Escola Politécnica da USP

Aluno: Ives Coelho Vieira NUSP: 10355551

PME 3380 - Modelagem de Sistemas Dinâmicos - Exs 22/10

1) $2\ddot{x} + 7\dot{x} + 3x = 0, \quad x(0) = x_0 \text{ e } \dot{x}(0) = 0$

* Pelas transformadas de Laplace:

$$2L\{\ddot{x}\} + 7L\{\dot{x}\} + 3L\{x\} = 0$$

$$2(s^2X(s) - sx(0)) + 7(sX(s) - x(0)) + 3X(s) = 0$$

$$2s^2X(s) + 7sX(s) + 3X(s) = 2sx_0 + 7x_0$$

$$\boxed{X(s) = \frac{x_0(2s+7)}{2(s+3)(s+1/2)}}$$

* Através das frações parciais:

$$\frac{x_0(2s+7)}{(s+3)(2s+1)} = \frac{A}{s+3} + \frac{B}{2s+1} = \frac{2sA + A + Bs + 3B}{(s+3)(2s+1)} \rightarrow \begin{cases} 2A + B = 2x_0 \\ A + 3B = 7x_0 \end{cases} \rightarrow \begin{matrix} A = -\frac{1}{5}x_0 \\ B = \frac{12}{5}x_0 \end{matrix}$$

$$\therefore \boxed{X(s) = -\frac{x_0}{5(s+3)} + \frac{12x_0}{5(2s+1)}}$$

↳ Aplicando a transformada inversa

$$\boxed{x(t) = -\frac{x_0}{5}e^{-3t} + \frac{6}{5}x_0 e^{-\frac{1}{2}t}}$$

2) $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u \quad u(t) = 1; \ \ddot{x}(0) = 2, \dot{x}(0) = 1, x(0) = 9, u(0) = 0, \dot{u}(0) = 0$

* Pela transformada de Laplace:

$$(s^3X(s) - \ddot{x}(0) - s\dot{x}(0) + s^2X(0)) + 2(s^2X(s) - \dot{x}(0) - sx(0)) + 7(sX(s) - x(0)) = (s^2U(s)) + 7(sU(s)) + 5U(s)$$

$$s^3X(s) + 2s^2X(s) + 7sX(s) - 2 - s - 9s^2 - 2 - 18s - 63 = s^2U(s) + 7sU(s) + 5U(s)$$

$$X(s)[s^3 + 2s^2 + 7s] = U(s)[s^2 + 7s + 5] + 9s^2 + 19s + 67$$

$$X(s) = \frac{U(s)(s^2+7s+5)}{s^3+2s^2+7s} + \frac{9s^2+19s+67}{s^3+2s^2+7s}$$

Resolvendo por frações parciais

Fazendo $U(s) = \frac{1}{s}$ $\rightarrow$ $\boxed{X(s) = \frac{9s^3 + 20s^2 + 79s + 5}{s^2(s^2+2s+7)}}$

$$X(s) = \frac{58}{49s} + \frac{5}{7s^2} + \frac{-67s-71}{49\left[(s+1)^2+6\right]}$$

→ Aplicando a transformada inversa

$$\boxed{x(t) = \frac{58}{49} + \frac{5t}{7} - \frac{67}{49} e^{-t} \cos(\sqrt{6}\, t) - \frac{4}{49\sqrt{6}} \operatorname{sen}(\sqrt{6}\, t)\, e^{-t}}$$

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