

POUSO DE FOGUETE

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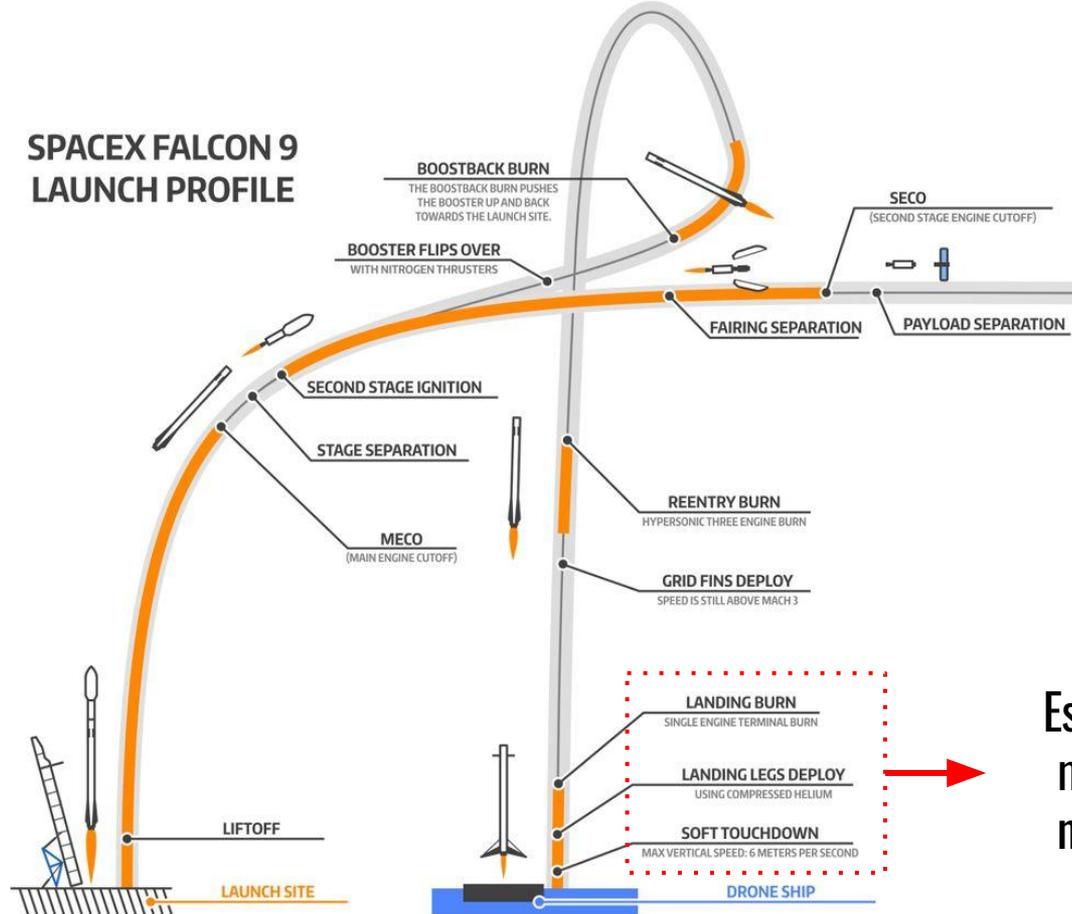
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Modelo Real

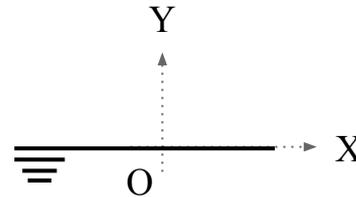
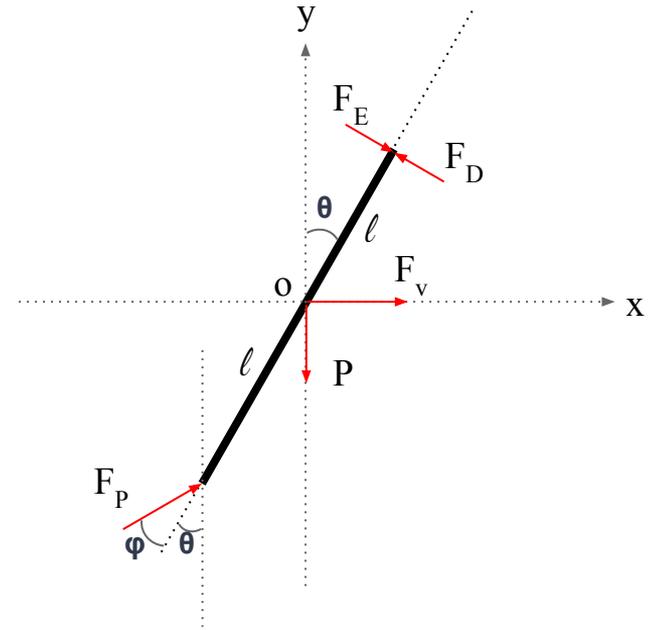
SPACEX FALCON 9 LAUNCH PROFILE



