present wherever agricultural production is a system of processes in parallel—overpopulation or no overpopulation.

To do away with unemployment proper is a difficult but not an intricate task. However, to do away with agricultural idleness is a well-nigh insoluble problem if one stops to think about it in detail. For should we try to find different agricultural activities which, if spliced, would completely eliminate the idleness of the farmer *and* his implements, we will discover an insuperable obstacle. Nature, as the silent partner of man, not only dictates to man when he should start an agricultural process, but also forbids him stopping the process until it is completed. In industry we can interrupt and start again almost any process whenever we please, but not so in agriculture. For this reason, trying to find agricultural processes that would fit exactly in the idleness periods of one another is a hopeless enterprise. The "romantic" Agrarians had their feet on the ground after all as they insisted on the beneficial role of the cottage *industry* as a complementary activity in underdeveloped agricultural economies. But even with cottage industries that would splice perfectly with the idleness periods of the human capital employed in agricultural activities, the capital proper would still remain idle over large intervals of time. The conclusion may be surprising, but it is inescapable. The predicament of agriculture as an economic activity is *overcapitalization*. Nothing need be added to see that this predicament holds the key to a rational economic policy for any underdeveloped agricultural economy.[63]

[62] An even more important reason for this difference will be discussed in Chapter X, Section 3, below.

[63] In such economies, overcapitalization is often aggravated by a land distribution such that the size of most farms is smaller than the optimum. Cf. my article "Economic Theory and Agrarian Economics," reprinted in AE, p. 394.

Two exceptions to the rule that the production function of a farm system is a functional such as (12) will help bring to the surface other important differences between the economy of the farm and that of the factory.

Take the case of the Island of Bali where, because the climate is practically uniform throughout the year, one can see all the activities (plowing, seeding, weeding, harvesting) performed at the same time on various fields. On a spot such as this, certainly, nothing stands in the way of growing rice by elementary processes arranged in line, by an open-air factory. The proper number of buffaloes, plows, sickles, flails, and villagers operating them could move over the entire field of a village, plowing, seeding, weeding, and so on, without any interruption, i.e., without any agent—land, capital, and labor—being idle at any time. The advantages of the factory system can in this case be easily pinpointed. First, the villagers would eat each day the rice sown that very day, as it were, because in a factory system, we remember, production is instantaneous. There would be no longer any need for the community to bear the specific burden of the loans for agricultural working capital which constitute everywhere the farmer's major headache. The overcapitalization of which I have just spoken will now appear as a palpable excess of capital to be used in other activities. For, as we would try to implement the factory system, we would be left with a residual of superfluous implements (and superfluous men) even if the older units of production were of the optimum size.

How tremendous the impact of the conversion from farm to factory on the cost of production may be is made crystal clear by the second exception. The exception is the system by which chickens are nowadays produced in the United States on practically every farm. With the use of the incubator, chickens are no longer produced in parallel as in the old system dictated by nature. A crop of chicken is ready for the market practically every day of the year, be it in August or in December. "Chicken farm" has thus become a misnomer: the situation calls for replacing it by "chicken factory." Because of the new system, a pound of chicken sells in the United States for less than a pound of any other kind of meat, while in the rest of the world, where the old system still prevails, chicken continues to be "the Sunday dinner." The famous "chicken war" of yesteryear would not have come about if the difference between the farm and the factory system in producing chickens had not been so great as to cover the shipping cost and the differential labor cost between the United States and Europe.

13. Internal Flows and Analysis. The analytical decomposition of a partial process into flow and fund coordinates bears on an incongruity associated with the Leontief input–output table. In fact, the incongruity goes back to Karl Marx—the first user of such a table. Thanks to Leontief's contribution, the input–output table no longer needs any introduction: it is now one of the most popular articles of the economist's trade. However, the point I wish to bring home requires that the relation between the input–output table and the ideas developed in this chapter should first be made as clear as possible.

TABLE 3
Economy E Represented in Process Form

	P_1	P_2	P_3	N	P_4
			Flow Coordinates		
C_1	x_1^*	$-x_{12}^*$	$-x_{13}^*$	$*$	$-x_{14}^*$
C_2	$-x_{21}^*$	x_2^*	$-x_{23}^*$	$*$	$-x_{24}^*$
C_3	$-x_{31}^*$	$-x_{32}^*$	x_3^*	$*$	$-x_{34}^*$
R	$-r_1^*$	$-r_2^*$	$-r_3^*$	r^*	$-r_4^*$
W	w_1^*	w_2^*	w_3^*	$-w^*$	w_4^*
			Fund Coordinates		
C_1	X_{11}^*	X_{12}^*	X_{13}^*	$*$	X_{14}^*
C_2	X_{21}^*	X_{22}^*	X_{23}^*	$*$	X_{24}^*
C_3	X_{31}^*	X_{32}^*	X_{33}^*	$*$	X_{34}^*
$\mathscr{C}$	$\mathscr{C}_1^*$	$\mathscr{C}_2^*$	$\mathscr{C}_3^*$	$*$	$*$
L	L_1^*	L_2^*	L_3^*	$*$	L_4^*
H	H_1^*	H_2^*	H_3^*	$*$	H^*

A very simple illustration will serve this purpose much better than the general structure commonly used in the studies of the Leontief input–output system. Thus, let E be a stationary economy surrounded by its natural environment N and consisting of three production sectors P_1, P_2, P_3 and one consumption sector P_4. To remain within the rationale of Leontief's own system, let us assume that each productive process P_i produces only one commodity C_i and that there is only one quality of natural resources R, of waste W, and of labor power H. For the same reason, each process will be represented by its flows and services over one year. The notations being the same as in the foregoing sections, this means that the coordinates are now $R(T = 1)$ and $H(T = 1)$, for instance, instead of $R(t)$ and $H(t)$—quantities instead of functions.[64] If for the convenience of diction, we use a star to show that a notation stands for the annual flow or service, the analytical representation of the five

[64] For the particular purposes intended by Leontief for his input–output system the fact that the seasonal rhythm of some processes is ignored in this simplified representation does not matter.

processes into which we have decomposed the whole actuality is laid out in Table 3.[65]

This table involves an algebraic key. Because of the tautological truth that every output flow of a process is an input flow of some other process(es)—and vice versa—every row of the flow matrix must add up to zero. For example, we must have $x_1^* = x_{12}^* + x_{13}^* + x_{14}^*$. We may therefore delete the elements $x_1^*, x_2^*, x_3^*, r^*, -w^*$ without discarding any information. However, for facility in reading the table we may write them in an additional column. And if, in addition, we change the sign of the other flow coordinates, we have simply transformed the flow matrix of our table into the input–output form shown in Table 4.[66]

TABLE 4
The Input–Output Table of Flows in Economy E

	P_1	P_2	P_3	N	P_4	Totals
C_1	$*$	x_{12}^*	x_{13}^*	$*$	x_{14}^*	x_1^*
C_2	x_{21}^*	$*$	x_{23}^*	$*$	x_{24}^*	x_2^*
C_3	x_{31}^*	x_{32}^*	$*$	$*$	x_{34}^*	x_3^*
R	r_1^*	r_2^*	r_3^*	$*$	r_4^*	r^*
W	$-w_1^*$	$-w_2^*$	$-w_3^*$	$*$	$-w_4^*$	$-w^*$

Two obvious points should be stressed now. The first is that an input–output table is only a scrambled form (according to some definite rules) of the corresponding flow matrix of the process representation. Consequently, the flow matrix and the input–output table are two completely equivalent forms. Given one of these forms, we can derive the other straightforwardly. The second point is that, because of the particular rules of scrambling, some boxes in any input–output table must always be empty. Such are the first four diagonal boxes of Table 4.[67]

[65] At this time, there is no need to separate each x_{ik}^* into a current input and a maintenance flow or each X_{ik}^* into the services of a store and of an equipment fund. Nor do we need to concern ourselves with the *stocks* of R and W in nature, except to note that because of the Entropy Law they have decreased and increased during the year by more than r^* and w^*.

