Exercícios - aula 22/10

1) $2\ddot{x}+7\dot{x}+3x=0 \xRightarrow{\mathcal{L}} 2s^2X-2x_0 s+7sX-7x_0+3X=0$

$$X(2s^2+7s+3)=x_0(2s+7)$$

$$\Rightarrow X(s)=\frac{x_0(2s+7)}{2s^2+7s+3}$$

Pólos: $2s^2+7s+3=0$

$49-4\cdot 2\cdot 3=25$

$\frac{-7\pm 5}{4}=-3$ ou $-1/2$

$$X(s)=\frac{(2s+7)x_0}{2(s+3)(s+1/2)}=\frac{x_0}{2}\left(\frac{-6+7}{-3+1/2}\cdot\frac{1}{(s+3)}+\frac{-1+7}{-1/2+3}\cdot\frac{1}{(s+1/2)}\right)=\frac{-x_0}{5(s+3)}+\frac{6x_0}{5(s+1/2)}$$

$$\Rightarrow X(t)=\frac{-x_0}{5}e^{-3t}+\frac{6x_0}{5}e^{-1/2}$$

2) $\dddot{x}+2\ddot{x}+7\dot{x}=\ddot{u}+7\dot{u}+5u \xRightarrow{\mathcal{L}} s^3X-s^2X(0)-s\dot{X}(0)-\ddot{X}(0)+2(s^2X-sX(0)-\dot{X}(0))+7(sX-X(0))=$

$$=s^2U-sU(0)-\dot{U}(0)+7(sU-U(0))+5U$$

Reorganizando a eq e substituindo $\ddot{x}(0)=2$, $\dot{x}(0)=1$, $x(0)=9$, $u(0)=1$, $\dot{u}(0)=0$:

$$X(s^3+2s^2+7s)=U(s^2+7s+5)+9s^2+18s+60$$

$$U(s)=\frac{1}{s} \Rightarrow X(s)=\frac{s^2+7s+5}{s^2(s^2+2s+7)}+\frac{9s^2+18s+60}{s(s^2+2s+7)}$$

$G(s)=\frac{X(s)}{U(s)}=\frac{s^2+7s+5}{s(s^2+2s+7)}$ pólos: $s=0$, $s=1+2{,}45j$, $s=-1-2{,}45j$ $\Rightarrow$ estável

Reescrevemos $X(s)$:

2) $\begin{cases} m_1\ddot{x}_1 = U_1 + k(x_2 - x_1) \\ m_2\ddot{x}_2 = U_2 - k(x_2 - x_1) \end{cases}$ $\quad \begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/m_1 & 0 & k/m_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1/m_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ \\ U_2 \end{bmatrix}$

$$\frac{9s^3 + 12s^2 + 67s + 5}{s^2(s^2+2s+7)} = \frac{a}{s} + \frac{b}{s^2} + \frac{cs + d}{s^2+2s+7}, \quad a = \frac{459}{49} \quad b = \frac{5}{7}, \ c = -18/49, \ d = -22/49$$

$$X(s) = \frac{459}{49s} + \frac{5}{7s^2} - \frac{18s/49 + 22/49}{s^2+2s+7} \xRightarrow{\mathcal{L}^{-1}(X(s))} x(t) = \frac{459}{9} + \frac{5}{7}t - \frac{18}{49}e^{-t}\cos(\sqrt{6}t) - \frac{4}{49\sqrt{6}}e^{-t}\operatorname{sen}(\sqrt{6}t)$$

Exercícios - aula 3/11