Esta parte será
modelada em
nosso projeto

Modelo físico e Hipóteses simplificadoras

- O foguete se desloca exclusivamente no plano vertical;
- Apenas o ângulo de arfagem será considerado;
- Foguete será considerado como uma barra homogênea;
- Diâmetro do foguete será muito menor que sua altura;
- Massa do foguete será constante durante todo o pouso;
- Considera-se os propulsores como pontuais;
- Força do vento será uma resultante horizontal em "o";
- Corpo infinitamente rígido;
- Efeitos da rotação terrestre desprezíveis;
- Forças de arrastamento serão desconsideradas;
- A balsa será fixa.



Equações Dinâmicas

- Teorema da Resultante;
- Teorema da Quantidade de Movimento.

$$\begin{cases} m\ddot{x} = F_v + F_{ps} + F_p \sin(\theta + \varphi) \\ m\ddot{y} = F_d \cos(\theta + \varphi) - F_{ps} \theta - P \\ J_o \ddot{\theta} = F_{ps} l - F_p \sin \varphi l \end{cases}$$

$$F_{ps} = F_e - F_d$$

Espaço de Estados

$$\begin{array}{lll} x_1 = x & \dot{x}_1 = x_2 & = f_1(x_2) \\ x_2 = \dot{x} & \dot{x}_2 = [F_v + F_{ps} + F_p \sin(\theta + \varphi)]/m & = f_2(F_v, F_{ps}, F_p, \theta, \varphi) \\ x_3 = y & \dot{x}_3 = x_4 & = f_3(x_4) \\ x_4 = \dot{y} & \dot{x}_4 = [F_p \cos(\theta + \varphi) - F_{ps} \theta - P] / m & = f_4(F_p, F_{ps}, P, \theta, \varphi) \\ x_5 = \theta & \dot{x}_5 = x_6 & = f_5(x_6) \\ x_6 = \dot{\theta} & \dot{x}_6 = [F_{ps} l - F_p \sin \varphi l] / J_o & = f_6(F_{ps}, F_p, \varphi) \end{array}$$

Linearização

- Expansão de Taylor e desprezando termos de ordem superior

$$f_1(x_2) = 0 + x_2$$

$$f_2(F_v, F_{ps}, F_p, \theta, \varphi) = 0 + \frac{1}{m}F_v + \frac{1}{m}F_{ps} + [\sin(\theta + \varphi)]/m \times F_p \\ + F_p \cos(\theta + \varphi)/m \times \theta + F_p \cos(\theta + \varphi)/m \times \varphi$$

$$f_3(x_2) = 0 + x_4$$

$$f_4(F_{ps}, F_p, P, \theta, \varphi) = 0 - \frac{\theta}{m}F_{ps} + \cos(\theta + \varphi)/m \times F_p - \frac{1}{m}P \\ - F_p \sin(\theta + \varphi)/m \times \theta - F_p \sin(\theta + \varphi)/m \times \varphi$$

$$f_5(x_6) = 0 + x_6$$

$$f_6(F_{ps}, F_p, \varphi) = 0 + \frac{l}{J_o}F_{ps} - \sin(\varphi)l/J_o \times F_p + F_p \cos(\varphi)/J_o \times \varphi$$

Matrizes essenciais

- Matrizes de estados, de entradas, de saídas e de alimentação direta

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & [\bar{F}_p \cos(\bar{\theta} + \bar{\varphi})]/m \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & [-\bar{F}_p \sin(\bar{\theta} + \bar{\varphi}) - \bar{F}_{ps}]/m \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1/m & 1/m & \sin(\bar{\theta} + \bar{\varphi})/m & [\bar{F}_p \cos(\bar{\theta} + \bar{\varphi})]/m & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\bar{\theta}/m & \cos(\bar{\varphi} + \bar{\theta})/m & -\bar{F}_p \sin(\bar{\varphi} + \bar{\theta})/m & -1/m \\ 0 & 0 & 0 & 0 & 0 \\ 0 & l/J_o & -\sin \bar{\varphi} l/J_o & \bar{F}_p \cos \bar{\varphi} l/J_o & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

