Nome: Yago Neves Yang   NUSP: 10772626

*PME3380 - Lista do dia 22/10:

1) $\underbrace{2\ddot{x}}_{2[s^2X(s) - sX(0) - \cancel{\dot{x}(0)}]} + \underbrace{7\dot{x}}_{7[sX(s) - x(0)]} + \underbrace{3x}_{3X(s)} = 0$; $x(0) = x_0$; $\dot{x}(0) = 0$ $\Rightarrow (2s^2 + 7s + 3)X(s) - (2s+7)x_0 = 0$

$\dot{x}(0) = 0$

$$\boxed{X(s) = \frac{(2s+7)x_0}{2s^2+7s+3}} //$$

$\Rightarrow 2s^2 + 7s + 3 = 0 \Rightarrow s = \frac{-7 \pm \sqrt{49-24}}{4}$; $s' = -1/2$, $s'' = -3$ $\Rightarrow 2s^2 + 7s + 3 = 2(s + 1/2)(s+3)$

$$X(s) = \frac{x_0}{2}\left[\frac{2s+7}{(s+1/2)(s+3)}\right] := \frac{x_0}{2}\left[\frac{P}{(s+1/2)} + \frac{Q}{(s+3)}\right]$$

$P = X(s)(s+1/2)\Big|_{s=-1/2} \Rightarrow P = \frac{12}{5}$

$Q = X(s)(s+3)\Big|_{s=-3} \Rightarrow Q = -\frac{2}{5}$

$$X(s) = \frac{x_0}{2}\left[\frac{12}{5(s+1/2)} - \frac{2}{5(s+3)}\right] = \frac{6}{5}x_0 \cdot \frac{1}{(s+1/2)} - \frac{1}{5}x_0 \cdot \frac{1}{(s+3)}$$

$$\boxed{x(t) = \frac{6}{5}x_0 . e^{-\frac{1}{2}t} - \frac{1}{5}x_0 . e^{-3t}} //$$

---

2) $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$; $\ddot{x}(0) = 2$; $\dot{x}(0) = 1$; $x(0) = 9$; $u(0) = 0$; $\dot{u}(0) = 0$

$5u \to 5U(s)$

$7\dot{u} \to 7[sU(s) - u(0)]$

$\ddot{u} \to s^2U(s) - su(0) - \dot{u}(0)$

$7\dot{x} \to 7[sX(s) - X(0)]$

$2\ddot{x} \to 2[s^2X(s) - sX(0) - \dot{x}(0)]$

$\dddot{x} \to s^3X(s) - s^2X(0) - s\dot{X}(0) - \ddot{X}(0)$

$\Rightarrow (s^3 + 2s^2 + 7s)X(s) - 9s^2 - s - 2 - 18s - 2 - 63 =$

$= (s^2 + 7s + 5)U(s)$

$$\Rightarrow X(s) = \frac{(s^2+7s+5)U(s)}{(s^3+2s^2+7s)} + \frac{(9s^2+19s+67)}{(s^3+2s^2+7s)}$$

$$\to G(s) = \frac{X(s)}{U(s)} = \frac{s^2+7s+5}{s^3+2s^2+7s}$$

$$u(t) = 1 \Rightarrow U(s) = \frac{1}{s} \Rightarrow X(s) = \frac{(s^2+7s+5)}{s(s^3+2s^2+7s)} + \frac{(9s^2+19s+67)}{(s^3+2s^2+7s)} = \frac{s^2+7s+5+9s^3+19s^2+67s}{s(s^3+2s^2+7s)}$$

$$\Rightarrow X(s) = \frac{9s^3+20s^2+74s+5}{s.s(s^2+2s+7)} = \frac{9s^3+20s^2+74s+5}{s^2(s+1+i\sqrt{6})(s+1-i\sqrt{6})} = \frac{P}{s} + \frac{Q}{s^2} + \frac{Rs+T}{s^2+2s+7}$$

$$X(s) = \frac{P}{s} + \frac{Q}{s^2} + \frac{Rs+T}{s^2+2s+7} \qquad Q = s^2X(s)\Big|_{s=0} \Rightarrow Q = \frac{5}{7}$$

$$P = \frac{d}{ds}\left[s^2X(s)\right]\Big|_{s=0} \Rightarrow P = \frac{9s^4+36s^3+155s^2+270s+508}{(s^2+2s+7)^2}\Bigg|_{s=0}$$

$$P = \frac{508}{49}$$

$$X(s) = \frac{508}{49}\cdot\frac{1}{s} + \frac{5}{7}\cdot\frac{1}{s^2} + \frac{Rs+T}{s^2+2s+7} = \frac{508(s^3+2s^2+7s)+35(s^2+2s+7)+49s^2(Rs+T)}{49s^2(s^2+2s+7)}$$

$$X(s) = \frac{49s^3R+508s^3+49s^2T+1051s^2+3626s+245}{49s^2(s^2+2s+7)}$$

$$\Rightarrow \frac{49s^3R+508s^3+49s^2T+1051s^2+3626s+245}{\cancel{s^2(s^2+2s+7)}} = \frac{9s^3+20s^2+74s+5}{\cancel{s^2(s^2+2s+7)}}\cdot 49$$

$$\Rightarrow 49R+508 = 9.45 \Rightarrow R = -\frac{67}{49} \ ; \Rightarrow 49T+1051 = 980 \Rightarrow T = -\frac{71}{49}$$

$$X(s) = \frac{508}{49}\cdot\frac{1}{s} + \frac{5}{7}\cdot\frac{1}{s^2} + \frac{(-67/49\,s - 71/49)}{s^2+2s+1+6} = \frac{508}{49}\cdot\frac{1}{s} + \frac{5}{7}\cdot\frac{1}{s^2} - \frac{1}{49}\left[\frac{67s+71}{(s+1)^2+6}\right]$$

$$X(s) = \frac{508}{49}\cdot\frac{1}{s} + \frac{5}{7}\cdot\frac{1}{s^2} - \frac{1}{49}\left[\frac{\overbrace{67s+67}^{\to 67(s+1)}+4}{(s+1)^2+6}\right] = \frac{508}{49}\cdot\frac{1}{s} + \frac{5}{7}\cdot\frac{1}{s^2} - \frac{67}{49}\cdot\frac{(s+1)}{(s+1)^2+6} - \frac{4}{49}\cdot\frac{1}{(s+1)^2+6}$$

$$\frac{4}{49\sqrt{6}}\cdot\frac{\sqrt{6}}{(s+1)^2+6}$$

$$\boxed{\therefore\ x(t) = \frac{508}{49} + \frac{5}{7}.t - \frac{67}{49}.e^{-t}\cos\sqrt{6}t - \frac{4}{49\sqrt{6}}\,e^{-t}\operatorname{sen}\sqrt{6}t}$$