[66] Leontief includes in the input–output table the "flow" of labor services which he regards as the "output" of the consumption sector. See Leontief, *The Structure of the American Economy: 1919–1939*, pp. 41 f and *passim*; Leontief *et al.*, *Studies in the Structure of the American Economy*, pp. 23, 55. I prefer to abide by the fundamental difference between flow (as a substance that crosses a boundary) and service (as an action performed by a fund element inside the boundary). Also in Table 3, H^* of column P_4 stands for the consumption activity of the entire population of E—which I believe to be the correct analytical representation of that process.

[67] These diagonal boxes correspond to *product* coordinates.

Many writers believe that a greater degree of generality is nevertheless reached if we fill these boxes with some elements.[68] The difficulty with this position is the question of what precisely corresponds to these diagonal elements in actuality. No one, to my knowledge, has put forward the thesis that an input–output table represents an entirely new conception of how a process may be represented analytically. This being so, the burning question is what place we should assign to the diagonal elements of an input–output table when we rescramble it into the matrix flow of the process form. If we add them to the marginal totals and treat such a sum as the product flow of the corresponding process, we are simply admitting that we did not follow the scrambling rules to the letter. The fact remains that no one seems to have thought of the issue raised by the diagonal elements in relation to the equivalence of the two forms. The sparse justifications offered for the input–output table in which the diagonal boxes are not necessarily empty have approached the issue from some side line.

The issue arose in connection with consolidation. Because the problem at hand does not always require that all production processes be explicitly distinguished in the analytical framework, the economist often consolidates several processes into a single process. This operation by itself raises no difficulty whatsoever. All we need to do in order to consolidate P_1 and P_2 into P_0 is to remove from our analytical picture the boundary that separates them. The effect on Table 3 is straightforward: the columns P_1 and P_2 are added horizontally into a new column P_0 that replaces the others.[69] This introduces, however, one dissonant feature: the consolidated process P_0 has *two* products C_1 and C_2,[70] a reason why economists generally do not stop here. We prefer to pair each process with one product only. In a sense, it seems natural that if we have consolidated several processes into one "metallurgical" industry, we should also aggregate

[68] E.g., O. Eckstein, "The Input–Output System: Its Nature and Its Uses," in *Economic Activity Analysis*, ed. O. Morgenstern (New York, 1954), pp. 45 ff; M. A. Woodbury, "Properties of Leontief-Type Input–Output Matrices" in that same volume, pp. 341 ff.

[69] This rule sounds like the ultrafamiliar rule for the addition of vectors. However, there is more to it. For instance, the flow coordinates x_1^* and $-x_{12}^*$ must be replaced by their sum $x_1^0 = x_1^* - x_{12}^*$ because the sum of the corresponding row must, as we have noted earlier, add up to zero. The reason why the fund coordinates X_{11}^* and X_{12}^* must also be replaced by their sum $X_{10}^* = X_{11}^* + X_{12}^*$ is, however, different: when a boundary that separates two processes is removed, the actions of the corresponding fund factors are obviously pooled together.

[70] The flow matrix of the new representation can nevertheless be transformed into an input–output table. Only, this table has one column less than Table 4. The point supplies an additional clarification of the relation between an input–output table and the representation in process form.

their products into one "metallurgical" product.[71] That is why in economics "consolidation" means *consolidation* of processes and *aggregation* of the corresponding products.

TABLE 5
The Consolidated Form of Table 3

	P_0	P_3	N	P_4
	Flow Coordinates			
C_0	x_0^* $(x_1^* + x_2^* - x_{12}^* - x_{21}^*)$	$-x_{03}^*$ $(-x_{13}^* - x_{23}^*)$	*	$-x_{04}^*$ $(-x_{14}^* - x_{24}^*)$
C_3	$-x_{30}^*$ $(-x_{31}^* - x_{32}^*)$	x_3^*	*	$-x_{34}^*$
R	$-r_0^*$ $(-r_1^* - r_2^*)$	r_3^*	r^*	r_4^*
W	w_0^* $(w_1^* + w_2^*)$	w_3^*	$-w^*$	w_4^*
	Fund Coordinates			
C_0	X_{00}^* $(X_{11}^* + X_{12}^* + X_{21}^* + X_{22}^*)$	X_{03}^* $(X_{13}^* + X_{23}^*)$	*	X_{04}^* $(X_{14}^* + X_{24}^*)$
C_3	X_{30}^*	X_{33}^*	*	X_{34}^*
$\mathscr{C}$	$\mathscr{C}_0^*$ $(\mathscr{C}_1^* + \mathscr{C}_2^*)$	$\mathscr{C}_3^*$	*	*
L	L_0^* $(L_1^* + L_2^*)$	L_3^*	*	L_4^*
H	H_0^* $(H_1^* + H_2^*)$	H_3^*	*	H^*

If we denote the aggregate commodity of C_1 and C_2 by C_0, the effect of the consolidation (in the above sense) of P_1 and P_2 is shown by Table 5. The rule is simple: we add the columns P_1 and P_2—as already explained—

[71] As I have said, the consolidation of P_1 and P_2 is a simple operation free from any snags. The opposite is true for the aggregation of several quanta into a single quantum. But this problem, the knottiest of all in economic analysis and especially in the applications of the input–output system, may be begged by the present argument without any risk.

TABLE 6
The Correct Consolidated Form of Table 4

	P_0	P_3	N	P_4	Totals
C_0	*	x^*_{03}	*	x^*_{04}	x^*_0
C_3	x^*_{30}	*	*	x^*_{34}	x^*_3
R	r^*_0	r^*_3	*	r^*_4	r^*
W	$-w^*_0$	$-w^*_3$	*	w^*_4	$-w^*$

and also the rows C_1 and C_2 in both the flow and the fund matrices of Table 3. An obvious, but crucial, point is that consolidation cannot destroy the algebraic key of the flow matrix: each row still adds up to zero. Consequently, we can transform the matrix flow of Table 5 into an input–output table by the same scrambling rules as before. The result, shown in Table 6, makes it abundantly clear why even after consolidation the proper diagonal boxes in an input–output table must still be empty. This vindicates the rule outlined by Leontief for the consolidation of an input–output table: after the addition of the corresponding columns and rows, the resulting diagonal element (if nonnull) must be suppressed and the row total modified accordingly.[72]

TABLE 7
The Incorrect Consolidated Form of Table 4

	P_0	P_3	N	P_4	Totals
C_0	$x^*_{12} + x^*_{21}$	x^*_{03}	*	x^*_{04}	$x^*_1 + x^*_2$
C_3	x^*_{30}	*	*	x^*_{34}	x^*_3
R	r^*_0	r^*_3	*	r^*_4	r^*
W	$-w^*_0$	$-w^*_3$	*	$-w^*_4$	$-w^*$