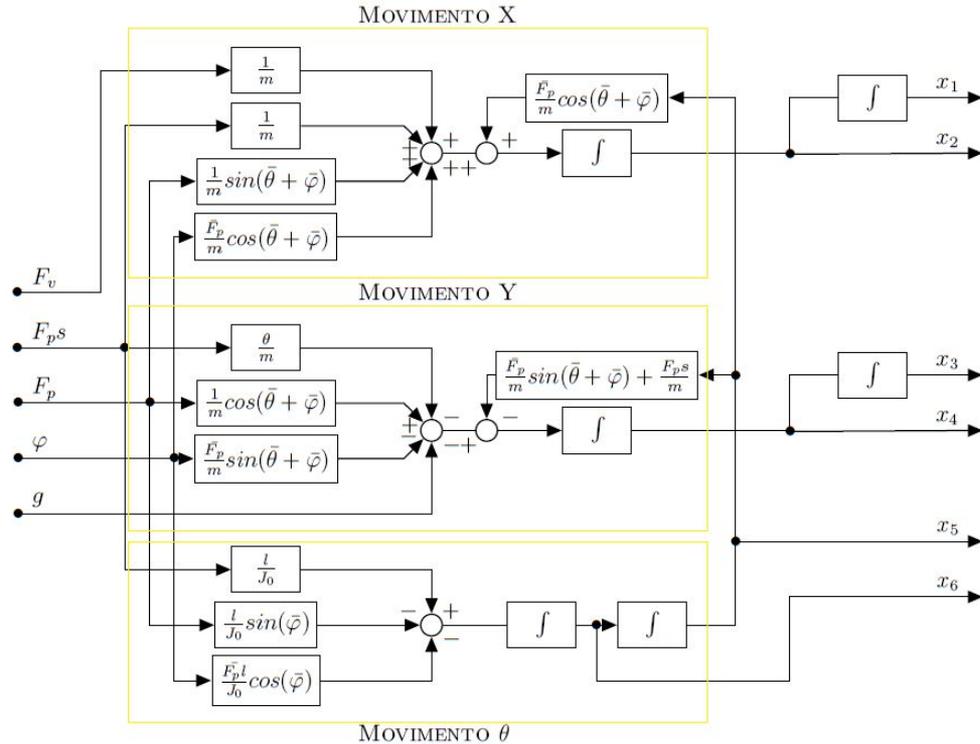
Matrizes essenciais

- Matrizes de estados, de entradas, de saídas e de alimentação direta

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & [\bar{F}_p \cos(\bar{\theta} + \bar{\varphi})]/m & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & [-\bar{F}_p \sin(\bar{\theta} + \bar{\varphi}) - \bar{F}_{ps}]/m & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1/m & 1/m & \sin(\bar{\theta} + \bar{\varphi})/m & [\bar{F}_p \cos(\bar{\theta} + \bar{\varphi})]/m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\theta/m & \cos(\bar{\varphi} + \bar{\theta})/m & -\bar{F}_p \sin(\bar{\varphi} + \bar{\theta})/m & -1/m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & l/J_o & -\sin \bar{\varphi} l/J_o & \bar{F}_p \cos \bar{\varphi} l/J_o & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}$$

Diagrama de Blocos



Parâmetros do modelo

- Massa do foguete: 25222 kg
- Altura do foguete: 44 m
- Diâmetro do foguete: 4,6 m
- Momento de inercia: $4,07 \times 10^6 \text{ kgm}^2$
- Gravidade e peso: $9,8 \text{ m/s}^2$ e 247.175,6 N
- Força do vento: $166,23 \times V_{\text{ar}}^2$
- Tempo de pouso: 20 segundos
- Velocidade vertical de pouso: 6 m/s
- Velocidade horizontal de pouso: 5,4 m/s
- Ângulo de arfagem para o pouso: 23°

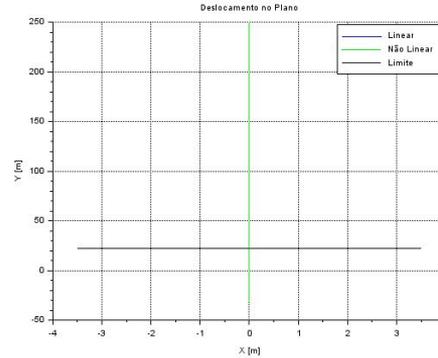


Linear Vs. Não Linear

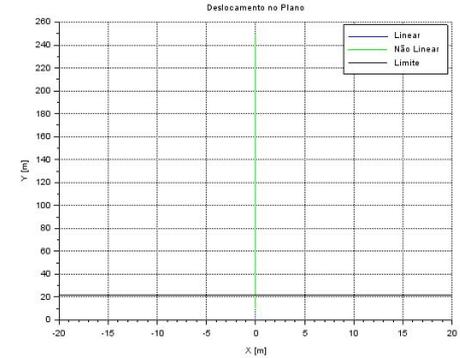
Deslocamentos no plano

- Caso 1: Queda livre vertical
- Caso 2: Queda vertical em equilíbrio
- Caso 3: Queda em desaceleração
- Caso 4: Queda em desaceleração com vento

