

Modelagem de Sistemas Dinâmicos

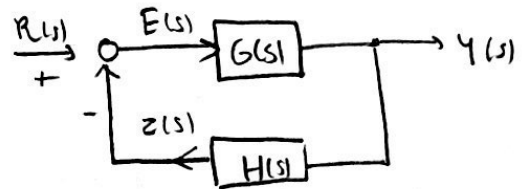
Lucas Hugo Matheo - Exercícios 03/11 - 10772911

① Nos slides da aula foi mostrado

$$Y = (I + GH)^{-1} G R ; (I + GH)^{-1} G = T$$

$$Y = TR$$

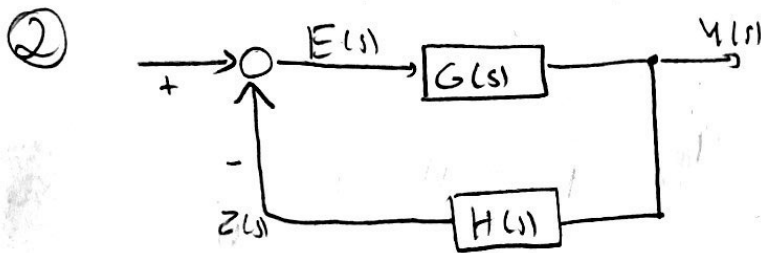
Do diagrama de blocos da FTMF



$$Z = HY \Rightarrow R - E = HY \Rightarrow R - G^{-1}Y = HY$$

$$R = Y(G^{-1} + H) \Rightarrow R = (I + HG)G^{-1}Y \Rightarrow (I + GH)^{-1}R = G^{-1}Y$$

$$\Rightarrow Y = G(I + HG)^{-1}R \Rightarrow \boxed{T = (I + GH)^{-1}G}$$



$$\rightarrow Z = HY \rightarrow Z = HG E = HG(R - Z) \rightarrow HGR = (I + HG)Z \rightarrow \underline{(I + HG)^{-1}HG = ZR^{-1}}$$

$$\rightarrow Y = GE \rightarrow H^{-1}Z = G(R - Z) \rightarrow GR = (H^{-1} + G)Z = (I + GH)H^{-1}Z \rightarrow$$

$$\underline{ZR^{-1} = H(I + GH)^{-1}G}$$

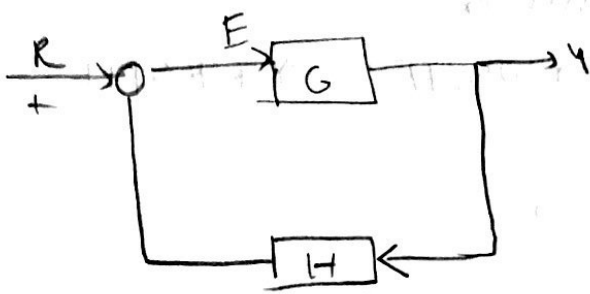
Assim, podemos juntar, com $HG = GH = L$

$$\boxed{\frac{Z}{R} = \frac{HG}{(I + GH)} = \frac{L}{1 + L}}$$

$$Z = HY = HG \overbrace{(R - Z)}^E \rightarrow Z(1 + HG) = HGR \Rightarrow \frac{Z}{R} = \frac{HG}{1 + HG}, HG = L$$

