

**PME 3380**

# **MODELAGEM DE SISTEMAS DINÂMICOS**

**JOÃO PAULO S. FLORES**  
10773892

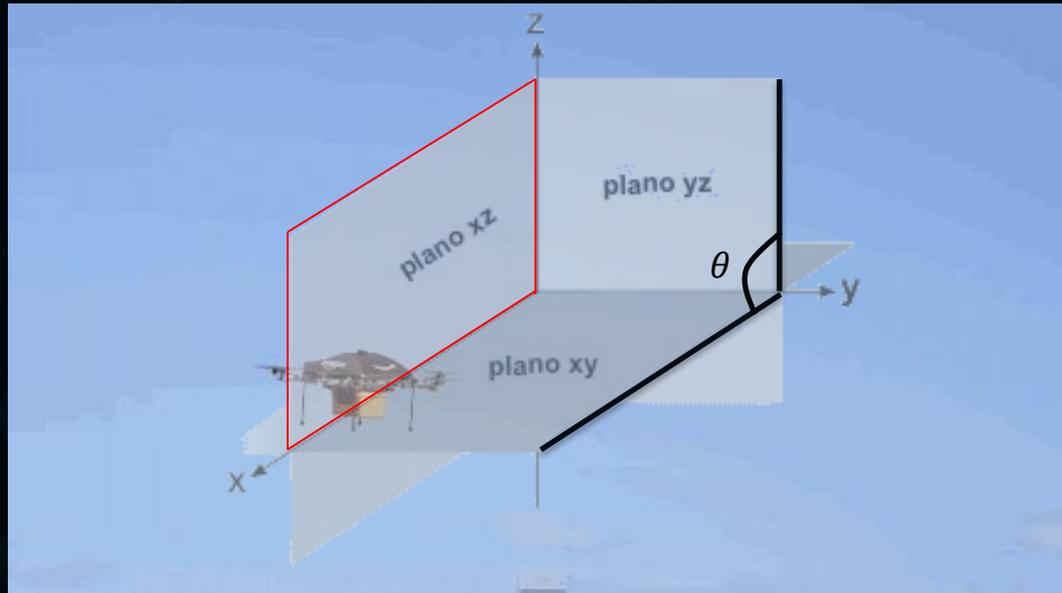
**JOSÉ ARTHUR S. GUERRERO**  
10791767

**SAMUEL ALVES DA S. JUNIOR**  
10769639

**WALLACE MOREIRA E SILVA**  
10823772



# OBJETIVO DO TRABALHO 2



- **QUADRICÓPTERO**
- **MOVIMENTAÇÃO NO PLANO XZ**
- **VARIAÇÃO NO ÂNGULO  $\theta$**

# REFERENCIAS BIBLIOGRÁFICAS 3



- CAVALLARO, Silvio Luis Hori. **Modelagem, simulação e controle de um VANT do tipo quadricóptero**. 2019. 153 f. Dissertação (Mestrado)



- CARLOS ESPIÚCA MONTEIRO, João. **Modelagem e Controle de um Veículo Quadrirrotor**. Orientador: Liu Hsu. 2015. Trabalho de Conclusão de Curso (Bacharel em Engenharia de Controle e Automação) - Escola Politécnica da Universidade Federal do Rio de Janeiro, [S. l.], 2015.



- DOMINGUES, J. M. B. Quadrotor prototype. Instituto Superior Tecnico, Universidade Tecnica de Lisboa, 2009.



- BALLAS, C. **Modelling and Linear Control of a Quadrotor**. Cranfield University, 2007.



- BRESCIANNI, T. **Modelling, Identification and Control of a Quadcopter Helicopter**. Dissertação (Mestrado em Engenharia Eletrônica). Department of Automatic Control, Lund University, Lund.

# INTRODUÇÃO 4



- **GYROPLANE No.1 (1907)**
- **RUSTON PROCTOR AERIAL TARGET (1916)**
- **OEMICHEN No.2 (1922)**
- **PARROT AR DRONE (2010)**