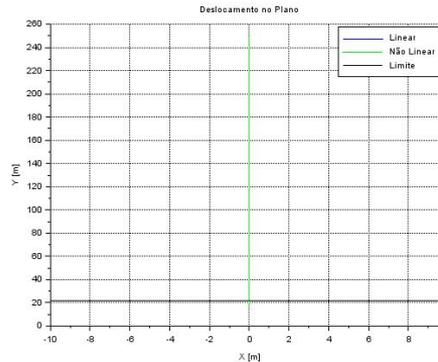
Caso 1



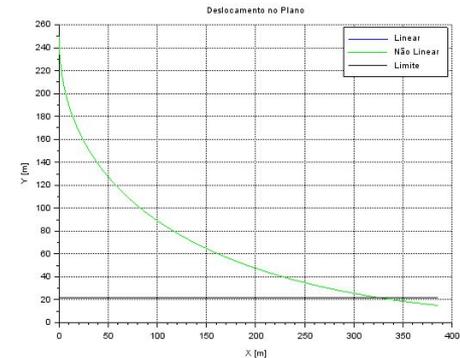
Caso 2



Caso 3

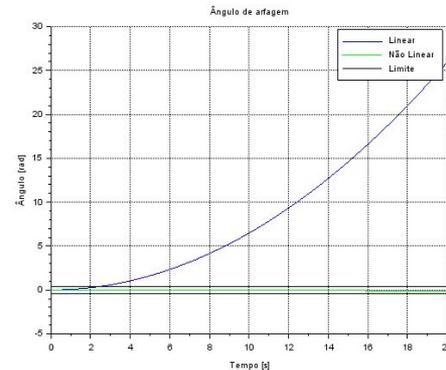
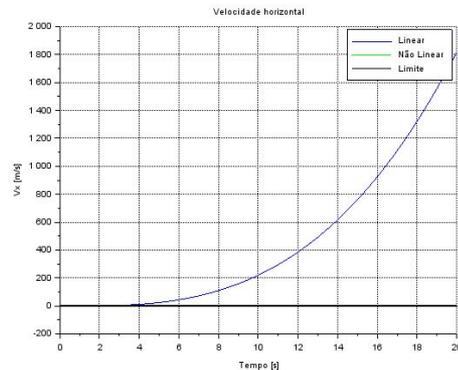
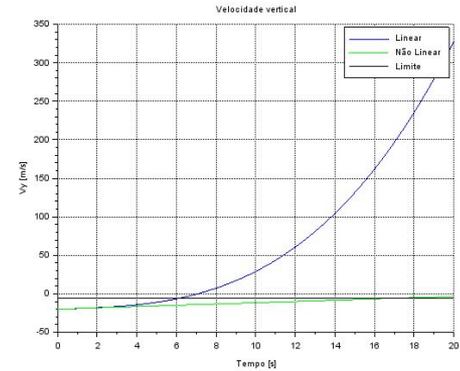
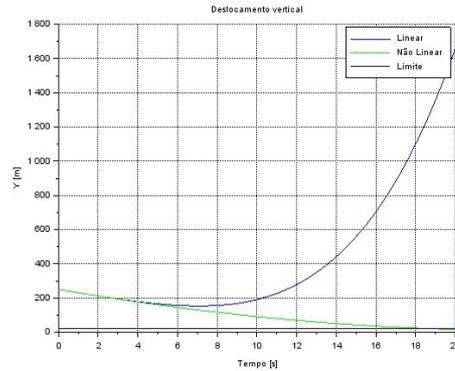
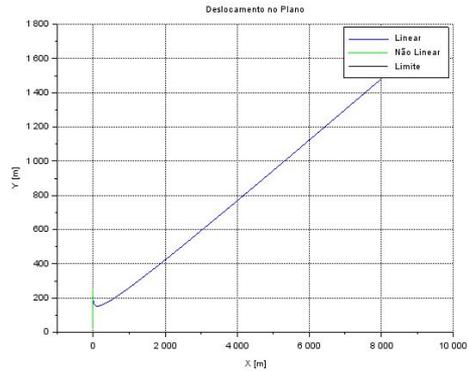