$$\Rightarrow \boxed{\frac{Z}{R} = \frac{L}{1 + L}}$$

③



$$Y = GE$$

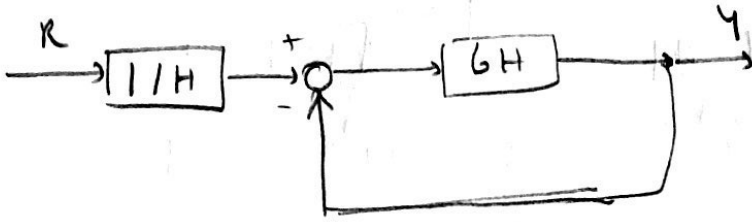
$$Y = G(R - Y)$$

$$Y = G(R - HY) = GR - GHY$$

$$GR = GHY + Y$$

$$GR = (GH + I)Y \rightarrow Y = \frac{GR}{GH + I}$$

$$Y = GR (GH + I)^{-1}$$

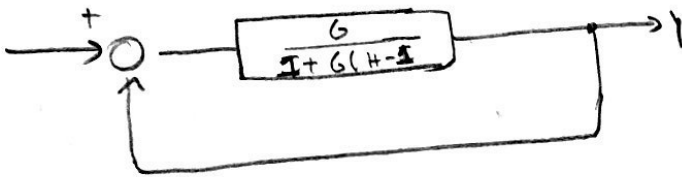


$$Y = GH \left(\frac{R - Y}{H} \right)$$

$$Y = GR - GHY \Rightarrow GR = Y(I + GH)$$

$$Y = GR (GH + I)^{-1}$$

④

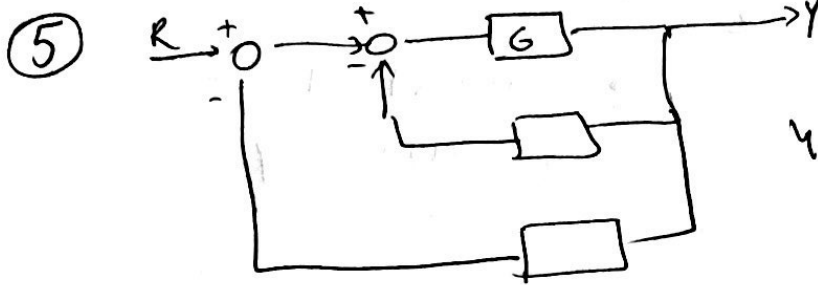


$$Y = (R - Y) \frac{G}{I + G(H-1)}$$

$$\frac{GR}{I + G(H-1)} = Y + \frac{GY}{I + G(H-1)} \rightarrow \frac{GR}{I + G(H-1)} = Y \left(1 + \frac{G}{I + G(H-1)} \right)$$

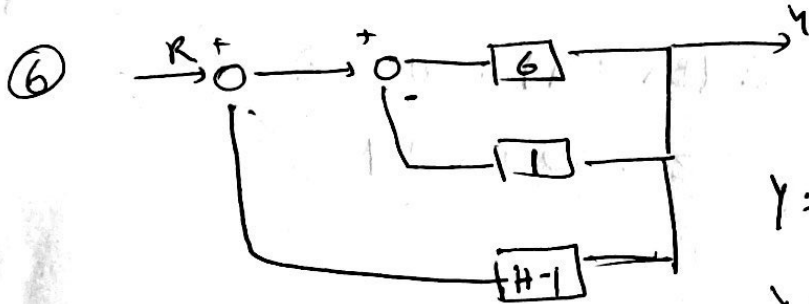
$$Y = \frac{GR}{1 + G(H-1)} \rightarrow Y = \frac{GR}{G + I + G(H-1)} \rightarrow Y = \frac{GR}{1 + GH}$$

