

Leonardo João de Oliveira  
 PME3380 - Modelagem de Sistemas Dinâmicos

1) Do diagrama, temos:

$$R - Z = E \Rightarrow R - HY = E \Rightarrow R - E = HY \Rightarrow R - G^{-1}Y = HY$$

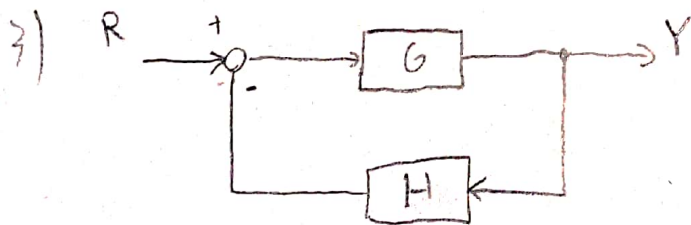
$$\Rightarrow R = (H + G^{-1})Y \Rightarrow R = (HG + I)G^{-1}Y \Rightarrow Y = G(HG + I)^{-1}R$$

Como  $Y = TR$ :

$$T = G(HG + I)^{-1} \Rightarrow T = G(I + HG)^{-1}$$

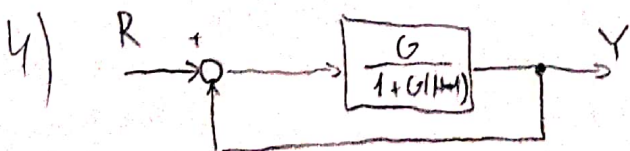
$$\therefore \boxed{G(I + HG)^{-1} = G(1 + L)^{-1}}$$

2) ?



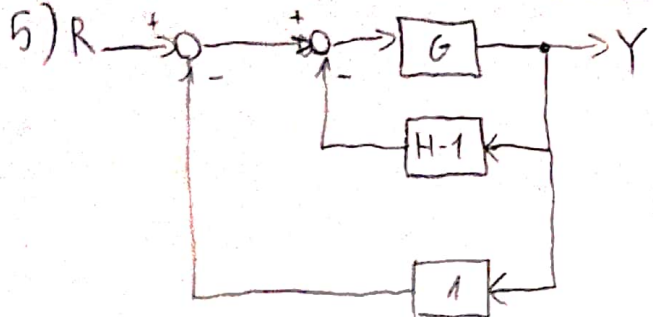
$$Y = GH \left( \frac{R}{H} - Y \right)$$

$$\Rightarrow Y = GR - GHY \Rightarrow Y(1 + GH) = GR \Rightarrow \boxed{\frac{Y}{R} = \frac{G}{1 + GH}}$$



$$Y = (R - Y) \left( \frac{G}{1 + G(H-1)} \right)$$

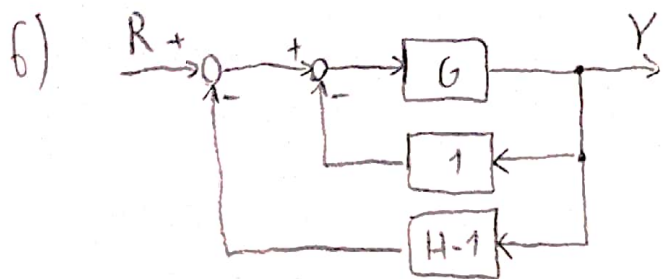
$$\Rightarrow Y \left( 1 + \frac{G}{1 + G(H-1)} \right) = \frac{RG}{1 + G(H-1)} \Rightarrow Y = \frac{RG}{1 + G(H-1)} \cdot \frac{1 + G(H-1)}{1 + G(H-1) + G} \Rightarrow \boxed{\frac{Y}{R} = \frac{G}{1 + GH}}$$



$$Y = G(R - Y) - GY(H-1)$$

$$\Rightarrow Y = GR - GX - GYH + GX$$

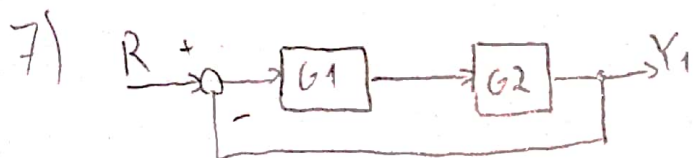
$$\Rightarrow \boxed{\frac{Y}{R} = \frac{G}{1+GH}}$$