Caso 4



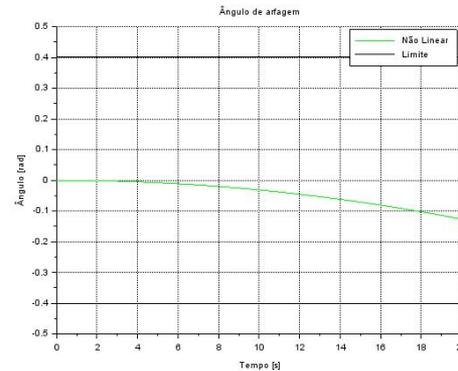
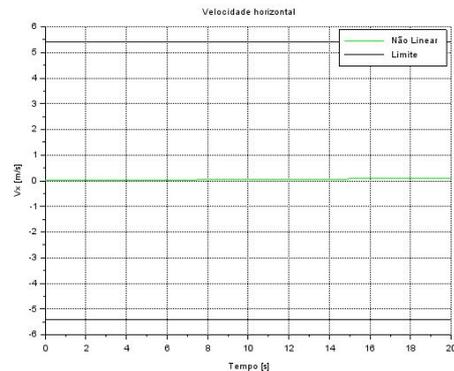
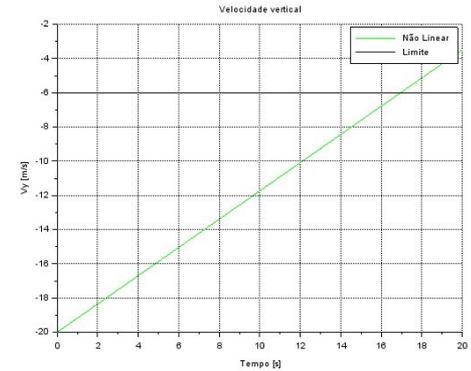
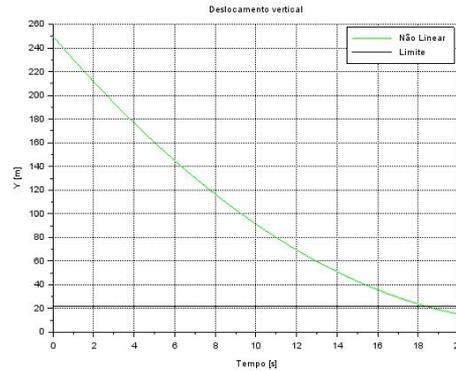
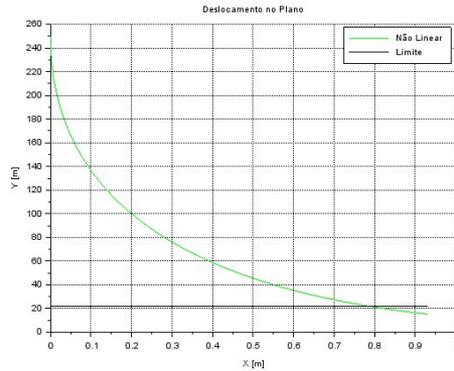
Linear Vs. Não Linear

- Caso 5: Caso em desaceleração com vento e resposta dos propulsores secundário



Linear Vs. Não Linear

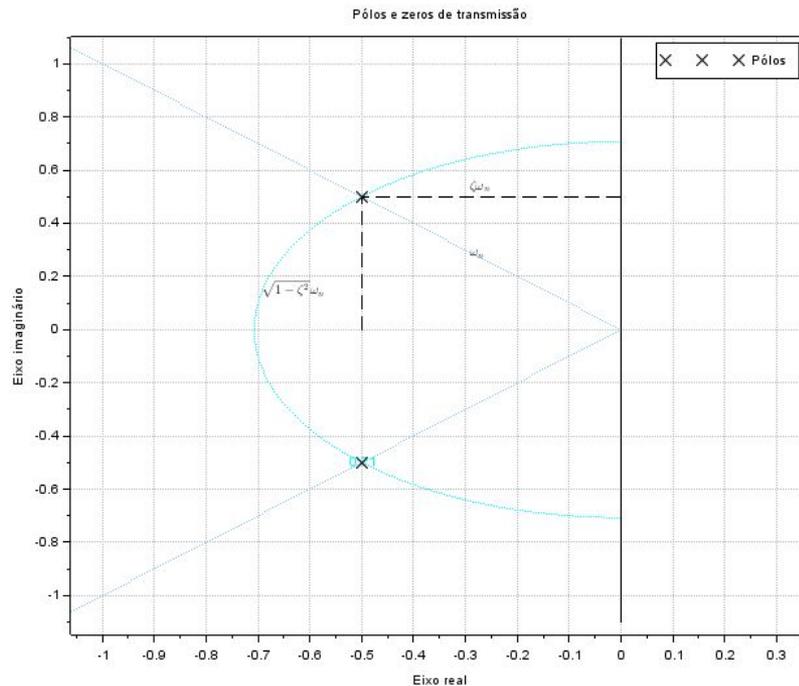
- Caso 5: Caso em desaceleração com vento e resposta dos propulsores secundário