$$Y = \frac{GR}{1 + GH}$$



$$Y = (R - Y - Y(H-1))G$$

$$Y = \frac{GR}{1+HG}$$



$$Y = [R - Y(H-1) - Y]G$$

$$Y + GY + GY(H-1) = R$$

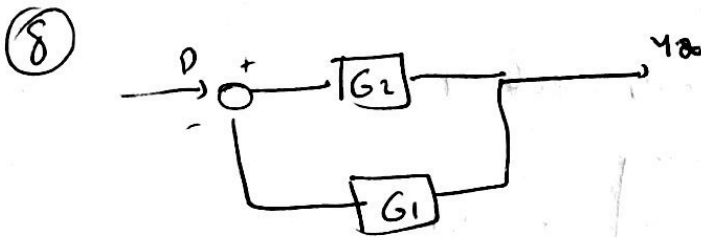
$$Y + GYH = RG$$

$$\frac{Y}{R} = \frac{G}{1+GH}$$



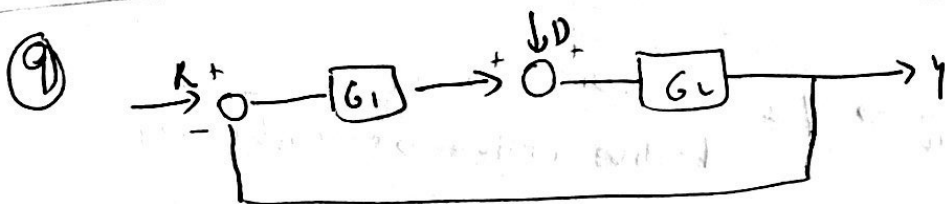
$$Y_1 = (R - Y_1)G_1G_2$$

$$Y_1 = \frac{RG_1G_2}{1+G_1G_2}$$



$$Y_2 = (D - Y_2)G_2$$

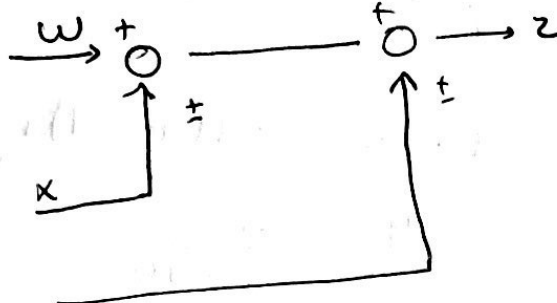
$$Y_2 = \frac{DG_2}{1+G_2}$$



$$Y = ((R - Y)G_1 + D)G_2$$

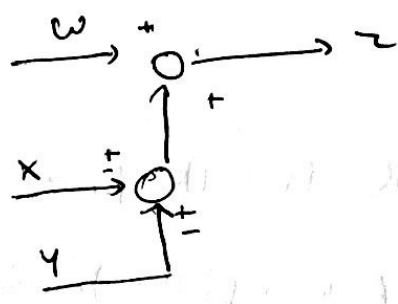
$$Y = \frac{(RG_1 + D)R}{1 + G_1G_2}$$

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$$z = (w \pm x) \pm y = z$$

$$z = w \pm x \pm y$$

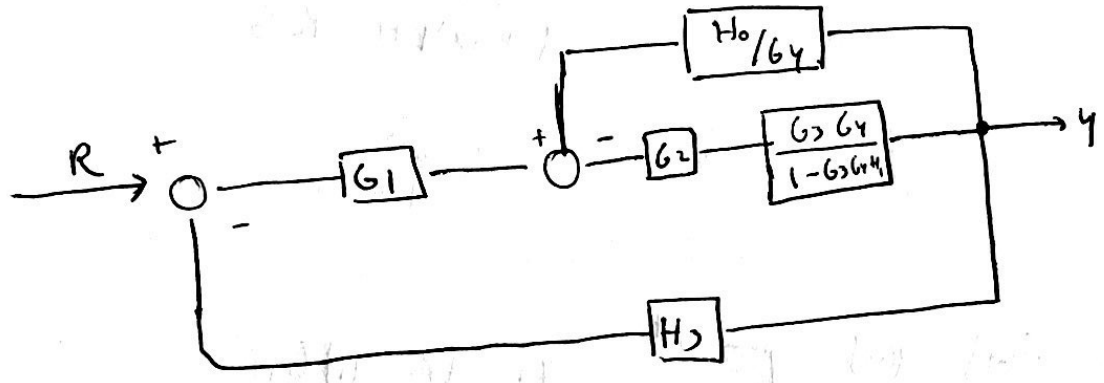


$$z = (\pm x \pm y) + w = z$$

$$z = w \pm x \pm y$$

equi
valentes

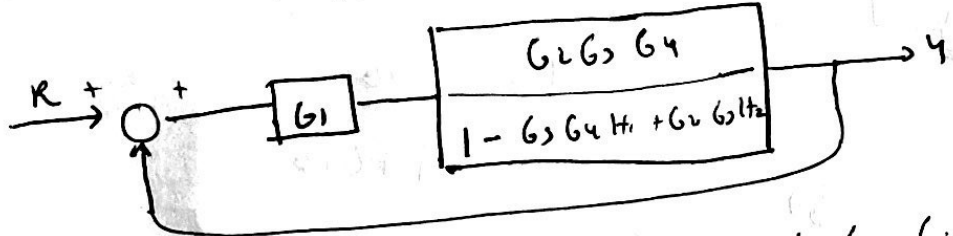
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$$Y = \left[(R - H_1 Y) G_1 - Y \frac{H_2}{G_4} \right] G_2 \frac{G_3 G_4}{1 - G_3 G_4 H_1}$$

$$Y = \frac{R G_1 G_2 G_3 G_4}{1 - H_1 G_3 G_4 + H_2 G_2 G_3 + H_3 G_1 G_2 G_3 G_4}$$

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$$Y = (R - Y) G_1 \frac{G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2} \rightarrow Y = \frac{R G_1 G_2 G_3 G_4}{1 - H_1 G_3 G_4 + H_2 G_2 G_3 + H_3 G_1 G_2 G_3 G_4}$$

$$\textcircled{13} \quad R \rightarrow \left[\frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_2} \right] \rightarrow Y$$

$$\frac{Y}{R} = \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_2}$$