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1- $2\ddot{x} + 7\dot{x} + 3x = 0 \quad x(0) = x_0 \quad \dot{x}(0) = 0$

$2\mathcal{L}[\ddot{x}] = 2[s^2X(s) - sx(0) - \dot{x}(0)]$

$7\mathcal{L}[\dot{x}] = 7[sX(s) - x(0)]$

$3\mathcal{L}[x] = 3X(s)$

$(2s^2 + 7s + 3)X(s) = (2s + 7)x_0 \Rightarrow X(s) = \dfrac{2s+7}{2s^2+7s+3} x_0$

$$\boxed{G(s) = \frac{1}{2s^2+7s+3}} \Rightarrow G(s) = \frac{1}{(s+1/2)(s+3)}$$

polos $-1/2$ e -3

$$X(s) = \left(\frac{a}{s+1/2} + \frac{b}{s+3}\right) x_0$$

$$a = \left.\frac{2s+7}{s+3}\right|_{s=-1/2} \Rightarrow a = \frac{8}{7/2} \Rightarrow a = \frac{16}{7}$$

$$b = \left.\frac{2s+7}{s+1/2}\right|_{s=-3} \Rightarrow b = \frac{13}{7/2} \Rightarrow b = \frac{26}{7}$$

$$X(s) = \left[\frac{16}{7(s+1/2)} + \frac{26}{7(s+3)}\right] x_0$$

$\downarrow \mathcal{L}^{-1}$

$$\boxed{x(t) = \left(\frac{16}{7} e^{-t/2} + \frac{26}{7} e^{-3t}\right) x_0}$$

2- $\dddot{x} + 2\ddot{x} + 7\dot{x} = \dot{u} + 7u + 5u$ $\quad \ddot{x}(0) = 2 \quad \dot{x}(0) = 1$

$x(0) = 9 \quad u(0) = 1$

$\dot{u}(0) = 0 \quad u(t) = 1$

$\mathcal{L}[\dddot{x}] = s^3X(s) - s^2\ddot{x}(0) - s\dot{x}(0) - x(0)$

$2\mathcal{L}[\ddot{x}] = 2[s^2X(s) - s\dot{x}(0) - x(0)]$

$7\mathcal{L}[\dot{x}] = 7[sX(s) - x(0)]$

$\mathcal{L}[\ddot{u}] = s^2U(s) - s\dot{u}(0) - u(0)$

$7\mathcal{L}[\dot{u}] = 7[sU(s) - u(0)]$

$5\mathcal{L}[u] = 5U(s)$

$(s^3 + 2s^2 + 7s)X(s) = (s^2 + 7s + 5)U(s) + s^2\ddot{x}(0) + s\dot{x}(0) + x(0) +$

$+ 2s\dot{x}(0) + 2x(0) + 7x(0) - u(0) - 7\dot{u}(0) \Rightarrow$

$\Rightarrow (s^3 + 2s^2 + 7s)X(s) = s + 7 + 5/s + s^2\ddot{x}(0) + 3s\dot{x}(0) + 10x(0) - 8u(0) \Rightarrow$

$\Rightarrow (s^3 + 2s^2 + 7s)X(s) = 2s^2 + 4s + 5/s + 90 \Rightarrow$

$\Rightarrow X(s) = \dfrac{2s^3 + 4s^2 + 90s + 5}{s^2(s^2 + 2s + 7)}$

$$\boxed{G(s) = \frac{1}{s^2 + 2s + 7}} \Rightarrow G(s) = \frac{1}{(s + 1 + j\sqrt{6})(s + 1 - j\sqrt{6})} \leftarrow \text{pólos complexos}$$

$$X(s) = \frac{C_1}{s} + \frac{C_2}{s^2} + \frac{C_3 + sC_4}{s^2 + 2s + 7}$$

$$C_2 = \left.\frac{2s^3 + 4s^2 + 90s + 5}{s^2 + 2s + 7}\right|_{s=0} \Rightarrow C_2 = \frac{5}{7}$$

$$C_1 = \left.\frac{d}{ds}\left(\frac{2s^3 + 4s^2 + 90s + 5}{s^2 + 2s + 7}\right)\right|_{s=0} \Rightarrow C_1 = \frac{620}{49}$$

$$X(s) = \frac{620}{49s} + \frac{5}{7s^2} + \frac{C_3 + sC_4}{s^2 + 2s + 7} \Rightarrow \frac{620}{49}(s^3 + 2s^2 + 7s) + \frac{5}{7}(s^2 + 2s + 7) +$$

$+ s^2(C_3 + sC_4) = 2s^3 + 4s^2 + 90s + 5 \Rightarrow s^3(49C_4 + 522) + s^2(49C_3 + 1079) = 0$

$\begin{cases} 49C_4 + 522 = 0 \Rightarrow C_4 = -522/49 \\ 49C_3 + 1079 = 0 \Rightarrow C_3 = -1079/49 \end{cases}$