Some economists, however, take exception to this rule and simply add the pertinent columns and rows without suppressing the diagonal element. They obtain Table 7 instead of Table 6. Perhaps this view is a faint echo of the rule for the addition of vectors which, as we have seen, works perfectly in the case of a process form representation. But if this is the case, the view ignores the essential fact that an input–output table is a scrambled arrangement of the other. Apparently, only one explicit reason has been offered in support of maintaining the diagonal elements after

[72] Leontief, *The Structure*, pp. 15 f, 189. Curiously, Leontief broke this rule himself. See note 76, below.

consolidation, namely, that the algebra works better if they are not suppressed.[73] About this, there can be no question: in algebra, terms may cancel each other but they are never just suppressed. Besides, if all flows are measured in money terms (as is often the case in applications), the grand total of the input–output table does not have to be changed. But the rub is that the algebra which works splendidly on a scrambled matrix is apt to be itself scrambled algebra in relation to the unscrambled, basic matrix.

To say only that "there is no difficulty connected with the definition of $[x^*_1 + x^*_2]$ and no need to eliminate items of the type $[x^*_{12} + x^*_{21}]$"[74] does not suffice to justify the form of Table 7. We need to know what corresponds to the item $x^*_{12} + x^*_{21}$ in actuality when we conceive the entire economy subdivided only into the processes listed in the consolidated input–output table. Analytical frameworks should not be superimposed in a confusing mesh. To explain, $x^*_1 + x^*_2$ represents indeed the combined product output of P_1 and P_2 but only in a framework which includes these processes explicitly. If they are consolidated into a single process P_0, there is no room in the resulting picture except for the product output of that process, namely, for $x^*_0 = x^*_1 + x^*_2 - x^*_{12} - x^*_{21}$—as shown by both Tables 5 and 6.

The point seems so simple that one can only wonder how it was possible to be set aside. I recall that the late League of Nations used to publish the foreign trade data for all countries in the world in the form of an input–output table identical in all respects with that made later famous by Leontief.[75] Of course, all the diagonal boxes were empty. Had there appeared a figure in the box corresponding to the export of Italy to Italy, everyone would have been certain that it was a typographical error! And let us think of such a statistical table consolidated so as to show the export between the continents of the world. Should we not consider it a typographical error if a figure would appear in the diagonal box for the export of Europe to Europe? The point is that in consolidating the table from countries into continents, the export between the European countries has to be suppressed. Clearly, such a consolidated table cannot include the "internal" European export any more than the export of the United States can include interstate commerce.

One is nevertheless greatly tempted to argue that we should place in

[73] R. Dorfman, P. A. Samuelson, and R. M. Solow, *Linear Programming and Economic Analysis* (New York, 1958), chaps. ix and x.

[74] *Ibid.*, p. 240. The expressions between square brackets are my apropos substitutions.

[75] See, for instance, *Memorandum on Balance of Payments and Foreign Trade Balances, 1910–1923*, League of Nations (2 vols., Geneva, 1924), I, 70 ff.

the diagonal box Europe-to-Europe the internal European export and that, similarly, we should regard the diagonal element $x_{12}^* + x_{21}^*$ in Table 7 as representing the *internal flow* of the consolidated process P_0. So great is this temptation that even Leontief, soon after insisting on the suppression of the diagonal elements, included such an element in one of his tables to represent an internal flow—"payments from business to business."[76] According to the analytical view of a process, however, flows are the elements that are especially associated with a crossing of a boundary. Consequently, once we have removed from our analytical picture the boundaries between the European countries or the boundary between P_1 and P_2, gone also must be the flows associated with them. Analytically, therefore, the term "internal flow" is a mismatch. Yet the use of the concept—under this or some other name—is so widespread that a direct proof of the analytical incongruity involved in it should be in order.

TABLE 8
The Input–Output Table of a Subdivided Canal

From/To	N	P_1	P_2	$\cdots$	P_{n-1}	P_n	Totals
N	w_0	w	*	$\cdots$	*	*	$w_0 + w$
P_1	*	w_1	w	$\cdots$	*	*	$w_1 + w$
P_2	*	*	w_2	$\cdots$	*	*	$w_2 + w$
.	.	.	.	$\cdots$	.	.	$\cdots$
P_{n-1}	*	*	*	$\cdots$	w_{n-1}	w	$w_{n-1} + w$
P_n	w	*	*	$\cdots$	*	w_n	$w_n + w$

Let us visualize a canal P through which water flows at a constant speed and let us decompose it into n partial canals by analytical boundaries drawn without any plan. Let $P_1, P_2, \cdots, P_n$ denote the partial canals and N the environment. The input–output table of the system is given by Table 8. On purpose, no assumption is made concerning the values of the coordinates w_i. If we now consolidate the P_i's back into P and do not suppress the diagonal elements, we obtain Table 9. And since we can take n as large as we may wish and since the value of w is independent of the number of subcanals, it follows that the internal flow of P—that is, $\sum_1^n w_i + (n-1)w$—may exceed any value we please. The internal flow should, therefore, be infinite. It is obvious that the same absurd conclusion obtains for any other process.

Another justification for the inclusion of diagonal elements invokes the common distinction between *gross* and *net* output flow. According to this view, the diagonal element $x_{12}^* + x_{21}^*$ of Table 7 is supposed to represent

[76] Leontief, *The Structure*, p. 18.

the difference between the gross output flow of P_0, $y_0^* = x_1^* + x_2^*$, and the net output flow, $x_0^* = x_{03}^* + x_{04}^*$. In explicit terms, that diagonal element represents the part of the flow of C_0 which is used by P_0 itself.[77] This interpretation thus takes us back to the same position—that a diagonal element stands for an internal flow.

TABLE 9
The Consolidated Form of Table 8

From/To	N	P	Totals
N	w_0	w	$w_0 + w$
P	w	$\sum_1^n w_i + (n-1)w$	$\sum_1^n w_i + nw$

Of course, *inside* any process there is something going on at all times, something flowing in the broad sense of the term. Inside a factory producing glass from sand, for instance, there is a continuous "flow" of sand, melted glass, rolled glass, etc. But this internal flow, as we have seen, is a fund category and, hence, is represented in the analytical picture of the factory by the process-fund $\mathscr{C}$, not by a flow coordinate. There is also a "flow" of clover seed in the process by which clover seed is produced or one of hammers in the process producing hammers. These, too, are funds that must be represented by a fund coordinate such as X_{11}^* of Table 3.[78] Perhaps, by insisting—as the flow complex does—on the inclusion of internal flows in an input–output table, we unwittingly seek to make room for such fund factors in a framework which seems so convenient but which normally includes only pure flows. That is, we seek to smuggle funds into a flow structure. In the end, we will find ourselves adding or subtracting flow and fund coordinates which, as we saw in Section 4, are heterogeneous elements. Unfortunately, the algebra will nonetheless work well most of the time—a cunning coincidence which should not be taken at its face value. Algebra cannot give us any warning signals on such matters. That is why the harm done by smuggling funds into the flow category is not likely to manifest itself on the surface. But below the skin of algebra, things may be distorted substantially.