# UTILIDADES



# MODELO FÍSICO

6

## ▣ CAD INICIAL

➤ CORPO CENTRAL

➤ BRAÇOS

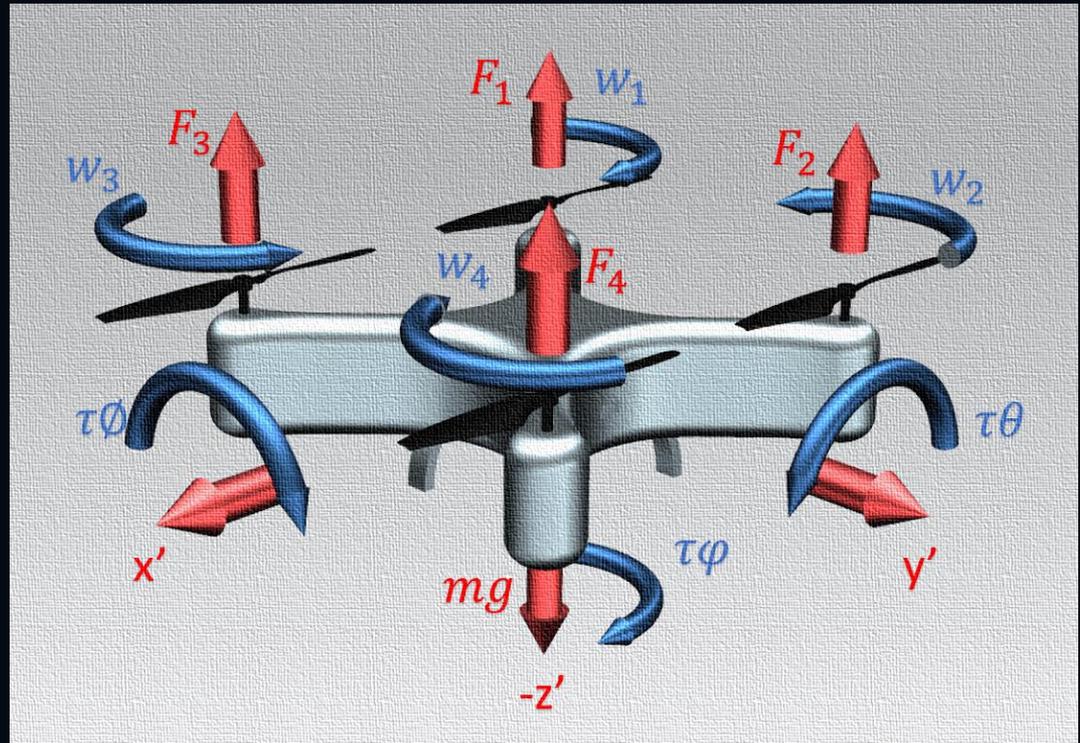


# MODELO FÍSICO

7

## □ CAD INICIAL

- REPRESENTAÇÃO DAS FORÇAS
- ROTAÇÃO DAS HÉLICES

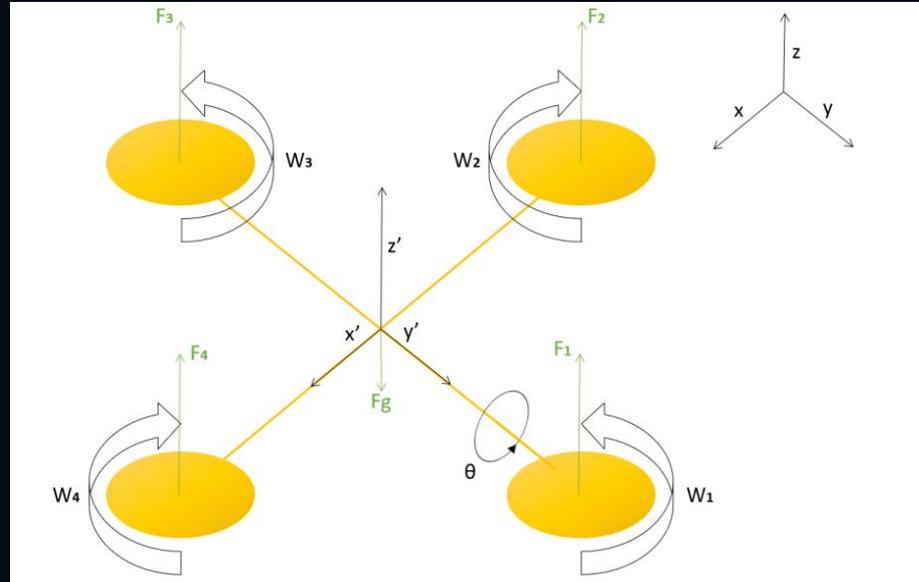


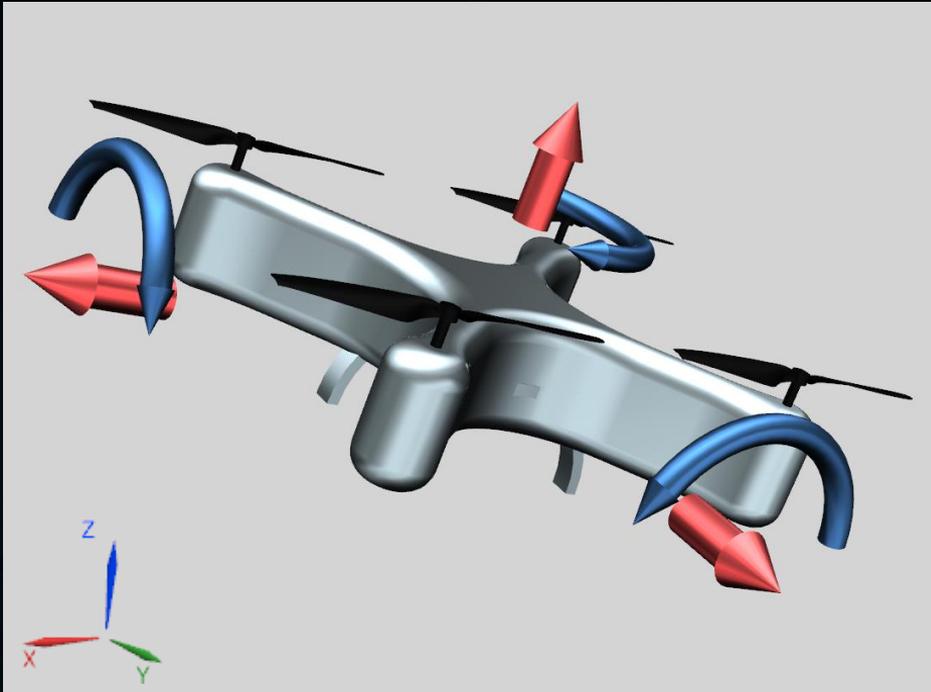
# MODELO FÍSICO

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## □ DIAGRAMA DE CORPO LIVRE

### ➤ REPRESENTAÇÃO DOS ESFORÇOS SOLICITANTES





- **HIPÓTESES**
- **3 GRAUS DE LIBERDADE**
- **REFERENCIAL**

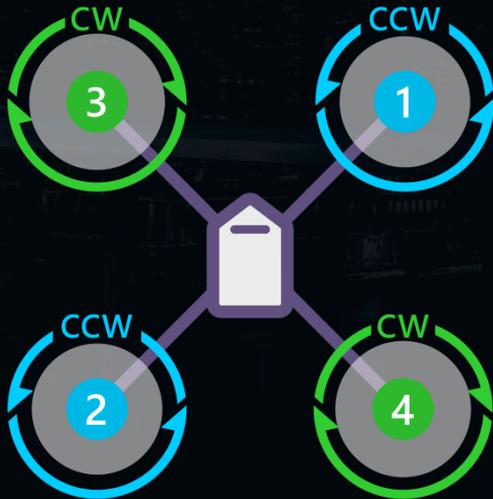
# HIPÓTESES SIMPLIFICADORAS

- **EFEITO SOLO DESPREZADO**
- **QUADRICÓPTERO SIMÉTRICO**
- **FORÇA DE ARRASTO SEMPRE CONTRÁRIA AO MOVIMENTO**
- **ÁREA DO QUADRIROTOR CONSTANTE**
- **VIBRAÇÃO DOS MOTORES DESPREZADA**
- **SEM ESCORREGAMENTO ENTRE HÉLICES E O ROTOR DO MOTOR**
- **4 MOTORES IDENTICOS**
- **VORTEX RING DESPREZADO**

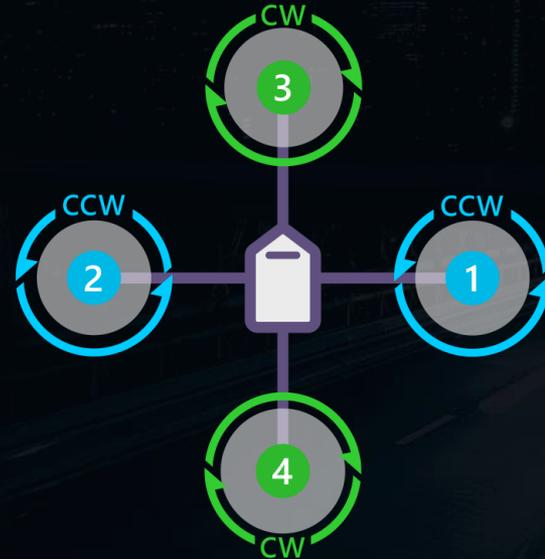
# TOPOLOGIA DO QUADRICÓPTERO

II

## CRUZADA



## POSITIVA

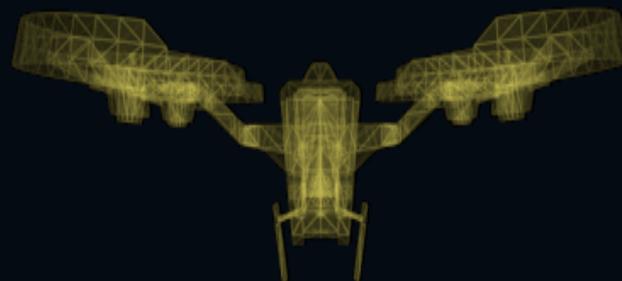
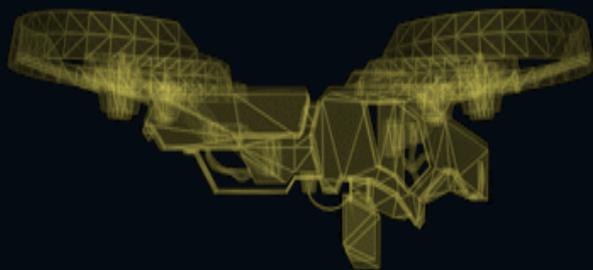


