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$$1) a) \det(sI - A) = \begin{vmatrix} s & -2 \\ 12 & s+4 \end{vmatrix} = s^2 + 4s + 24$$

$$\Rightarrow \text{POLOS} \Rightarrow \begin{cases} p_1 = -2 + \sqrt{5}i \\ p_2 = -2 - 2\sqrt{5}i \end{cases} \rightarrow \text{SISTEMA INSTÁVEL}$$

$$G(s) = C (sI - A)^{-1} B + D$$

$$\Rightarrow G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s & -2 \\ 12 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow G(s) = \frac{2}{s^2 + 4s + 24} \Rightarrow \begin{array}{l|l} s^2 & 1 & 24 \\ s^1 & 4 & 0 \\ s^0 & 24 & 0 \end{array}$$

$$b) \omega_n = \sqrt{2^2 + (2\sqrt{5})^2} = \sqrt{4 + 20} = \sqrt{24} = 2\sqrt{6} \text{ rad/s}$$

$$\Rightarrow \omega_n = \sqrt{\frac{k}{m}} \Rightarrow k = m \omega_n^2 = 24 \text{ N/m}$$

$$\Rightarrow \zeta = \frac{c}{m \omega_n} = \frac{2}{2\sqrt{6}} = \frac{\sqrt{6}}{6} \Rightarrow \boxed{c = 2 \sqrt{6} \text{ Km} = 2 \text{ Ns/m}}$$

$$c) \omega_d = \omega_n \sqrt{1 - \zeta^2} = 4,46 \text{ rad/s}$$

$$\omega_r = \omega_n \sqrt{1 - 2\zeta^2} = 4 \text{ rad/s}$$

$$\Rightarrow M_{r, \text{deg}} = \exp\left(\frac{-\pi \zeta}{\sqrt{1 - \zeta^2}}\right) = 0,24 \text{ s}$$

$$\Rightarrow M_{r, \text{res}} = \frac{-m(\omega_0^2)}{\zeta \omega_n} = 1,955$$

$$d) \Phi(\Delta t) = e^{A \Delta t}; \Delta t = 0,5$$

$$\Rightarrow \Phi(\Delta t) = \begin{bmatrix} -0,0976 & 0,1294 \\ -0,7766 & -0,3595 \end{bmatrix}$$

$$\Rightarrow \bar{\Phi}(s) = (sI - A)^{-1} = \begin{bmatrix} s & -2 \\ 12 & s+4 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{s+4}{s^2+4s+24} & \frac{2}{s^2+4s+24} \\ \frac{-12}{s^2+4s+24} & \frac{s}{s^2+4s+24} \end{bmatrix}$$

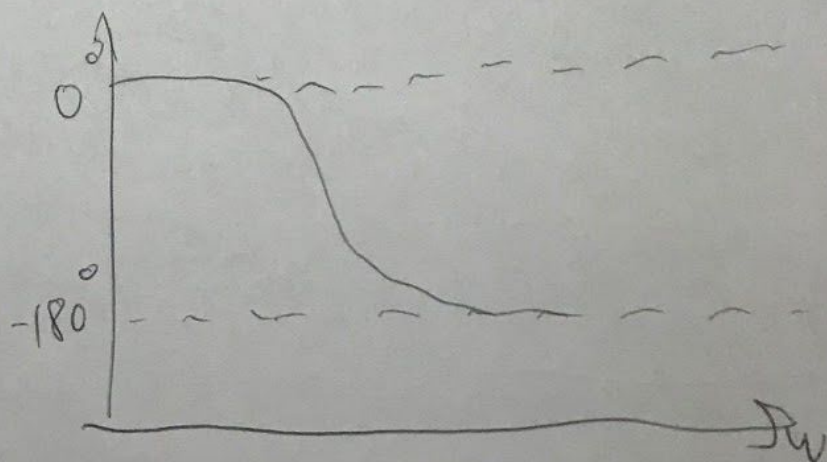
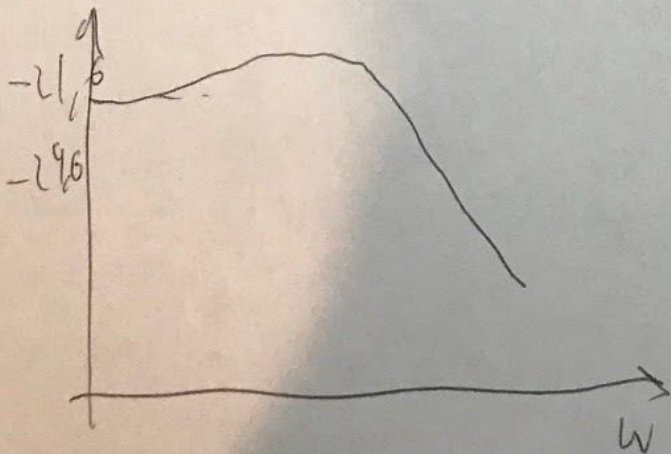
$$e) T = A^{-1}(\Phi - I) \Rightarrow$$

$$\Rightarrow T = \begin{bmatrix} 0,248 & 0,091 \\ -0,549 & 0,065 \end{bmatrix} \Rightarrow$$

$$\Rightarrow x(t) = \Phi x + T B u = \begin{bmatrix} 0,051 \\ 0,065 \end{bmatrix}$$

$$f) G(s) = \frac{2}{s^2+4s+24} \Rightarrow G(j\omega) = \frac{2}{24\left(11-\frac{\omega}{24}\right)^2 + \frac{\omega}{6}j}$$

$$\Rightarrow K_B = \sqrt{12}$$



$$g) G(s) = \frac{2}{s^2 + 4s + 29}$$

$$h) G_F = G \cdot G_1 = \frac{2s + 4}{(s+12)(s^2 + 4s + 29)}$$

$$2) a) Y_1 = E_1 \cdot G_1 = G_1 (U_1 - G_3 Y_2) = G_1 (U_1 - G_3 G_4 (U_2 - G_2 Y_1)) = 0$$

$$\Rightarrow Y_1 = \frac{G_1 U_1 - G_1 G_3 G_4 U_2}{1 - G_1 G_2 G_3 G_4}$$

$$b) E_1 = U_1 - G_3 Y_2 \Rightarrow$$

$$\Rightarrow E_1 = U_1 - G_3 G_4 U_2 + G_3 G_4 G_1 G_2 E_1$$

$$\Rightarrow \frac{E_1}{U_1} = \frac{1}{1 - G_1 G_2 G_3 G_4} \quad ; \quad \frac{E_1}{U_2} = \frac{1}{1 - G_1 G_2 G_3 G_4}$$

$$3) a) (s^3 + 0,4511s^2 + 0,3015s + 0,0589)s = 0$$

$$\Rightarrow \begin{cases} s = 0 \\ s = -0,366 \\ s = -0,0426 \pm 0,518i \\ s = -0,0426 \mp 0,518i \end{cases}$$

$$b) z_1 = 0; z_2 = 2,2812; z_3 = -27339; z_4 = 1,57 \cdot 10^{-3}$$

$$d) \begin{array}{c|cc} s^3 & 1 & 0,3015 \\ s^2 & 0,4511 & 0,0589 \\ s^1 & 0,0823 & 0 \\ s^0 & 0,0589 & 0 \end{array} \Rightarrow \text{ESTÁVEL}$$

$$g) \omega_n = 0,5199 \text{ rad/s} \\ \omega_r = 0,519 \text{ rad/s}$$

$$h) M_p, \text{deg} = \exp\left(\frac{-\xi \pi}{\sqrt{1-\xi^2}}\right) = 0,7$$