[77] This viewpoint appears in Leontief, *The Structure*, Tables 5, 6, and 24 (pocket), where several diagonal boxes are filled with data. See also the tables in his "The Structure of Development," *Scientific American*, September 1963, pp. 148–166.

[78] I feel it necessary to go on and point out that the clover seed used in producing clover fodder is, on the contrary, a flow, not a fund element. If the difference may seem perplexing, it is undoubtedly because of our money fetishism—a harmful fetishism this is—of thinking of every economic variable in money terms by preference. But the puzzle should disappear if we observe that to repeat the process of raising clover fodder a farmer has to exchange some fodder for seed, that is, he must go through another process—the market for seed and fodder.

The most convincing illustration is supplied by Marx's endeavor to explain the pricing system in the capitalist system by his labor doctrine of value. The crucial element in his argument is the simple "diagram" by which he represented the economic process analytically, and which, at bottom, is an input–output table. The source of Marx's well-known predicament, I contend, is the internal flow by which he represented the hammers used to hammer hammers—to use again my metaphor. But since Marx was completely sold on the idea that economics must be a *dialectical* science (in the strict sense), it was in order for him not to distinguish between flow and fund and, hence, to substitute an internal flow for an analytical fund. In a strictly dialectical approach of any strain, Being is Becoming. However, for his diagram of simple reproduction Marx turned to analysis and, at that point, he mixed dialectics with analysis—a fact of which he was not aware, apparently. The object lesson of the difficulties he encountered thereafter is clear. If one decides to make dialectics his intellectual companion, one must also be careful not to mix dialectics with analysis. The stern commandments of analysis can be neither circumvented nor disobeyed.

14. Marx's Diagram of Simple Reproduction versus a Flow–Fund Model. As we may recall, in Marx's analytical diagram the economy is divided into two departments, P_1 and P_2, producing capital goods and consumer goods, respectively, and two consumption sectors, P_3 and P_4, of the workers and of the capitalists.[79] The notations used in Table 10 are the

TABLE 10

The Input–Output Table of Marx's Diagram of Reproduction

	P_1	P_2	P_3	P_4	Totals
G_1	c_1	$c_2 = v_1 + s_1$	*	*	$\omega_1 = c_1 + v_1 + s_1$
G_2	*	*	$v_1 + v_2$	$s_1 + s_2$	$\omega_2 = c_2 + v_2 + s_2$
H	v_1	v_2	*	*	$v_1 + v_2$

familiar ones: v_i and s_i represent the *flow* of consumption goods accruing to the workers and the capitalists associated with department P_i. The term c_2 stands for the maintenance *flow* of G_1 necessary to keep the capital of P_2 constant; and c_1, the troublesome item, stands for the *internal flow* of capital goods in P_1, i.e., the flow of capital goods consumed in the

[79] It may be worth pointing out here that the current practice of putting all households in the same analytical bag is a regrettable regress from Marx's analysis which, by separating the households of the capitalists from those of the workers, kept the social dimension in the center of economic analysis. Economics has indeed drifted away from political economy to become almost entirely a science of management.

production of capital goods. All terms are expressed in labor value terms.[80] If one still needs to expose the heterogeneity of the terms composing ω_i, one may cite Sweezy's cacophony in explaining that the total value is obtained by adding "the constant capital engaged [in production with] the income of the capitalist [and] the income of the worker."[81] Marx also took it that the total capital of the capitalist consists only of constant capital and variable capital. Clearly, in a steady-going *industrial* process wages as well as the current input and maintenance flows are paid out of the simultaneous product flow. There is thus no need for assuming that the capitalist owns also some working capital, unless we wish to make the diagram more realistic and bring in a store fund of money to take care of irregular fluctuations in the operations (Section 9). But such a fund does not necessarily stand in the same ratio with every category of payments. There is one way, however, to make some analytical sense of Marx's diagram, which is strongly suggested by countless elaborations in *Capital*. In all probability, the process Marx had in mind in setting up his diagram was an agricultural, not an industrial process. For we should not forget that he borrowed his diagram from François Quesnay, who by his famous *Tableau économique* sought to depict the economics of agricultural production.[82] In this alternative, c_1 is analogous to the corn used as seed at the beginning of the process and ω_1 is the gross output of corn at the end of the process. The diagram would thus represent a system of elementary processes which are arranged *in series* and in which there is no durable fund, whereas the industrial system which Marx wanted to analyze is a system *in line* in which capital is a self-maintaining fund at all times. But we must pass on.

Marx's basic tenets are well-known: (1) competition brings about the equality of values and prices in the sense that quantities of equal value sell for equal amounts of money; (2) the workers are paid their value, i.e., their standard of subsistence, regardless of how many hours the capitalists may force them to work each day;[83] (3) competition also equalizes in all departments the rate of labor exploitation

[80] Marx, *Capital*, II, 458–460, and Paul Sweezy, *The Theory of Capitalist Development* (New York, 1942), pp. 75–79. I have included the row H in Table 10 because, like Leontief, Marx treated the services of labor as a flow category (although, as I have pointed out earlier, he steadily avoided the term service).

[81] Sweezy, *Theory*, pp. 76 f.

[82] Cf. K. Marx and F. Engels, *Correspondence, 1846–1895* (New York, 1935), pp. 153–156. As I have pointed out in my "Economic Theory and Agrarian Economics," reprinted in AE, p. 384, even Marx's law of surplus value—relation (26), below—reflects the tithe system in agriculture.

[83] Implicit in these two tenets is the principle mentioned in Section 6 above, that the value of a fund's service is completely taken care of by the maintenance flow of that fund.

(26) $$s_1/v_1 = s_2/v_2.$$

On this basis, Marx claimed to be able to explain the pricing mechanism by which the rates of profit of all departments are equalized in the *capitalist system*. But as he in the end found out, if (26) is true, then the equality of the rates of profit,

(27) $$s_1/(c_1 + v_1) = s_2/(c_2 + v_2),$$

does not obtain unless the organic composition of capital is the same in both departments, i.e., unless

(28) $$v_1/c_1 = v_2/c_2.$$

This relation expresses in fact a general technological law which cannot possibly be accepted.[84] As a result, Marx was compelled to admit that prices cannot reflect values and proposed a rule for determining "the prices of production" corresponding to a given diagram. The rule consists of redistributing the total surplus value, $s = s_1 + s_2$, between the two departments in such a way as to bring about the equality of the rates of profit. But Marx offered no economic explanation of why and how the production prices would be brought about. The same is true of the numerous rescuers of latter days who tried to conjure away the analytical impasse by far-fetched reinterpretations and, often, highly complicated algebra.[85] But turning in circles is inevitable as long as we cling to Marx's flow complex. Let us then abandon this complex and see what we may be able to do if, instead, we use our flow–fund model for probing Marx's argument about value.

Table 11 represents in process form the same structure as that which Marx had in mind. It assumes that the working day, δ, is the same in both departments, that the working class receives only its daily standard subsistence V, and that the scales of production are adjusted so that the product flow of P_1 is just sufficient for the maintenance of the capital fund K_2 of P_2. The other notations are self-explanatory: K_1 is the capital fund of P_1, and n_1, n_2 are the numbers of homogeneous workers employed in the two departments, $n = n_1 + n_2$. We can always choose the unit of G_2 in such a way that δx_2 be equal to the total labor time δn, in which case the labor value of that unit is unity. This convention yields $x_2 = n$.