# ARFAGEM

12

SIDE

FRONT



PITCH

## Matriz de Rotação Análise do sistema a partir de um referencial inercial

$$R_t = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\cos\varphi & -\text{sen}\theta \\ \text{sen}\varphi\text{sen}\theta\cos\psi - \cos\varphi\text{sen}\psi & \cos\varphi\cos\psi + \text{sen}\varphi\text{sen}\theta\text{sen}\psi & \text{sen}\varphi\cos\theta \\ \text{sen}\varphi\text{sen}\psi - \cos\varphi\text{sen}\theta\text{sen}\psi & \cos\varphi\cos\psi\text{sen}\theta - \text{sen}\varphi\text{sen}\psi & \cos\varphi\cos\theta \end{bmatrix}$$

# ANALISE CINEMÁTICA

- Distância

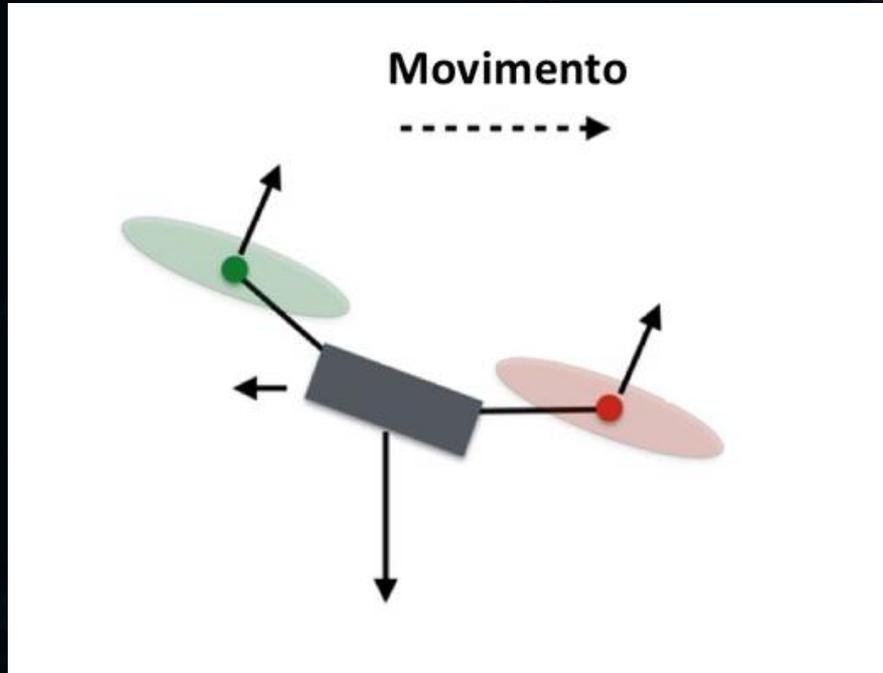
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_T \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

- Velocidade

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = R_T \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

- Ângulos

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\varphi \tan\theta & \cos\varphi \tan\theta \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi \sec\theta & \cos\varphi \sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$



## ▣ FORÇAS ATUANTES

- FORÇAS DE ARRASTO
- FORÇA GRAVITACIONAL
- FORÇA DOS MOTORES

# ANÁLISE DINÂMICA

16

- Teorema do Baricentro 
$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{1}{M} \begin{bmatrix} 0 \\ 0 \\ f_g \end{bmatrix} + [R_T] \frac{1}{M} \begin{bmatrix} 0 \\ 0 \\ F_i \end{bmatrix} + \frac{1}{M} \begin{bmatrix} F_{arr,x} \\ F_{arr,y} \\ F_{arr,z} \end{bmatrix}$$

- Equilíbrio de Forças do sistema 
$$\begin{bmatrix} -Mg \sin \theta - F_{arr,x} \\ Mg \sin \varphi \cos \theta - F_{arr,y} \\ -Mg \cos \varphi \cos \theta + F - F_{arr,z} \end{bmatrix} = M \left( \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \end{bmatrix} \right)$$

- Equilíbrio dos Momentos 
$$\begin{bmatrix} T_\varphi \\ T_\theta \\ T_\psi \end{bmatrix} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \left( \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right)$$

# ESPAÇO DE ESTADOS

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$$\mathbf{X} = [x \quad z \quad u \quad w \quad \theta \quad q]^T$$

$$\mathbf{U} = [T_\theta \quad F \quad 1]^T$$

## Equações Diferenciais Translacionais

$$\dot{x} = u(\cos\psi \cos\theta) + v(\cos\theta \sin\varphi) - w(\sin\theta)$$

$$\dot{y} = u(\cos\psi \sin\theta \sin\varphi - \sin\psi \cos\varphi) + v(\cos\varphi \cos\psi + \sin\varphi \sin\theta \sin\psi) + w(\cos\theta \sin\varphi)$$

$$\dot{z} = u(\sin\psi \sin\varphi + \cos\psi \sin\theta \cos\varphi) + v(\sin\psi \cos\varphi \sin\theta - \cos\psi \sin\varphi) + w(\cos\theta \cos\varphi)$$

$$\dot{u} = \frac{-1}{M} \rho A C_d u^2 - g \sin\theta + (rv - qw)$$

$$\dot{v} = \frac{-1}{M} \rho A C_d v^2 + g \sin\varphi \cos\theta + (pw - ru)$$

$$\dot{w} = \frac{-1}{M} \rho A C_d w^2 + g \cos\varphi \cos\theta + \frac{F}{M} + (qu - pv)$$

## Equações Diferenciais Rotacionais

$$\dot{\varphi} = p + (q \operatorname{sen} \varphi + r \operatorname{cos} \varphi) \operatorname{tg} \theta$$

$$\dot{\theta} = (q \operatorname{cos} \varphi - r \operatorname{sen} \varphi)$$

$$\dot{\psi} = (q \operatorname{sen} \varphi + r \operatorname{cos} \varphi) \operatorname{sec} \theta$$

$$\dot{p} = \frac{T_{\varphi}}{I_{xx}} + \frac{qr(I_{yy} - I_{zz})}{I_{xx}}$$

$$\dot{q} = \frac{T_{\theta}}{I_{yy}} + \frac{qr(I_{zz} - I_{xx})}{I_{yy}}$$

$$\dot{r} = \frac{T_{\psi}}{I_{zz}} + \frac{qr(I_{xx} - I_{yy})}{I_{zz}}$$

## EXPANSÃO EM SÉRIE DE TAYLOR DE 2ª ORDEM

$$f(x, u) = f(x_0, u_0) + \left[ \frac{\delta f(x, u)}{\delta x} \right]_{x_0, u_0} (x - x_0) + \left[ \frac{\delta f(x, u)}{\delta u} \right]_{x_0, u_0} (u - u_0) + \frac{1}{2!} \left[ \frac{\delta^2 f(x, u)}{\delta^2 x} \right]_{x_0, u_0} (x - x_0)^2 + 2 \frac{\delta^2 f(x, u)}{\delta x \delta y} \Big|_{x_0, u_0} + \frac{\delta^2 f(x, u)}{\delta^2 u} \Big|_{x_0, u_0} (u - u_0)^2 \Big]$$