Análise no Domínio de Laplace

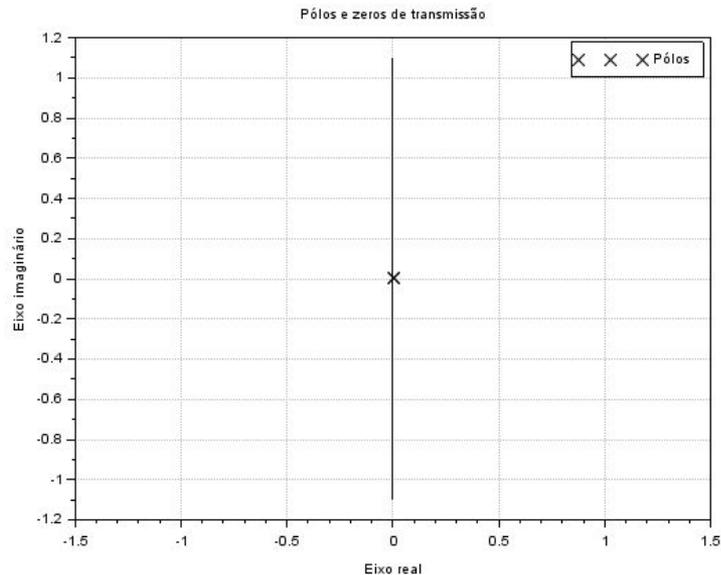
- Transformada de Laplace
- Pólos e Zeros
- Estabilidade ditada pela posição dos pólos da equação característica

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st} dt$$



Análise no Domínio de Laplace

- Movimento Translacional X
 - 2 pólos na origem



$$\frac{X}{F_V}(s) = \frac{1}{ms^2}$$

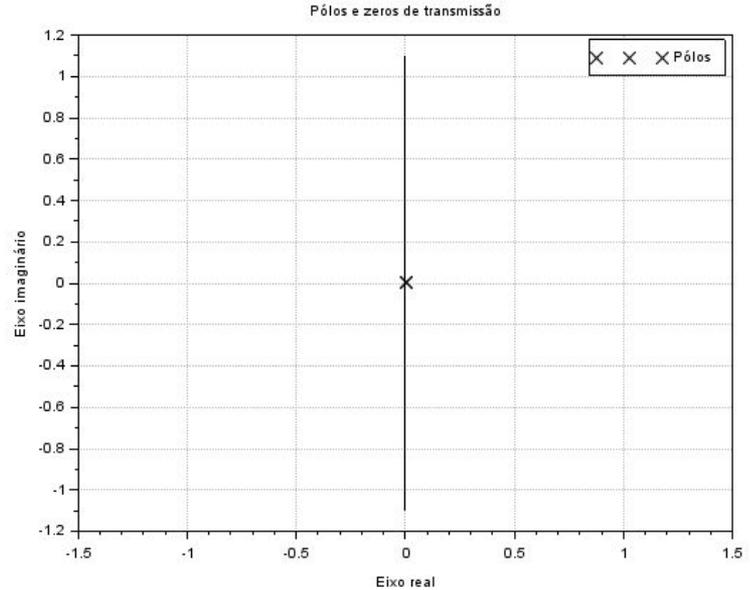
$$\frac{X}{F_{PS}}(s) = \frac{1}{ms^2}$$

$$\frac{X}{F_P}(s) = \frac{\text{sen}(\bar{\theta} + \bar{\varphi})}{ms^2}$$

$$\frac{X}{\Phi}(s) = \frac{\bar{F}_p \cos(\bar{\theta} + \bar{\varphi})}{ms^2}$$