$$Y = GR - G(H-1)Y - GY$$

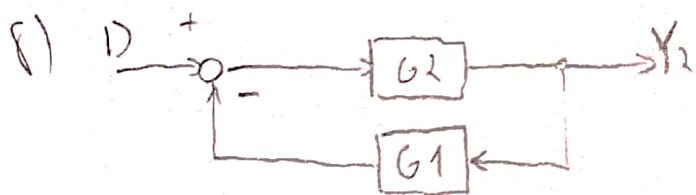
$$\Rightarrow Y = GR - GHY + GX - GY$$

$$\Rightarrow \boxed{\frac{Y}{R} = \frac{G}{1+GH}}$$



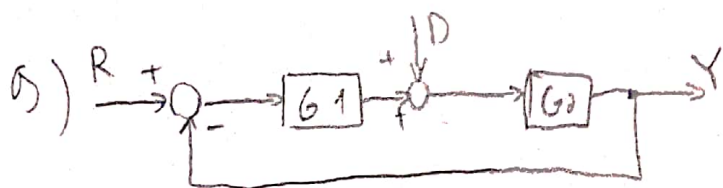
$$G_1 G_2 (R - Y_1) = Y_1$$

$$\Rightarrow \boxed{Y_1 = \frac{G_1 G_2 R}{1 + G_1 G_2}}$$



$$G_2 (D - G_1 Y_2) = Y_2 \Rightarrow G_2 D - G_1 G_2 Y_2 = Y_2$$

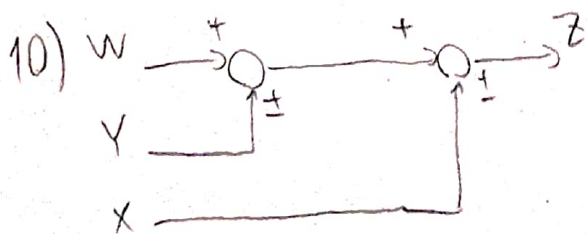
$$\Rightarrow \boxed{Y_2 = \frac{G_2 D}{1 + G_1 G_2}}$$



$$G_2 [G_1 (R - Y) + D] = Y$$

$$\Rightarrow G_1 G_2 R - G_1 G_2 Y + G_2 D = Y \Rightarrow Y = \frac{G_1 G_2 R}{1 + G_1 G_2} + \frac{G_2 D}{1 + G_1 G_2}$$

Percebe-se que, de fato  $Y = Y_1 + Y_2$ , conforme calculado anteriormente.



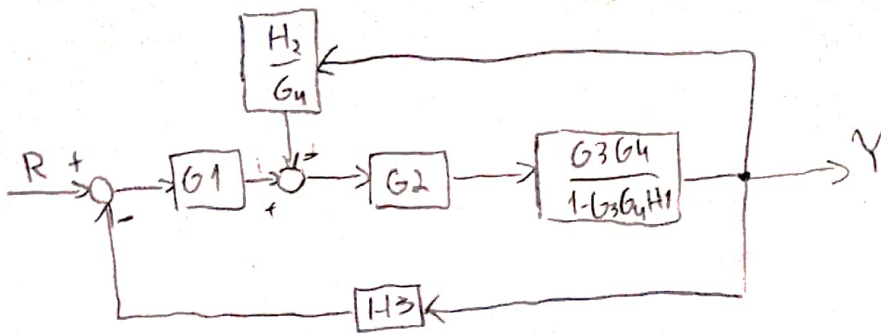
$$W + (\pm X \pm Y) = Z$$

$$\Rightarrow \boxed{Z = W \pm X \pm Y}$$

$$(W \pm Y) \pm X = Z$$

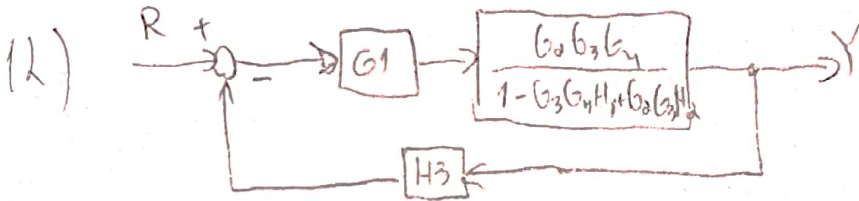
$$\Rightarrow \boxed{Z = W \pm X \pm Y}$$

11)



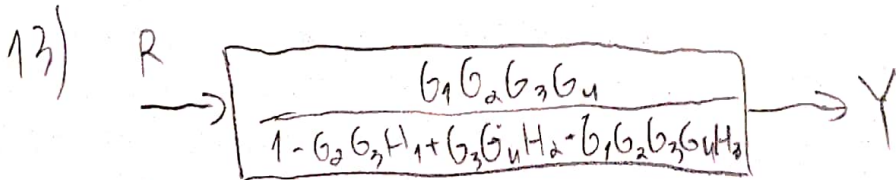
$$\frac{G_3 G_4}{1 - G_3 G_4 H_1} \left\{ G_2 \left[ G_1 (R - H_3 Y) - \frac{H_2}{G_4} Y \right] \right\} = Y$$

$$\Rightarrow \frac{Y}{R} = \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3}$$



$$\frac{G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2} \left[ G_1 (R - H_3 Y) \right] = Y$$

$$\Rightarrow \frac{Y}{R} = \frac{G_1 G_2 G_3 G_4}{1 - G_2 G_3 H_1 + G_3 G_4 H_2 + G_1 G_2 G_3 G_4 H_3}$$



$$\frac{G_1 G_2 G_3 G_4}{1 - G_2 G_3 H_1 + G_3 G_4 H_2 + G_1 G_2 G_3 G_4 H_3} R = Y$$

$$\Rightarrow \frac{Y}{R} = \frac{G_1 G_2 G_3 G_4}{1 - G_2 G_3 H_1 + G_3 G_4 H_2 + G_1 G_2 G_3 G_4 H_3}$$