*$u_0, w_0$  e  $\theta_0$*

# ESPAÇO DE ESTADOS

21

## CINEMÁTICA TRANSACIONAL

$$\dot{x} = u \cos \theta_0 - w \sin \theta_0 - \dot{\theta} (u_0 \sin \theta_0 + w_0 \cos \theta_0) + \theta_0 (w_0 \cos \theta_0 + u_0 \sin \theta_0)$$

$$\dot{z} = u \sin \theta_0 + w \cos \theta_0 + \dot{\theta} (u_0 \sin \theta_0 - w_0 \cos \theta_0) + \theta_0 (w_0 \cos \theta_0 - u_0 \sin \theta_0)$$

## CINEMÁTICA ROTACIONAL

$$\dot{\theta} = q$$

# ESPAÇO DE ESTADOS

22

## DINÂMICA TRANSLACIONAL

$$\dot{u} = \frac{-2u_0 u \rho A_x C_{dx}}{M} - \theta g - q w_0 + \frac{u_0^2 \rho A_x C_{dx}}{M}$$

$$\dot{w} = \frac{-2w_0 w \rho A_z C_{dz}}{M} - \theta g \sin \theta_0 + q u_0 + \frac{w_0^2 \rho A_z C_{dz}}{M} + g \theta_0 \sin \theta_0 + g \cos \theta_0 + \frac{F}{M}$$

## DINÂMICA ROTACIONAL

$$\dot{q} = \frac{T_\theta}{I_{yy}}$$

## MATRIZ A

$$A = \begin{bmatrix}
 0 & 0 & \cos \theta_0 & -\sin \theta_0 & -(u_0 \sin \theta_0 + w_0 \cos \theta_0) & 0 \\
 0 & 0 & \sin \theta_0 & \cos \theta_0 & (u_0 \cos \theta_0 - w_0 \sin \theta_0) & 0 \\
 0 & 0 & \frac{-2u_0 A_x C_{dx} \rho}{M} & 0 & -g & -w_0 \\
 0 & 0 & 0 & \frac{-2w_0 A_z C_{dz} \rho}{M} & -g \sin \theta_0 & u_0 \\
 0 & 0 & 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0
 \end{bmatrix}_{x_0, u_0}$$

## MATRIZ B

$$B = \begin{bmatrix} 0 & 0 & \theta_0(w_0 \cos \theta_0 + u_0 \sin \theta_0) \\ 0 & 0 & \theta_0(w_0 \cos \theta_0 - u_0 \sin \theta_0) \\ 0 & 0 & \frac{u_0^2 A_x C_{dx} \rho}{M} \\ 0 & \frac{1}{M} & \frac{w_0^2 A_z C_{dz} \rho}{M} + g \theta_0 \sin \theta_0 + g \cos \theta_0 \\ 0 & 0 & 0 \\ \frac{1}{I_{yy}} & 0 & 0 \end{bmatrix}$$

## MATRIZ C

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

# PARÂMETROS 26

## PROPRIEDADE

## VALOR



**Massa [kg]**

**0,76**



**Área superior e inferior [m<sup>2</sup>]**

**0,013**



**Área lateral [m<sup>2</sup>]**

**0,0086**



**Momento de inércia  $I_{xx}$  e  $I_{yy}$  [kg. m<sup>2</sup>]**

**0,0098**



**Momento de inércia  $I_{zz}$  [kg. m<sup>2</sup>]**

**0,0198**

# PARÂMETROS 27

## PROPRIEDADE

## VALOR



**Coeficiente de arrasto  $C_{dx}$  e  $C_{dy}$**

**0,52**



**Coeficiente de arrasto  $C_{dz}$**

**0,27**



**Aceleração da gravidade [ $m/s^2$ ]**

**9,81**



**Densidade do ar [ $kg/m^3$ ]**

**1,2**

# PARÂMETROS 28

## PROPRIEDADE

## VALOR



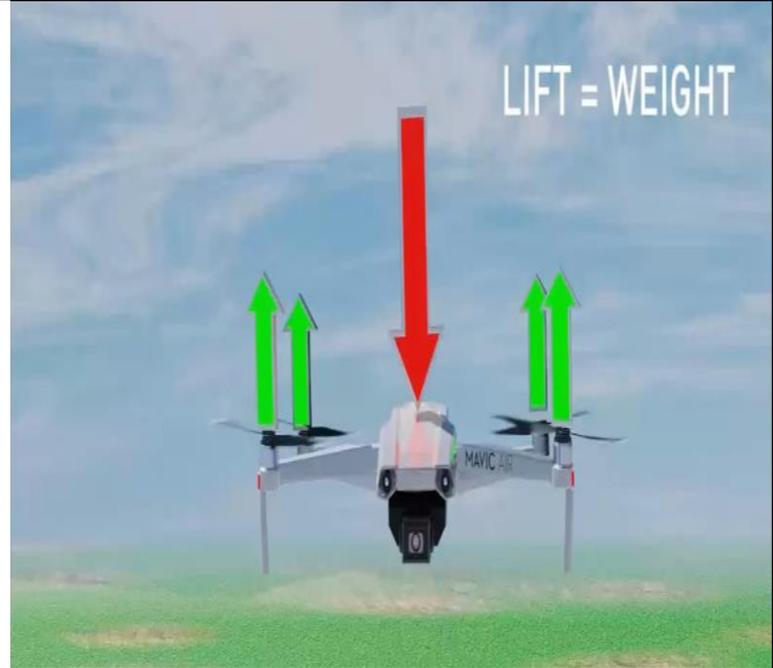
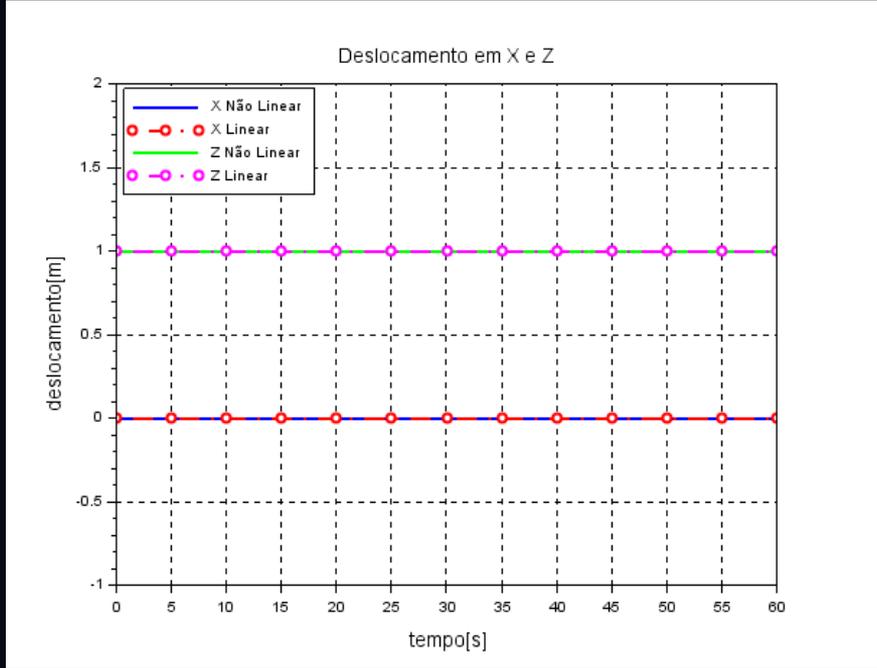
<b>FORÇA MÁXIMA [N]</b>	<b>8.27</b>
<b>FORÇA DE SUSTENTAÇÃO [N]</b>	<b>7.46</b>