[84] Marx himself denounced it in *Capital*, vol. III, chap. viii. See also Sweezy, *Theory*, pp. 69 f.

[85] To my knowledge, all these solutions are concerned only with the flow diagram. For Marx's rule see *Capital*, vol. III, chap. ix, and Sweezy, *Theory*, pp. 107–115. One of the highly praised alternative solutions, by L. von Bortkiewicz, is presented in Sweezy, pp. 115–125.

There remain only two unknowns to be determined: δ_0, the length of the "normal" working day (the necessary labor, in Marx's terminology), and p_0, the *value* of G_1. Not to depart from Marx's line of reasoning, we must compute p_0 in the absence of any labor exploitation.

TABLE 11
A Two-Department Economy

	P_1	P_2	P_3	P_4
		Flow Coordinates		
G_1	δx_1	$-\delta x_1$	*	*
G_2	*	δx_2	$-V$	$-(s_1 + s_2)$
		Fund Coordinates		
G_1	δK_1	δK_2	*	*
H	δn_1	δn_2	*	*

If there is no exploitation—by which, with Marx, we must mean that $s_1 = s_2 = 0$—from the last flow row of Table 11 we obtain the normal working day,

(29) $$\delta_0 = nv/x_2 = v,$$

where $v = V/n$ is the daily wage of the worker. If δ^* is the greatest numbers of hours a worker can work daily without impairing his biological existence, the last relation shows that the workability of the system represented by Table 11 requires that $\delta^* - v \geq 0$. The fact that labor is productive, in the sense that under any circumstances it can produce more than its standard subsistence, invites us to assume that $\delta_0 < \delta^*$ and, hence,

(30) $$\delta - v > 0$$

for any δ, $\delta_0 < \delta < \delta^*$. The equality between price and cost (with no share for the services of capital) yields for each department

(31) $$\delta_0 x_1 p_0 = n_1 v, \qquad \delta_0 x_2 = \delta_0 x_1 p_0 + n_2 v.$$

By (29), from the first of these conditions we obtain

(32) $$p_0 = n_1/x_1,$$

a value which satisfies the second condition as well, since $x_2 = n$.

Next, let us assume—also with Marx—that the capitalists can impose a working day δ, $\delta_0 < \delta \leq \delta^*$, and still pay the workers the same *daily* wages.[86] In this case, from the cost equations (with G_1 priced at p_0) we obtain

$$s_1^0 = n_1(\delta - \delta_0), \qquad s_2^0 = n_2(\delta - \delta_0). \tag{33}$$

Per worker, therefore, the rate of exploitation is the same, $(\delta - \delta_0)$, in both departments, and Marx's law of surplus value (26) is vindicated. However, the rates of profit in the two departments being $r_1^0 = n_1(\delta - \delta_0)/p_0K_1$ and $r_2^0 = n_2(\delta - \delta_0)/p_0K_2$, they, again, cannot be equal unless the fund factors are combined in the same proportion in both departments, i.e., unless

$$n_1/K_1 = n_2/K_2, \tag{34}$$

which is tantamount to Marx's (28).

One factual element should be now brought into our abstract analysis. Capital goods are produced in the same manner as biological species. Occasionally, one "species" of capital goods evolves from another such species. That is, new capital species are produced by mutations. The first stone hammer was produced only by labor out of some materials supplied by the environment; the first bronze hammer was produced by labor aided by a substantial number of stone hammers. But in a stationary economy there can be no mutation: hammers (or machines) are reproduced by the same kind of hammers (or machines). Now, the role of capital is not only to save labor but also to amplify man's meager physical power. It stands to reason, then, that *on the whole* it takes more machines per man to make machines than to use these last machines in producing consumer goods. The fact, I contend, is fairly transparent and within a stationary two-sector economy perhaps an *a priori* synthetic judgment.[87]

[86] The notion that the *wage rate* should be set so as to allow the worker only his daily maintenance at the "regular" working day was very old by Marx's time: "for if you allow double, then he works but half so much as he could have done, and otherwise would; which is a loss to the *Publick* of the fruit of so much labor." *The Economic Writings of Sir William Petty*, ed. C. H. Hull (2 vols., Cambridge, Eng., 1899), I, 87 (my italics). This idea, found also in the works of François Quesnay, implies a unit elasticity of the supply of hours of work and, clearly, differs from Marx's own explanation. For what may bring the workers, of that and later times, to have such a supply schedule, see my article "Economic Theory and Agrarian Economics" (1960), reprinted in AE, p. 383.

[87] Whether the same judgment is true for *any* capital goods industry compared with *any* consumer goods industry constitutes an entirely different issue. To decide it, we need an accurate estimation of every K/H^* (in our case, K_i/n_i). But, for the reasons explained in Section 10, above, the best available censuses of manufactures do not provide us with the necessary data. Nor is the usual classification of industries suitable for this particular purpose. If the nineteen basic manufacturing industries (of the United States classification) are ranked according to the following *brute* capital-labor ratios—fixed capital per worker, capital invested per production worker, horsepower per worker, and fixed capital per wage and salary dollar—the rankings display no striking parallelism. For whatever significance it might have, I should add that the industries of apparel, textiles, leather, furniture, and printing usually are at the bottom of every ranking. Only the food industry tends to be slightly above the median.

This means that

$$n_1/K_1 < n_2/K_2 \tag{35}$$

is the only case in actuality. Hence, always $r_1^0 < r_2^0$. Consequently, as long as the capital goods sell at their value p_0, the owners of the means of production will certainly shift their capital from P_1 to P_2. As a result, the decreased production of P_1 will no longer suffice to maintain the increased capital fund of P_2 constant. Ultimately, *the whole fund of constant capital of the economy will dwindle away*.[88]

But before this would come about, the capitalists of the department P_2 will naturally compete for the increasingly scarce maintenance flow of G_1. Competition—which, we may remember, is a fundamental condition in Marx's argument—must necessarily bring an increase in the price of the capital goods. This increase may put an end to the flight of capital from P_1 to P_2 and, ipso facto, to the gradual shrinking of the capital fund of the economy. To check this conclusion by algebra, let p be the money price of G_1 at which there would be no incentive for any shift of means of production from one department to the other. This price must obviously bring about the equality of the two rates of profit, $s_1/pK_1 = s_2/pK_2$. After some algebraic manipulations, this condition yields

$$p = p_0 + \frac{(\delta - \delta_0)(n_2K_1 - n_1K_2)}{\delta x_1(K_1 + K_2)}. \tag{36}$$

In view of (30) and (35), this formula shows that, while everything else continues to sell at its labor value (in Marx's sense), capital goods must sell at more than their labor value.[89] The only exception is the case of $\delta = \delta_0$, which entails $p = p_0$ and $s_1 = s_2 = 0$. But in this case the capitalists would eat up their capital anyhow.[90] Of course, if $\delta > \delta_0$ and

[88] Because of (35), any shift of capital from P_1 to P_2 calls for an increase in employment. It would seem therefore that there should be also an increase in the total wage bill. However, if we interpret analytically Marx's assumption of the reserve army combined with the idea that the working class receives exactly its standard subsistence, the wage rate is not a datum. Instead, it is determined by the historically determined constant V and the size of the employment, $v = V/n$. Cf. my article "Mathematical Proofs of the Breakdown of Capitalism," reprinted in AE, p. 400.

