Aula 22/10

$$2\ddot{x} + 7\dot{x} + 3x = 0$$

$$\begin{cases} x(0) = x_0 \\ \dot{x}(0) = 0 \end{cases}$$

$$\Rightarrow 2\mathcal{L}\{\ddot{x}\} + 7\mathcal{L}\{\dot{x}\} + 3\mathcal{L}\{x\} = 0$$

$$\Rightarrow \mathcal{L}\{\ddot{x}\} = s^2X(s) - sx(0) - \dot{x}(0)^{0}$$

$$\mathcal{L}\{\dot{x}\} = sX(s) - x(0)$$

$$\mathcal{L}\{x\} = X(s)$$

$$\Rightarrow 2(s^2X(s) - sx_0) + 7(sX(s) - x_0) + 3X(s) = 0$$

$$\therefore X(s)(2s^2 + 7s + 3) - x_0(2s + 7) = 0$$

$$\therefore X(s) = \frac{x_0(2s+7)}{2(s+3)(s+\frac{1}{2})}$$

Logo:

$$X(s) = \frac{x_0(2s+7)}{(s+3)(2s+1)} = \frac{A}{s+3} + \frac{B}{2s+1} = \frac{2A\cdot s + A + B\cdot s + 3B}{(s+3)(2s+1)}$$

$$\therefore 2x_0 s + 7x_0 = s(2A+B) + (A+3B)$$

$$\begin{cases} 2x_0 = 2A + B \\ 7x_0 = A + 3B \end{cases} \rightarrow 12x_0 = 5B$$

$$B = \frac{12x_0}{5}$$

$$A = -\frac{x_0}{5}$$

$$\therefore X(s) = \frac{-x_0}{5(s+3)} + \frac{12x_0}{5(2s+1)}$$ → transformada

$$x(t) = -\frac{x_0}{5}\cdot\mathcal{L}^{-1}\left(\frac{1}{s+3}\right) + \frac{6x_0}{5}\cdot\mathcal{L}^{-1}\left(\frac{1}{s+\frac{1}{2}}\right)$$

$$x(t) = \frac{-x_0}{5}\cdot e^{-3t} + \frac{6x_0}{5}\cdot e^{-\frac{1}{2}t}$$

$$\dddot{x} + 2\ddot{x} + 7\dot{x} = \dot{u} + 7u + 5u$$

$$\begin{cases} \ddot{x}(0) = 2 \\ \dot{x}(0) = 1 \\ x(0) = 9 \end{cases} \qquad \begin{cases} \dot{u}(0) = 0 \\ u(0) = 0 \end{cases}$$

$$\mathcal{L}\{\ddot{x}\} = s^3 x(s) - \ddot{x}(0) - s\dot{x}(0) - s^2 x(0)$$

$$\mathcal{L}\{\ddot{x}\} = s^2 x(s) - \dot{x}(0) - s x(0)$$

$$\mathcal{L}\{\dot{x}\} = s x(s) - x(0)$$

~~$\mathcal{L}\{x\} = x(s)$~~

$$\mathcal{L}\{\ddot{u}\} = s^2 u(s) - \dot{u}(0) - s u(0)$$

$$\mathcal{L}\{\dot{u}\} = s\,u(s) - u(0)$$

$$\mathcal{L}\{u\} = u(s)$$

$$s^3 x(s) - 9s^2 - s - 2 + 2s^2 x(s) - 18s - 2 + 7s x(s) - 63 =$$

$$= s^2 u(s) + 7s\,u(s) + 5\,u(s)$$

$$x(s)\,(s^3 + 2s^2 + 7s) - 9s^2 - 19s - 67 = u(s)\,(s^2 + 7s + 5)$$

$$\therefore \quad x(s) = \frac{u(s)\,[s^2 + 7s + 5] + [9s^2 + 19s + 67]}{s^3 + 2s^2 + 7s}$$

* $u(t) = 1$

$\hookrightarrow \quad u(s) = \frac{1}{s}$

$$x(s) = \frac{9s^3 + 20s^2 + 74s + 5}{s^2\,(s^2 + 2s + 7)}$$

$$x(s) = \frac{508}{49s} + \frac{5}{7s^2} - \frac{67s - 71}{49[(s+1)^2 + 6]}$$

$$x(t) = \frac{508}{49} \cdot \mathcal{L}^{-1}\left(\frac{1}{s}\right) + \frac{5}{7} \cdot \mathcal{L}^{-1}\left(\frac{1}{s^2}\right) - \frac{67}{49} \cdot \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2 + 6}\right\}$$

$$x(t) = \frac{508}{49} + \frac{5}{7}t - \frac{67}{49}\,e^{-t}\cos(t\sqrt{6}) - \frac{4}{49\sqrt{6}} \cdot e^{-t}\sin(t\sqrt{6})$$