# VOO PARAIDO

FORÇA
SUSTENTAÇÃO [N]

ÂNGULO
0°

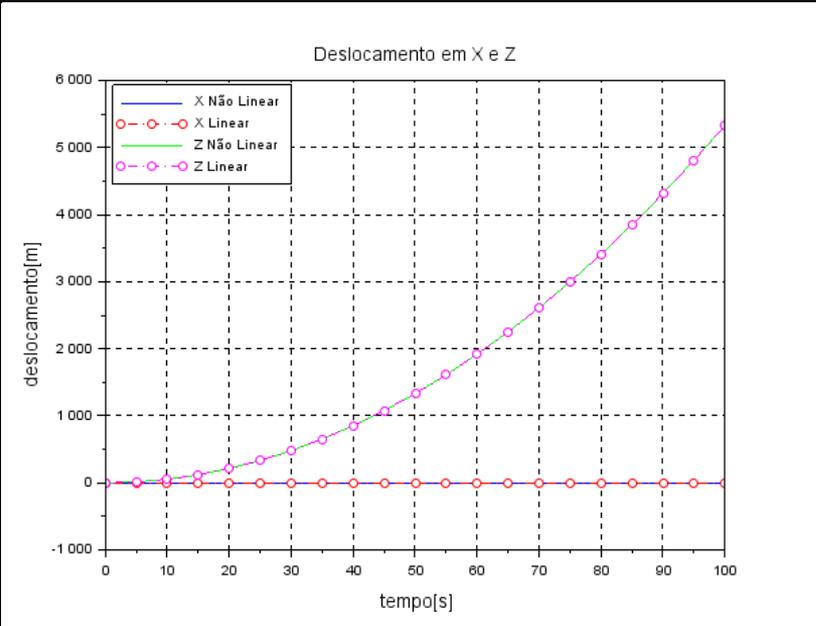
29



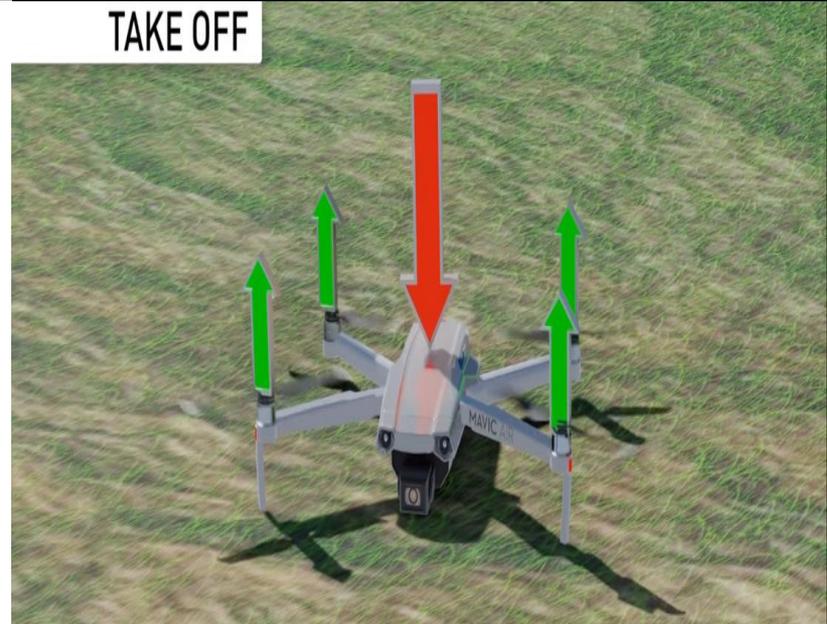
# MOVIMENTO APENAS EM Z

FORÇA	ÂNGULO
MÁXIMA [N]	0°

30



TAKE OFF



# MOVIMENTO APENAS EM X

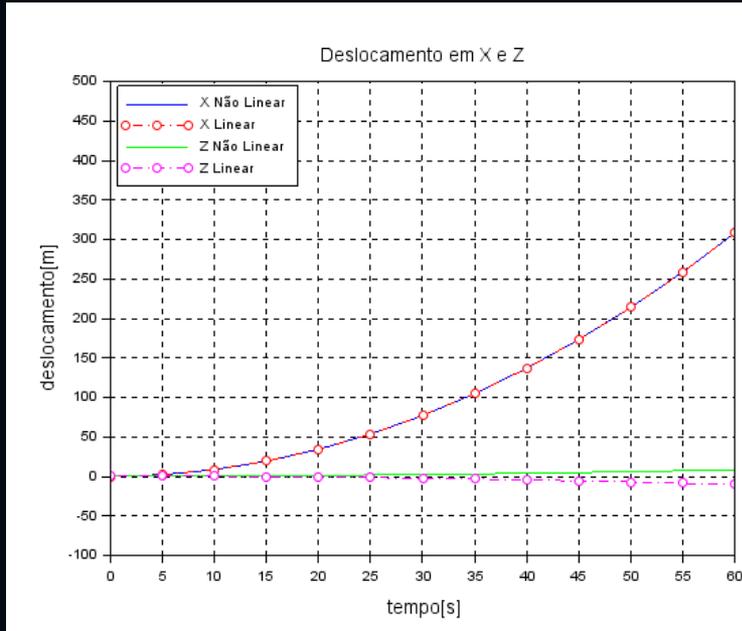
FORÇA

$$\frac{Mg}{\cos(\theta)}$$

ÂNGULO

$1^\circ$

31



# MOVIMENTO EM X E Z

FORÇA

$\theta_0$