[89] Because px_1 represents a money transfer from department P_2 to P_1, we should expect the total surplus value $s = s_1 + s_2$ to remain the same for any value of p—a fact which is easily checked by algebra. Also, my solution, in contrast with that by Bortkiewicz, does not require a reevaluation of the wage bills; hence it is much more in the spirit of Marx's.

[90] To avoid a possible misunderstanding, I may note that this statement does not contradict the proposition that a zero rate of interest is compatible with any trend of capital accumulation. In the model considered here the working class cannot save, because it receives only its standard subsistence. (This, again, does not preclude that each member of the worker class may save for old age at zero interest within that class.) The point is that, in this situation, a title to the means of production could not possibly find a buyer among the income earners: its market value would be zero, smaller than that of a piece of scrap paper.

the inequality (35) is reversed, capital goods should sell at *less* than their value. Were they to sell at p_0, all capital would move into the producer goods industries and the economy would die because machines would be used to make only machines. The fact that this reversed world, in which the consumer goods industries are more capital intensive than the others, can exist only on paper sharpens the general conclusion of this section.

Needless to insist, within a scheme of simple reproduction in which the capital fund is a datum we cannot entertain the question of how and why capital has been accumulated. The only problem that we can entertain is how that fund can be maintained. If the means of production are not owned by some individuals, then it is tautological that the whole production flow of consumer goods must accrue to the workers (provided no other institutional claim exists on it). The normal working day is, in this case, determined by the preferences of the whole population between leisure and real income at the prevailing technical rate v/x_2. As a price of account, G_1 must be reckoned at p_0. The system can then go on reproducing itself indefinitely. If, on the contrary, the means of production are owned by some individuals who, as we have seen, can only transform them into a flow of consumer goods, the maintenance of the capital fund requires that the working day should be longer than the normal working day. Otherwise the owners would eat up their capital (alternatively, the other institutional claimants would starve). A further condition for the reproduction of the system is that the share of the flow of consumer goods accruing to the owners must be proportional to the value of the capital invested in each line of production. In turn, this condition brings in some hard facts of technology, namely, that in the sector in which capital goods are reproduced they participate in a higher ratio to labor than in the sector in which consumer goods are produced.[91] This is the ultimate reason why capital goods must sell at a higher price than their labor value established according to Marx's own rationale.

15. Commodities, Processes, and Growth. We have thus far considered only the analytical representation of steady-going processes, that is, of processes that *reproduce* themselves. We have not touched the question of how such a process may come into existence. Were we concerned with steady-going mechanical systems involving only locomotion, we could dispose of this question either by assuming—as Aristotle did—a Prime

[91] Of course, a stationary economy without ownership of capital could go on indefinitely even in the reversed world. An unsuspected difficulty emerges, however, if instead of a stationary we consider a dynamic system: the "normal" world is dynamically unstable and the "reversed" world stable! See my paper "Relaxation Phenomena in Linear Dynamic Models" (1951), reprinted in AE, pp. 310 f, for an analysis of each case according to whether $n_1K_2 - n_2K_1$ is greater than, equal to, or less than 0.

Mover which set them into motion at the beginning of Time or by simply acknowledging their existence—as Newton did—through the Law of Inertia (Newton's First Law). But in economics we cannot dodge the question in this manner. Economic processes, even the steady-going ones, are set in motion and kept so by man. More pointedly, *economic processes are produced just as commodities are*. Think of a factory. Is not a textile factory, for instance, just as much the "product" of man's economic activity as an ell of cloth is? Ever since the economic evolution of mankind reached the phase in which man used commodities to produce commodities, the production of more commodities has had to be preceded by the production of additional processes. On the other hand, to produce an additional process implies the use of some commodities already available. In a down-to-earth view, *investment* is the production of additional processes, and *saving* is the allocation of already available commodities to this production.

Needless to say, none of the analytical representations considered in the preceding sections offer room for this important side of man's economic activity: the production of processes. These representations describe reproductive processes already produced. But the fact which I wish to bring to the reader's attention is that, as far as one may search the economic literature, all dynamic models (including those concerned with *growth*) allow for the production of commodities but not for that of processes. The omission is not inconsequential, be it for the theoretical understanding of the economic process or for the relevance of these models as guides for economic planning. For one thing, the omission is responsible for the quasi explosive feature which is ingrained in all current models of dynamic economics—as I shall show in a while.

But there is another reason why—the literature of economic dynamics notwithstanding—a dynamic model is useless for throwing any light on the problem of how growth comes about, which includes the problem of how growth itself may grow faster.[92] Just as a stationary model by itself implies a Prime Mover at minus infinity on the time scale, so a dynamic model implicitly assumes a Prime Planner which set the system growing at the origin of Time. A parallel from mechanics will set in sharp focus the issue as I see it. Let us imagine a ball moving (without friction) on a horizontal table according to the Law of Inertia, i.e., in a linear uniform motion. According to the same law, this system cannot change *by itself* its reproductive manner of moving. Only an *external* force—say, the gravitational force that comes into play as soon as the ball reaches the edge of the

[92] As J. R. Hicks, "A 'Value and Capital' Growth Model," *Review of Economic Studies*, XXVI (1959), 173, indicted the dynamic models, they allow only the selection of the starting point on a pre-selected growth path.

table—can cause its motion to become accelerated. By contrast, an economic steady-going system has *within itself* the power to move faster, in a word, to grow. A second (and far more important) difference is this: the ball does not have to move slower for a while in order to acquire a greater velocity under the influence of the gravitational force. A steady-going economic process, on the contrary, must, like the jumper, back up some distance in order to be able to jump. And my point is that in a dynamic model this backing up is thrown to minus infinity on the time scale.

To illustrate in detail the preceding remarks, I shall refer to that dynamic system which, in my opinion, is the most explicitly outlined of all, the Leontief system. The simplicity of its framework will also keep irrelevant issues from cluttering the argument. For the same reason, I wish to consider the simplest case, namely, that of a system consisting of two productive processes P_1 and P_2 producing commodities C_1 and C_2, respectively. With the notations of Table 3 (Section 13), the characteristic assumption of all Leontief systems (static or dynamic) is that for every process that may produce C_i, the input coefficients

$$a_{ki} = x^*_{ki}/x^*_i, \qquad B_{ki} = X^*_{ki}/x^*_i, \tag{37}$$

are constant.[93] To render this assumption more explicit, we may write

$$\begin{aligned} x^*_1 &= x_1\delta_1 a_{11}, & x^*_{21} &= x_1\delta_1 a_{21}, & x^*_2 &= x_2\delta_2 a_{22}, & x^*_{12} &= x_2\delta_2 a_{12}, \\ X^*_{11} &= x_1\delta_1 B_{11}, & X^*_{21} &= x_1\delta_1 B_{21}, & X^*_{12} &= x_2\delta_2 B_{12}, & X^*_{22} &= x_2\delta_2 B_{22}, \end{aligned} \tag{38}$$

where δ_i is the working day of P_i and x_i is a pure number measuring the *scale* of P_i in relation to the corresponding unit-scale process. The unit-scale processes are:

$$\begin{aligned} &P^0_1\,(a_{11} = 1,\ -a_{21};\ B_{11},\ B_{21}), \\ &P^0_2\,(-a_{12},\ a_{22} = 1;\ B_{12},\ B_{22}). \end{aligned} \tag{39}$$

P^0_1, for instance, describes the process *capable* of producing a flow rate of one unit of C_1 per unit of time.[94] Consequently, a_{ik} is a *flow rate*, and B_{ik}

[93] Leontief, *Studies in the Structure of the American Economy*, pp. 18, 56. In his dynamic system, Leontief leaves out even the labor input. The reason is, perhaps, the same as that which led me to write the basic relations of the production function in the form of (18) in Section 9, above.