Análise no Domínio de Laplace

- Movimento Translacional Y
 - 2 pólos na origem
 - 3 pólos na origem



$$\frac{Y}{F_{PS}}(s) = -\frac{\bar{\theta}}{ms^2}$$

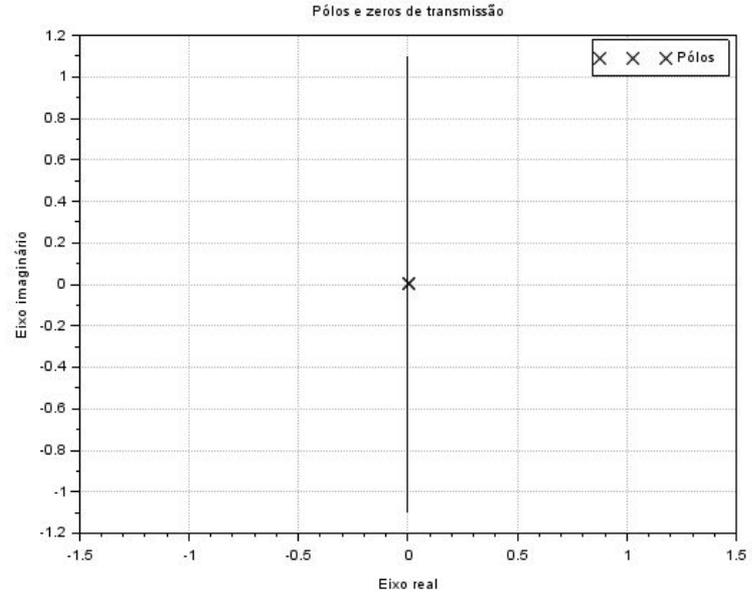
$$\frac{Y}{F_P}(s) = \frac{\cos(\bar{\theta} + \bar{\varphi})}{ms^2}$$

$$\frac{Y}{\Phi}(s) = -\frac{\bar{F}_p \text{sen}(\bar{\theta} + \bar{\varphi})}{ms^2}$$

$$Y(s) = -\frac{g}{s^3}$$

Análise no Domínio de Laplace

- Movimento Rotacional θ
 - 2 pólos na origem



$$\frac{\Theta}{F_{PS}}(s) = \frac{l}{J_0 s^2}$$

$$\frac{\Theta}{F_P}(s) = -\frac{\text{sen}(\bar{\varphi})l}{J_0 s^2}$$

$$\frac{\Theta}{\Phi}(s) = \frac{\bar{F}_p \text{cos}(\bar{\varphi})l}{J_0 s^2}$$

Retorno ao Domínio do tempo

- Aplicação da Transformada Inversa de Laplace nas funções de transferência anteriores

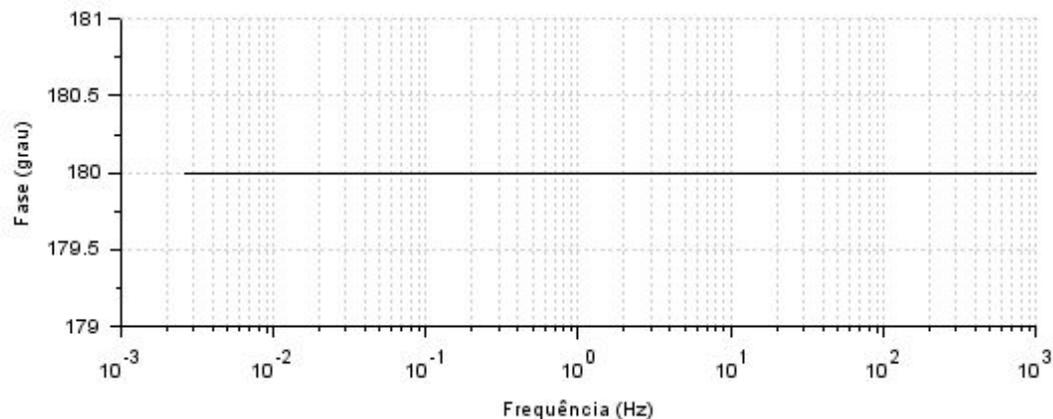
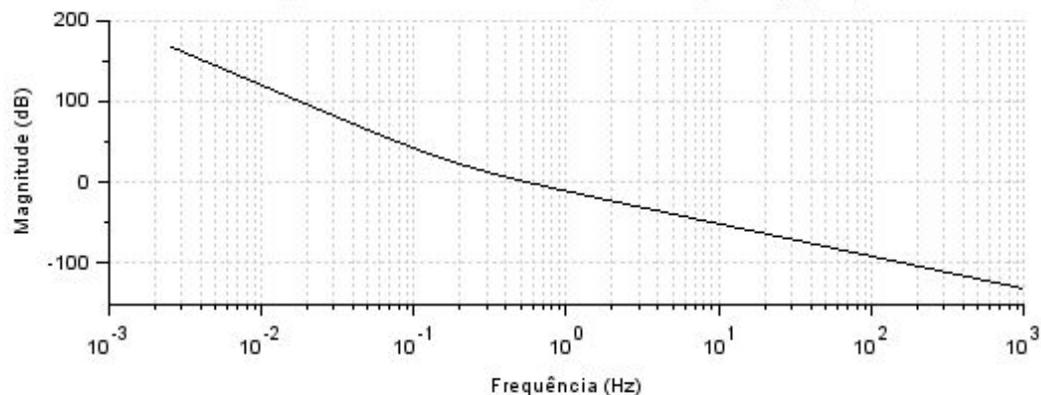
$$\mathcal{L}^{-1} \left[\frac{1}{s^n} \right] = \frac{t^{n-1}}{(n-1)!}$$