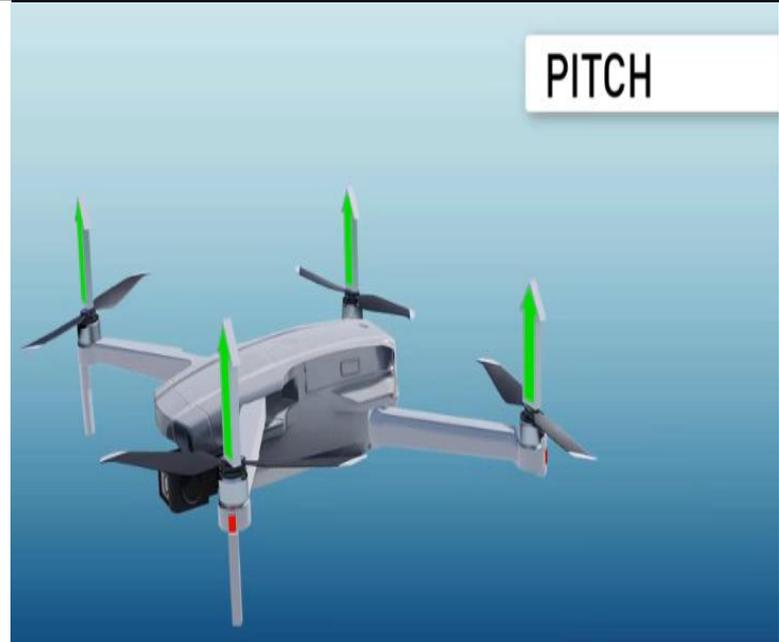
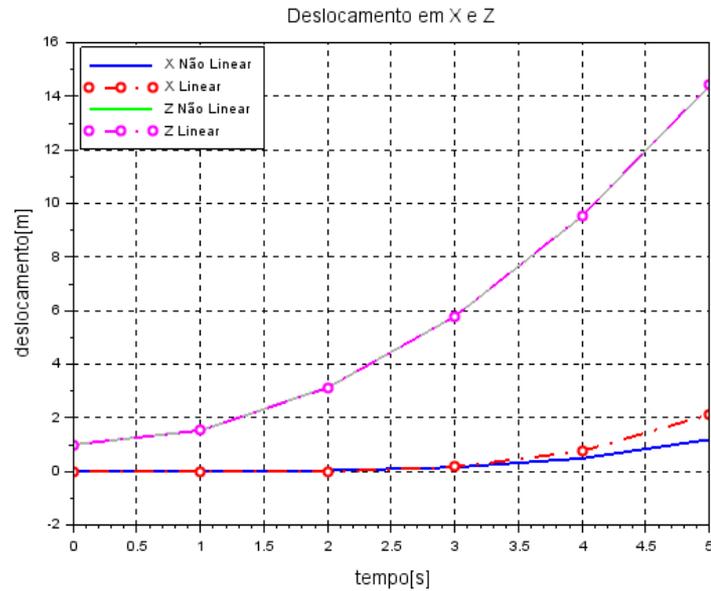
TORQUE

MÁXIMA

$2^\circ$

$1e^{-4}$  [N/m]

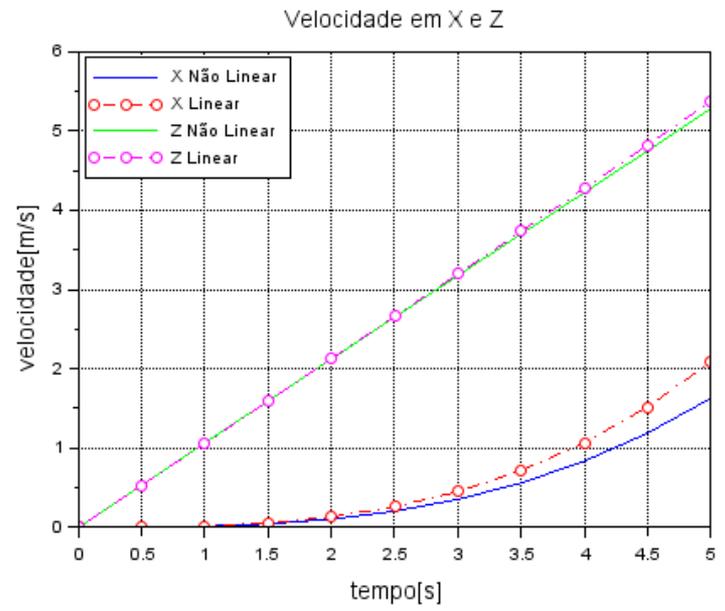
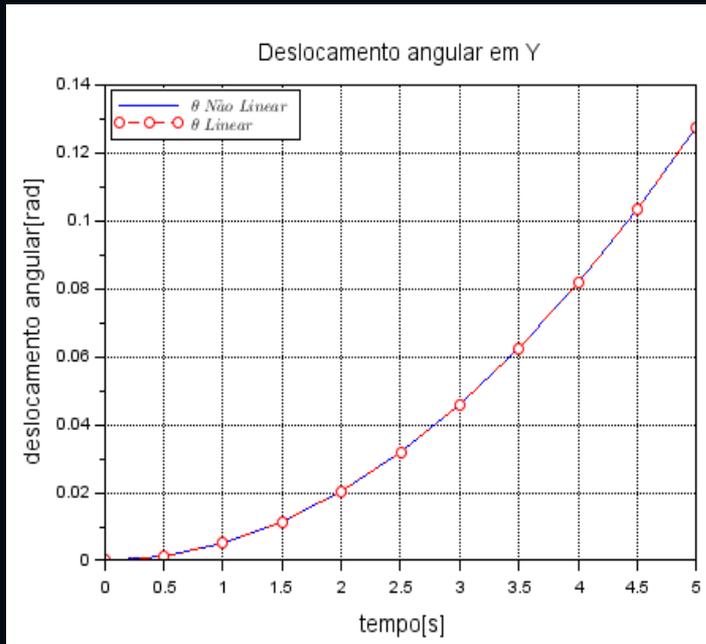
32



# MOVIMENTO EM X E Z

FORÇA	$\theta_0$	TORQUE
MÁXIMA	$2^\circ$	$1e^{-4}$ [N/m]

33



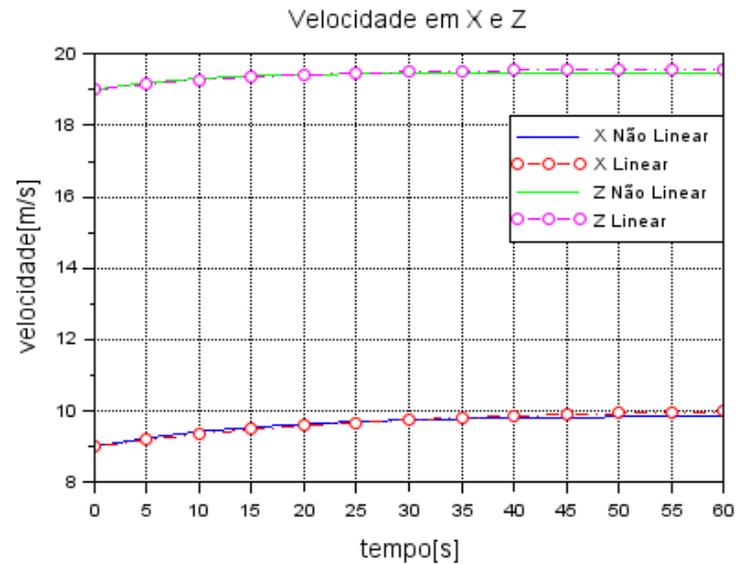
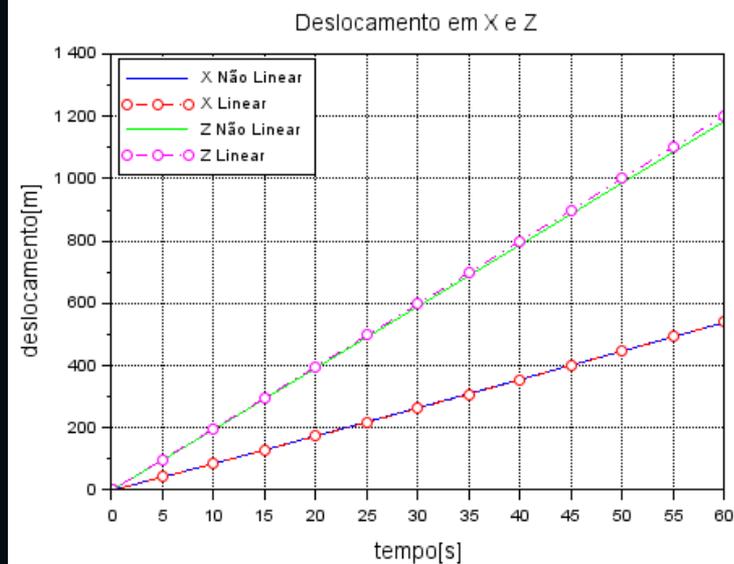
# MOVIMENTO COM ARRASTO