[94] As it should be clear by now, I take exception to Leontief's view—*The Structure*, p. 211, *Studies*, p. 12—that static analysis or short-run analysis may completely disregard the fund coordinates B_{ik}. True, in the short run the extant funds are supposed to remain fixed. Now, if the constancy of coefficients (37) is assumed, the short-run variations can come only from a change in the δ_i's or the capacity utilized (which is tantamount to a change in the x_i's). Hence, it is important to know what each P_i can produce at full and continuous utilization of the extant capacity (which is determined by the extant funds, not by the observed flow coefficients, a_{ik}). This maximum capacity cannot be exceeded no matter how much labor power we transfer to P_i—a point which is generally ignored in the practical applications of the Leontief static system.

a *fund*. Since the δ_i's do not appear explicitly in Leontief's presentation, we may assume that, like every Neoclassical economist, he took it that they have the same and invariable value.[95] For the following argument, it does not matter if we take the same position and, in addition, assume $\delta_1 = \delta_2 = 1$.

Given the scale x_i, the flow rate of net product (y_1, y_2) which the system is capable of producing is determined by the well-known system of relations[96]

$$a_{11}x_1 - a_{12}x_2 = y_1, \qquad -a_{21}x_1 + a_{22}x_2 = y_2, \tag{40}$$

which is subject to the indispensable condition

$$a = a_{11}a_{22} - a_{12}a_{21} > 0. \tag{41}$$

Let us now assume that we plan for the increases in the flow rates of net product

$$\Delta y_1 \geq 0, \qquad \Delta y_2 \geq 0, \qquad \Delta y_1 + \Delta y_2 > 0. \tag{42}$$

These increases require the increases Δx_1 and Δx_2 in the *scales* of P_1 and P_2. They are determined by the system

$$a_{11}\Delta x_1 - a_{12}\Delta x_2 = \Delta y_1, \qquad -a_{21}\Delta x_1 + a_{22}\Delta x_2 = \Delta y_2. \tag{43}$$

These last increases, in turn, require some increases in the existing funds $B_1 = x_1B_{11} + x_2B_{12}$, $B_2 = x_1B_{21} + x_2B_{22}$, namely,

$$\Delta B_1 = B_{11}\Delta x_1 + B_{12}\Delta x_2, \qquad \Delta B_2 = B_{21}\Delta x_1 + B_{22}\Delta x_2. \tag{44}$$

To accumulate these additional funds, a part of the flow of net product must be accumulated (instead of consumed) over some period Δt. During this period, therefore, the flow rate of net product available for consumption is

$$z_1 = y_1 - \frac{\Delta B_1}{\Delta t}, \qquad z_2 = y_2 - \frac{\Delta B_2}{\Delta t}. \tag{45}$$

[95] In one place, *The Structure of the American Economy*, p. 160, Leontief does allude to the possibility of the working day to vary, but only from one industry to another. I should also mention that in the decomposition (38) a_{ik} is not a time-free coordinate, but B_{ik} is so. Of course, the a_{ik}'s are *numerically* equal to a time-free coordinate, namely, to "the physical amounts of $[C_i]$ absorbed by industry $[P_k]$ per unit of its own output"—as Leontief has it in *The Structure*, pp. 188 f, and *Studies*, p. 18.

[96] In these relations $a_{11} = 1$, $a_{22} = 1$. Because a_{11} and a_{22} are dimensional coefficients (not pure numbers), I included them explicitly so as to allow us to check at a glance the dimensional homogeneity of these and the subsequent relations.

If Δx_1 and Δx_2 are eliminated from (43) and (44), the last relations become

$$(46)\qquad \begin{aligned} z_1 &= y_1 - \frac{a_{22}B_{11} + a_{21}B_{12}}{a}\left(\frac{\Delta y_1}{\Delta t}\right) - \frac{a_{11}B_{12} + a_{12}B_{11}}{a}\left(\frac{\Delta y_2}{\Delta t}\right), \\ z_2 &= y_2 - \frac{a_{22}B_{21} + a_{21}B_{22}}{a}\left(\frac{\Delta y_1}{\Delta t}\right) - \frac{a_{11}B_{22} + a_{12}B_{21}}{a}\left(\frac{\Delta y_2}{\Delta t}\right), \end{aligned}$$

or briefly,

$$(47)\qquad \begin{aligned} z_1 &= y_1 - M_{11}\left(\frac{\Delta y_1}{\Delta t}\right) - M_{12}\left(\frac{\Delta y_2}{\Delta t}\right), \\ z_2 &= y_2 - M_{21}\left(\frac{\Delta y_1}{\Delta t}\right) - M_{22}\left(\frac{\Delta y_2}{\Delta t}\right). \end{aligned}$$

This system shows, first, that once we have chosen Δy_1, Δy_2, there is a lower limit to Δt, i.e., to how quickly we may reach the chosen level $y_1^1 = y_1 + \Delta y_1$, $y_2^1 = y_2 + \Delta y_2$. Conversely, if Δt is chosen there is an upper limit to Δy_1 and Δy_2. Secondly, (47) shows that no matter how small Δy_1 and Δy_2 and how large Δt are chosen, the system must drop to a lower level of consumption before pulling itself up to a higher level. Obviously, we may diminish this drop by using the additional funds as they are accumulating, but we cannot avoid it.

Let us then consider a succession of periods Δt and assume that the funds saved during each period are invested at the end of the period. For each period there obtains a system analogous to

$$(48)\qquad \begin{aligned} z_1^i &= y_1^i - M_{11}\left(\frac{\Delta y_1^i}{\Delta t}\right) - M_{12}\left(\frac{\Delta y_2^i}{\Delta t}\right), \\ z_2^i &= y_2^i - M_{21}\left(\frac{\Delta y_1^i}{\Delta t}\right) - M_{22}\left(\frac{\Delta y_2^i}{\Delta t}\right), \end{aligned}$$

where $y_k^{i+1} = y_k^i + \Delta y_k^i$, $y_k^0 = y_k$. The systems (48) allow us to determine step by step the sequences $[y_k^i]$ from appropriately chosen sequences $[z_k^i]$, and conversely. The picture of how a steady-going process may become a growing process is thus clear.

