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Ex 22/10 - PME 3380

(1) $2\ddot{x} + 7\dot{x} + 3x = 0$

$$\begin{cases} x(0) = x_0 \\ \dot{x}(0) = 0 \end{cases}$$

$\downarrow$

$$2(s^2 X - s x_0) + 7(sX - x_0) + 3X = 0$$

- $X = \dfrac{x_0(2s+7)}{2s^2+7s+3} = \dfrac{x_0(2s+7)}{2(s+3)(s+1/2)}$

- $X = \dfrac{A}{s+3} + \dfrac{B}{2s+1} = \dfrac{2As + A + Bs + 3B}{(s+3)(2s+1)}$

$$\begin{cases} A = -x_0/5 \\ B = 12x_0/5 \end{cases}$$

Aplicamos a transformada inversa:

$$\boxed{x(t) = -\frac{x_0}{5} e^{-3t} - \frac{6x_0}{5} e^{-t/2}}$$

(2) $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$

$$\begin{cases} \ddot{x}(0) = 2 \\ \dot{x}(0) = 1 \\ x(0) = 9 \\ u(0) = 0 \\ \dot{u}(0) = 0 \\ u(1) = 1 \end{cases}$$

$\downarrow$

$$(s^3X - 2 - s - 9s^2) + 2(s^2X - 1 - 9s) + 7(sX - 9) = (s^2U) + 7sU + 5U$$

$$X(s) = U(s) \cdot \frac{s^2 + 7s + 5}{s^3 + 2s + 7s} + \frac{9s^2 + 19s + 67}{s^3 + 2s^2 + 7s}$$

$$\{\, U(s) = 1/s$$

$$X(s) = \frac{9s^3 + 20s^2 + 74s + 5}{s^2(s^2 + 2s + 7)} = \frac{508}{49s} + \frac{5}{7s^2} - \frac{67s + 71}{49[(s+1)^2 + 6]}$$

Com isso, substituímos e obtemos:

$$x(t) = \frac{508}{49} + \frac{5t}{7} - \frac{67}{49} e^{-t} \cos(\sqrt{6}\,t) - \frac{4}{96\sqrt{6}} e^{-t} \operatorname{sen}(\sqrt{6}.t)$$