$u_0$   
2.8 m/s

$w_0$   
5.7 m/s

$\theta_0$   
2°

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# FUNÇÕES DE TRANSFERENCIA

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## SEM ARRASTO

$$\begin{bmatrix} \frac{49.05}{s^4} & 0 & 0 \\ 0 & \frac{1.31}{0.067s - s^2} & \frac{-9.81}{s^2} \\ \frac{49.05}{s^3} & 0 & 0 \\ 0 & \frac{1.31}{0.067 + s} & \frac{-9.81}{s} \\ \frac{5}{s^2} & 0 & 0 \\ \frac{5}{s} & 0 & 0 \end{bmatrix}$$

**POLOS NULOS COM ALTA  
MULTIPLICIDADE  
SISTEMA INSTÁVEL**

# FUNÇÕES DE TRANSFERENCIA

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## COM ARRASTO

$$\begin{array}{ccc} \frac{3.06+44.12s-57.94s^2}{0.003s^3+0.1s^4-s^5} & \frac{-0.046}{0.064s-s^2} & \frac{0.017+0.41s+0.20s^2}{0.0025s+0.10s^2-s^3} \\ \frac{0.21+5.25s+25.99s^2}{0.003s+0.1s^4+s^5} & \frac{1.31}{0.064s-s^2} & \frac{-0.38-9.60s+0.2s^2}{0.003s+0.1s^2+s^3} \\ \frac{49.05-28.5s}{0.039s^2-s^3} & 0 & \frac{0.055}{0.039+s} \\ \frac{1.71+14s}{0.064s^2-s^3} & \frac{1.31}{0.064+s} & \frac{-9.63}{0.064+s} \\ \frac{c}{s^2} & 0 & 0 \\ 0 & 0 & 0 \end{array}$$

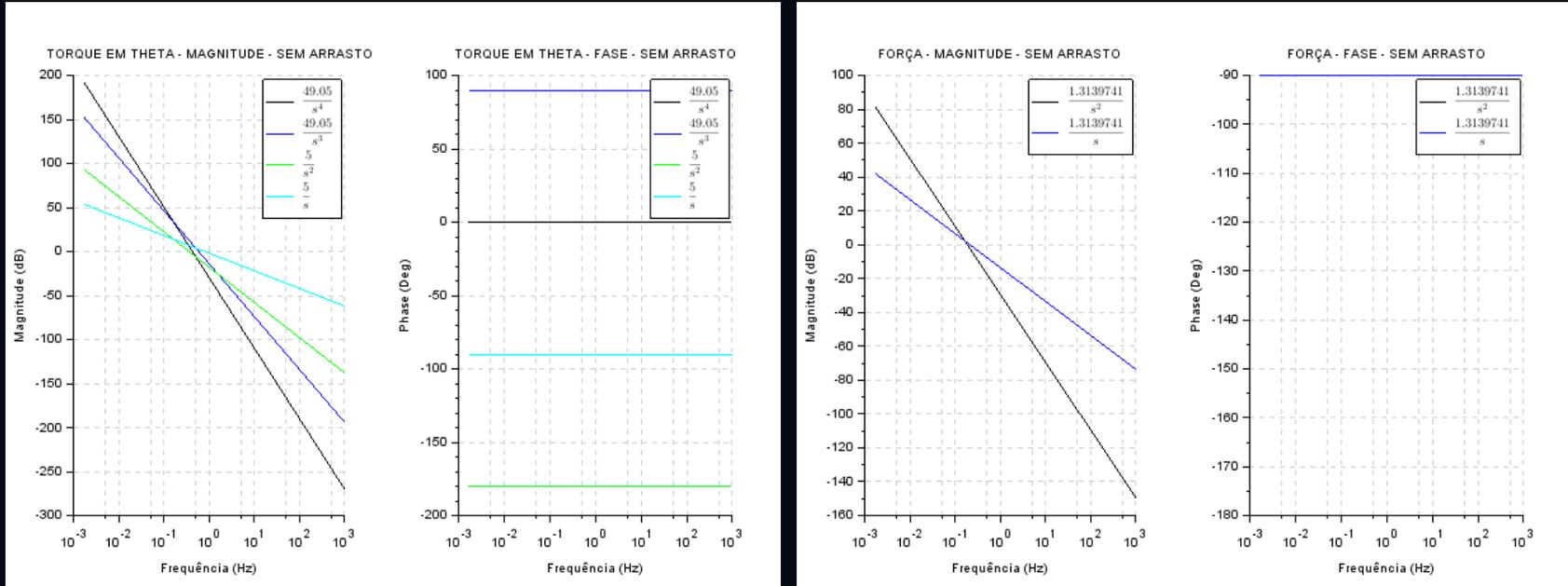
**POLOS NULOS COM ALTA MULTIPLICIDADE**  
**SISTEMA INSTÁVEL**

# CRITÉRIO DE ROUTH-HURWITZ

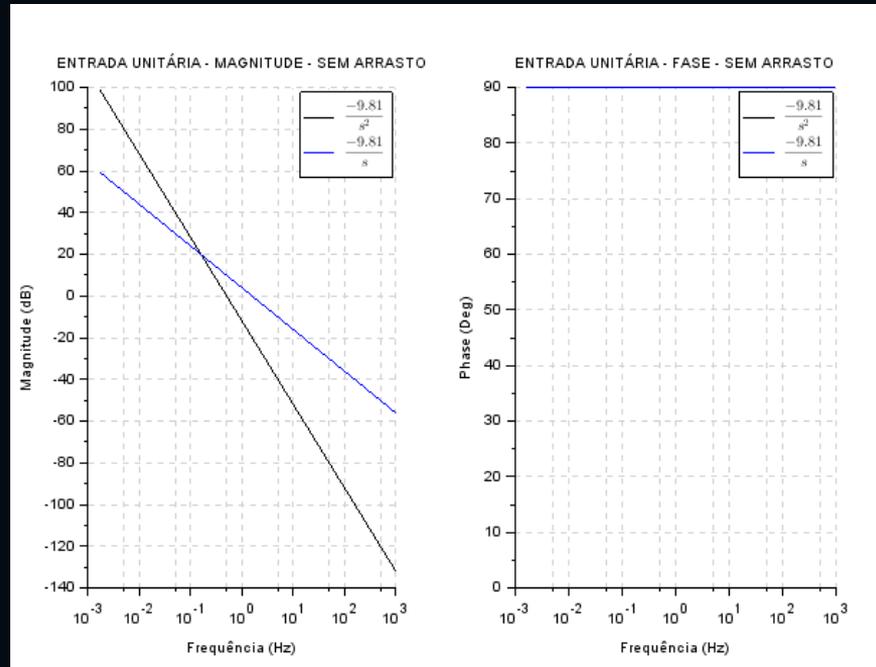
Linha				
1	$s^n$	$a_0$	$a_2$	$a_4$
2	$s^{n-1}$	$a_1$	$a_3$	$a_5$
3	$s^{n-2}$	$b_1$	$b_2$	$b_3$
4	$s^{n-3}$	$c_1$	$c_2$	$c_3$
5	$s^{n-4}$	$d_1$	$d_2$	$d_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n - 1$	$s^1$	$y_1$	$y_2$	
$n$	$s^0$	$z_1$		