If we pass to the limit by choosing an increasingly smaller Δt, (48) becomes

$$(49)\qquad \begin{aligned} z_1(t) &= y_1(t) - M_{11}\dot{y}_1(t) - M_{12}\dot{y}_2(t), \\ z_2(t) &= y_2(t) - M_{21}\dot{y}_1(t) - M_{22}\dot{y}_2(t), \end{aligned}$$

where the dot indicates the derivative with respect to t.[97] In this case,

[97] The standard form used by Leontief (*Studies*, pp. 56 f) can be derived from (49) if y_1 and y_2 are replaced by their values given by (40). My preference for (49) is that it directly compares the net product with the consumption level.

too, we can determine the functions $z_k(t)$ if $y_1(t)$ and $y_2(t)$ are given; this is simple enough. But the main application of (49) or of any other dynamic system concerns the case in which we choose arbitrarily the z_k's and use the system to determine the y_k's.[98] Calculus teaches us that given the z_k's, the general solutions of (49) involve two arbitrary constants. These constants, as Leontief advises us, can be determined by the initial conditions $y_1(0) = y_1^0$ and $y_2(0) = y_2^0$. This advice is fully correct provided that at the chosen origin, $t = 0$, the actual process was *already* an accelerated one, i.e., a dynamic process. In case the process comes from the past as a stationary one, there are some restrictions on the choice of the z_k's, the most important being that $z_k(0) < y_k(0)$—to allow for the "drop" of which I have spoken above.[99]

As it should be clear from the foregoing analysis, the dynamic models involve a peculiar assumption to which practically no attention has been paid. The assumption is that as soon as the necessary funds have been saved the level of net product *instantaneously* jumps to $(y_1 + \Delta y$, $y_2 + \Delta y_2)$. As a result, the net product starts to increase the very moment the old level of consumption is decreased. This is the quasi explosive feature of the dynamic models to which I have alluded earlier. Indeed, if this assumption were true in actuality, we could bring about a fantastic growth of any economy by merely decreeing, say, one day of the week during which no commodities should flow into the consumption sector (all other things being kept as before). The reason why we cannot achieve this tour de force is that an increase in the product flow requires that some additional *processes* be first created. Also, as we have seen in Section 9, above, a process can start producing a product flow only after it is primed, i.e., only after its process-fund $\mathscr{C}$ is completed. And both to build a process out of commodities and to prime it require some duration in addition to the time necessary for the accumulation of the funds ΔB_1 and ΔB_2. Specifically, after we have accumulated the additional funds $B_{11}\Delta x_1$ and $B_{21}\Delta x_1$ during the interval Δt, we must wait an additional

[98] *Ibid.*, pp. 57–60.

[99] Dynamic systems such as (49) conceal unpleasant surprises. This is why even the condition just mentioned is not always sufficient to sustain growth continuously. The point is simply illustrated by a system involving only one commodity, in which case (49) reduces to $z(t) = y - M\dot{y}$ or to $z(t) = y - \dot{y}$ if M is chosen as the unit of time. The solution that transforms a steady-going system y^0 into a growing one is

$$y(t) = y^0e^t - e^t\int_0^t e^{-t}z(t)\,dt,$$

for $t \geq 0$. The necessary and sufficient condition that y should be always increasing is that $y(t) > z(t)$. Let us also note that, in contrast with the movement of the ball of our earlier example, $\dot{y}(t)$ cannot have the same value for $t = 0$ as the speed of the previous system up to that point.

time interval τ_1 before the additional product flow of P_1 becomes available. And, a point which deserves stressing, τ_1 covers the time needed to build and prime the new processes, just as the necessary savings $B_{11}\Delta x_1$, $B_{21}\Delta x_1$ must include not only the ordinary equipment of that process but also its process-fund $\mathscr{C}_1$. The consequence is that in the chain of systems (48) we can no longer write $y_k^{i+1} = y_k^i + \Delta y_k^i$. That is not all. Accumulation of stocks may be regarded as locomotion, which goes on continuously in time. But building a process is an event which cannot be reduced to a point in time. Consequently, though nothing stands in the way of making Δt tend toward zero in the modified system (48), it would mess up things completely if we were to make τ_1 and τ_2, too, tend toward zero. These lags, therefore, must appear explicitly in the new system, which is now better expressed in terms of $x_1(t)$ and $x_2(t)$:

$$(50)\quad \begin{aligned} z_1(t) &= a_{11}x_1(t) - a_{12}x_2(t) - B_{11}\dot{x}_1(t - \tau_1) - B_{12}\dot{x}_2(t - \tau_2), \\ z_2(t) &= -a_{21}x_1(t) + a_{22}x_2(t) - B_{21}\dot{x}_1(t - \tau_1) - B_{22}\dot{x}_2(t - \tau_2). \end{aligned}$$

The quasi explosive feature of the Leontief dynamic system (49) *as a planning tool* is thus eliminated. In particular, if we apply (50) to changing a steady-going economic process into a growing process or to increasing the growth of an already growing system, the solution will be such that no increase in the output of P_1 or P_2 will appear before some time interval (the smaller of τ_1 and τ_2) has elapsed after the beginning of the new saving.[100]

But even in a *growing* process there need not necessarily be any waiting for growth. A lag between accumulation and the increased output exists because each additional process, too, is the product of an elementary process and because the completion of an elementary process requires duration—the time of production. The reason for the lag is, therefore, the same as that which we have found to work in the case of small-shop production, namely, a low rate of demand.[101] However, with economic development an economy may reach the point when it finds advantageous the building of a system Π_1 that produces processes P_1 and P_2 *in line* just as a factory produces commodities in line. Once the process Π_1 is built, the economy can produce processes P_1 and P_2 *without any waiting*. What is true for a factory producing commodities "instantaneously" must hold

[100] The analytical advantages of the lag systems over the purely dynamical ones have been repeatedly stressed in the literature: e.g., Leontief, *Studies*, pp. 82 f; J. D. Sargan, "The Instability of the Leontief Dynamic Model," *Econometrica* XXVI (1958), 381–392. But the fact that their solutions do not possess the analytical simplicity of the purely dynamic systems has made their study less profitable and has deterred their use in concrete applications. On the issue of the stability of the Leontief dynamic system see also my paper cited in note 91, above.

[101] Cf. Sections 7 and 11, above.

for a factory producing processes. The economy can therefore grow at a constant speed which is determined by the scale of Π_1. There will be waiting only if the economy wants to grow at a higher speed. To grow at a higher speed requires an increase in the scale of Π_1 which can be achieved only by elementary processes in series, unless the economy includes a process Π_2 that produces processes Π_1 in line. Should this be the case, the economy can grow at a constant acceleration (constantly increasing speed) without waiting. On paper, there is no limit to this analytical algorithm.

The world of facts, however, does not seem to quite fit into this Π-model. Even in the most advanced economies we do not find factories that build factories that build factories that build factories.... However, in these economies we find a complex and extensive net of enterprises that are continuously engaged in building factories not quite in line but almost so. They are the general contracting firms, the building enterprises, the construction firms, and so on. Because of the necessity of dispersing their activity over a large territory, these enterprises do not possess a factory in the narrow sense of the term. Yet these organizations operate severally or in association essentially like a factory—a flexible factory, but still a factory.

In conclusion, I wish to submit that it is this Π-sector that constitutes the fountainhead of the growth and further growth which seems to come about as by magic in the developed economies and which, precisely for this reason, has intrigued economists and puzzled the planners of developing economies. By a now popular metaphor, we speak of the "take-off" of a developing economy as that moment when the economy has succeeded in creating within itself the motive-power of its further growth. In light of the foregoing analysis, an economy can "take off" when and only when it has succeeded in developing a Π-sector. It is high time, I believe, for us to recognize that the essence of development consists of the organizational and flexible power to create new processes rather than the power to produce commodities by materially crystallized plants. Ipso facto, we should revise our economics of economic development for the sake of our profession as a pure and practical art.