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1. $2\ddot{x} + 7\dot{x} + 3x = 0$; $x(0) = x_0$; $\dot{x}(0) = 0$

$2(s^2X - s x_0) + 7(sX - x_0) + 3X = 0.$

$2s^2X + 7sX + 3X = 2s x_0 + 7x_0 \Rightarrow X = \dfrac{x_0(2s+7)}{2(s+3)(s+1/2)}$

$X = \dfrac{2As + A + Bs + 3B}{(s+3)(2s+1)} \Rightarrow A = -\dfrac{x_0}{5}$; $B = \dfrac{12x_0}{5}$

$\mathcal{L}\{e^{ax}\} = \dfrac{1}{s-a} \Rightarrow x(t) = -\dfrac{x_0}{5}e^{-3t} + \dfrac{6x_0}{5}e^{-t/2}$

2. $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$; $\ddot{x}(0) = 2$; $\dot{x}(0) = 1$; $x(0) = 9$; $u(0) = \dot{u}(0) = 0$;

$(s^3X - 2 - s - 9s^2) + 2(s^2X - 1 - 9s) + 7(sX - 9) = (s^2U) + 7(sU) + 5U$ $\quad u(t) = 1$

$X(s) = U(s)\cdot\dfrac{s^2+7s+5}{s^3+2s^2+7s} + \dfrac{9s^2+19s+67}{s^3+2s^2+7s}$

Para $u(t) = 1$, $U(s) = 1/s \Rightarrow X(s) = \dfrac{9s^3+20s^2+74s+5}{s^2(s^2+2s+7)}$

$\mathcal{L}^{-1}\{1/s\} = 1$, $\mathcal{L}^{-1}\{1/s^2\} = t$, $\mathcal{L}^{-1}\left\{\dfrac{s+1}{(s+1)^2+6}\right\} = e^{-t}\cos(\sqrt{6}t)$, $\mathcal{L}^{-1}\left\{\dfrac{1}{(s+1)^2+6}\right\} = \dfrac{1}{\sqrt{6}}e^{-t}\operatorname{sen}(\sqrt{6}t)$

$x(t) = \dfrac{508}{49} + \dfrac{5t}{7} - \dfrac{67e^{-t}\cos(\sqrt{6}t)}{49} - \dfrac{4e^{-t}\operatorname{sen}(\sqrt{6}t)}{96\sqrt{6}}$