$$\left\{ \begin{array}{l} x(t) = \frac{t}{m} [F_V + F_{PS} + F_P [\text{sen}(\bar{\theta} + \bar{\varphi})] + \Phi [\bar{F}_p \text{cos}(\bar{\theta} + \bar{\varphi})]] \\ y(t) = \frac{t}{m} [-F_{PS} \bar{\theta} + F_P \text{cos}(\bar{\theta} + \bar{\varphi}) - \Phi \bar{F}_p \text{sin}(\bar{\theta} + \bar{\varphi}) - \frac{tP}{2}] \\ \theta(x) = \frac{t}{J_0} [F_{PS} l - F_P \text{sen}(\bar{\varphi})l + \Phi \bar{F}_p \text{cos}(\bar{\varphi})l] \end{array} \right.$$

Estudo dos Diagramas de Bode

- Natureza assintoticamente estável

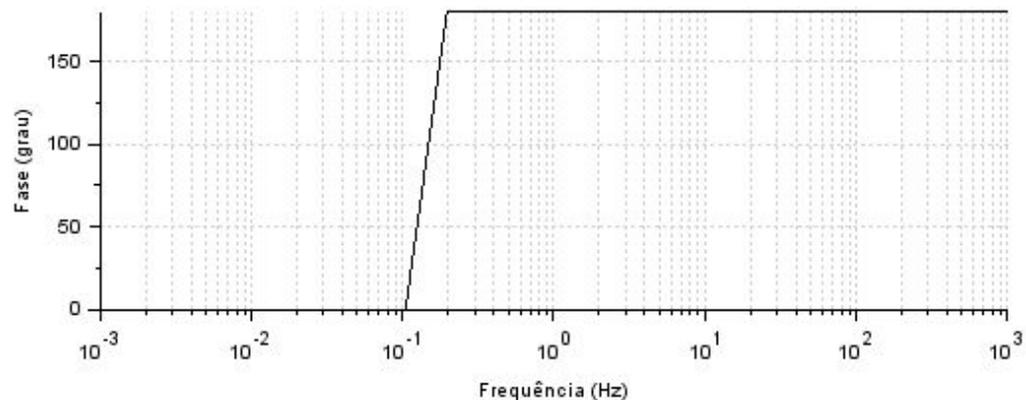
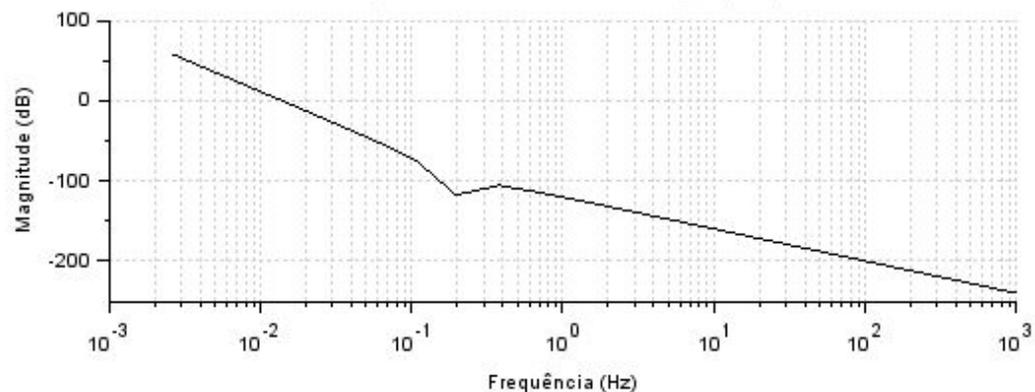
*Diagramas de Bode : $Fp * \cos(\theta + \varphi)/m/s^2$*



Estudo dos Diagramas de Bode

- Degrau e mudança na fase

Diagramas de Bode : $Fps/m/s^2$



Conclusão

Referências

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