

①  $2\ddot{x} + 7\dot{x} + 3x = 0 \quad x(0) = 0 ; \dot{x}(0) = 0$

$$2(s^2 x(s) - s x(0) - \dot{x}(0)) + 7(s x(s) - x(0)) + 3x(s) = 0$$

$$x(s) (2s^2 + 7s + 3) - 2(s x(0)) - 7(x(0)) = 0$$

$$x(s) = \frac{2s x_0 + 7x_0}{2s^2 + 7s + 3} = \frac{x_0(2s + 7)}{2(s+3)(s + \frac{1}{2})}$$

$$x(s) = \frac{A}{s+3} + \frac{B}{s+\frac{1}{2}} \Rightarrow 2As + A + Bs + 3B = 2s x_0 + 7x_0$$

$$2A + B = 2x_0 \Rightarrow B = \frac{12}{5} x_0$$

$$A + 3B = 7x_0 \Rightarrow A = -\frac{1}{5} x_0$$

$$x(s) = \frac{-x_0}{5(s+3)} + \frac{12x_0}{5(2s+1)} \Rightarrow x(t) = -\frac{x_0}{5} (e^{-3t}) + \frac{12x_0}{5} (e^{-\frac{1}{2}t})$$

②  $\ddot{x} + 2\dot{x} + 7x = \ddot{u} + 7\dot{u} + 5u$

$$\dot{x}(0) = 2 ; x(0) = 1 ; u(0) = 0 ; \dot{u}(0) = 0$$

$$s^3 x(s) - s^2 x(0) - s \dot{x}(0) - \ddot{x}(0) + 2(s^2 x(s) - s x(0) - \dot{x}(0)) + 7(s x(s) - x(0)) = s^3 u(s) - s^2 u(0) - s \dot{u}(0) - \ddot{u}(0) + 7(s u(s) - u(0)) + 5u(s)$$

$$= s^2 u(s) - s u(0) - \dot{u}(0) + 7(s u(s) - u(0)) + 5u(s)$$

$$x(s) (s^3 + 2s^2 + 7s) = u(s) (s^2 + 7s + 5) + 9s^2 + 19s + 67$$

$$x(s) = u(s) \left( \frac{s^2 + 7s + 5}{s^3 + 2s^2 + 7s} \right) + \left( \frac{9s^2 + 19s + 67}{s^3 + 2s^2 + 7s} \right)$$

$$u(0) = 1 \Rightarrow u(s) = \frac{1}{s}$$

$$x(s) = \frac{s^2 + 7s + 5 + s(9s^2 + 19s + 67)}{s^2(s^2 + 2s + 7)} = \frac{9s^3 + 20s^2 + 74s + 5}{s^2(s^2 + 2s + 7)}$$

$$x(s) = \frac{508}{49s} + \frac{5}{7s^2} + \frac{-67s - 71}{49((s+1)^2 + 6)}$$

$$x(t) = \frac{508}{49} + \frac{5}{7} t - \frac{67}{49} e^{-t} \cos(\sqrt{6}t) - \frac{4}{49\sqrt{6}} e^{-t} \sin(\sqrt{6}t)$$