*caso exista coeficientes nulos no Polinômio a ser analisado  
( $a_i = 0$ ), pode – se considerar o sistema instável*

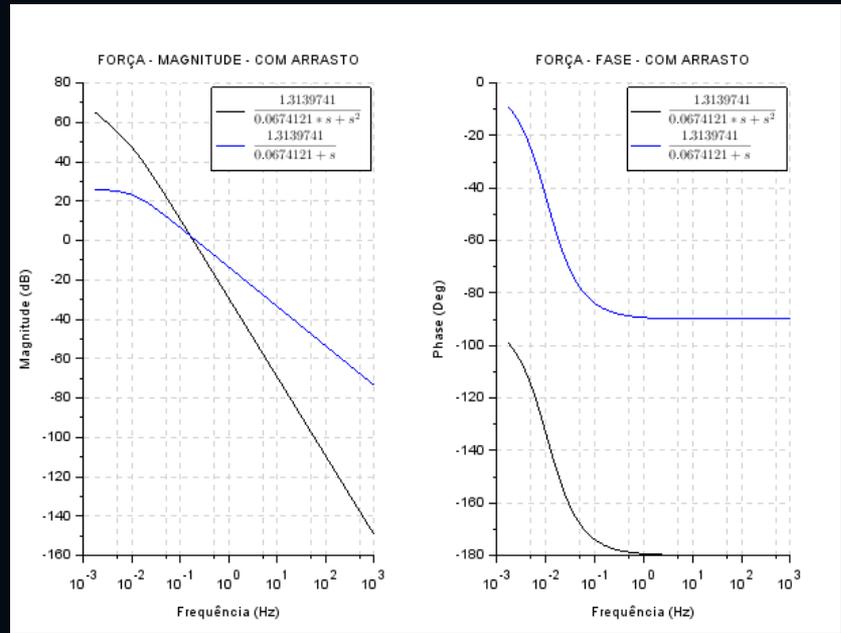
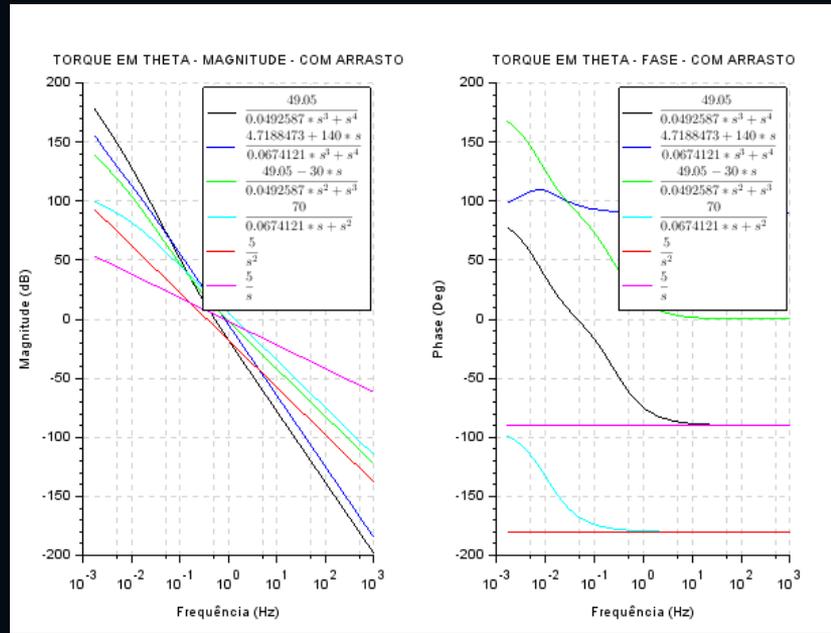
# DIAGRAMA DE BODE – SEM ARRASTO



# DIAGRAMA DE BODE – SEM ARRASTO

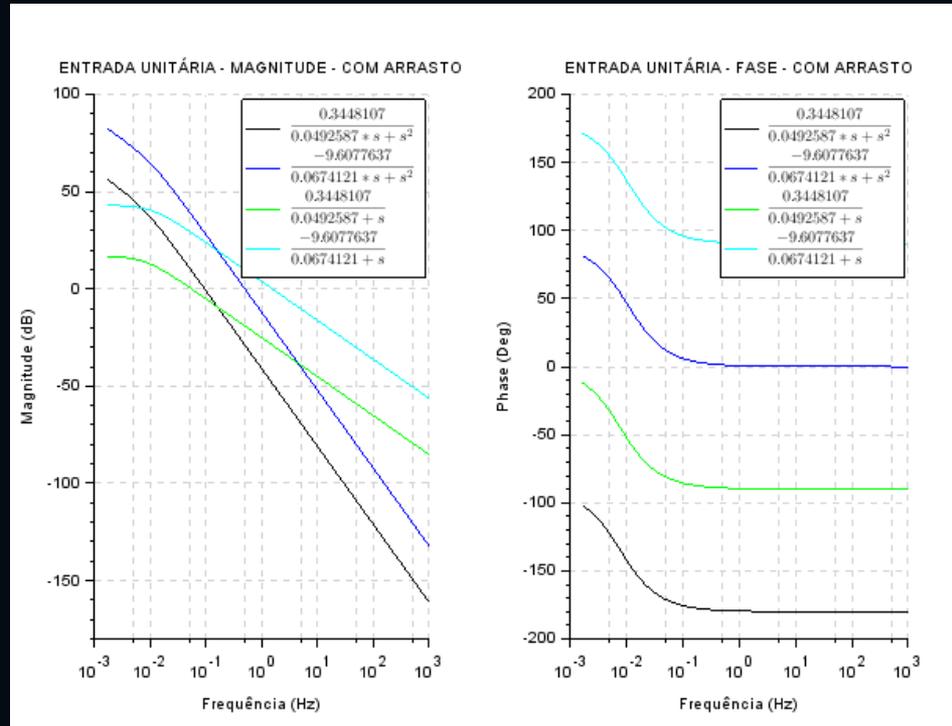


# DIAGRAMA DE BODE – SEM ARRASTO



# DIAGRAMA DE BODE – COM ARRASTO

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# CONCLUSÃO