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PME3380 - Modelagem de Sistemas Dinâmicos

Exercícios Cular 22/10

$$1) \quad 2\ddot{x} + 7\dot{x} + 3x = 0, \quad \text{com} \quad \begin{cases} x(0) = x_0 \\ \dot{x}(0) = 0 \end{cases}$$

$$\Rightarrow 2(s^2 X(s) + s \cdot x(0)) + 7(sX(s) - \dot{x}(0)) + 3X(s) = 0$$

$$\Rightarrow (2s^2 + 7s + 3)X(s) = (2s + 7)x_0 \Rightarrow X(s) = \frac{2s + 7}{2s^2 + 7s + 3} x_0 \quad \begin{cases} s_1 = -\frac{1}{2} \\ s_2 = -3 \end{cases}$$

$$\Rightarrow X(s) = \frac{(2s + 7)x_0}{2(s + 3)(s + \frac{1}{2})} = \left( \frac{A}{s + 3} + \frac{B}{s + \frac{1}{2}} \right) \frac{x_0}{2} \quad \rightarrow \text{Resolvendo para } A \text{ e } B: \begin{cases} A = -\frac{2}{3} \\ B = \frac{10}{3} \end{cases}$$

$$\Rightarrow X(s) = -\frac{x_0}{5(s + 3)} + \frac{6x_0}{5(s + \frac{1}{2})} \xrightarrow{\mathcal{L}^{-1}} \boxed{x(t) = -\frac{x_0}{5} e^{-3t} + \frac{6x_0}{5} e^{-\frac{t}{2}}}$$

$$2) \quad \ddot{x} + 2\dot{x} + 7x = \ddot{u} + 7\dot{u} + 5u, \quad \text{com} \quad \begin{cases} \ddot{x}(0) = 2; \dot{x}(0) = 1; x(0) = 9 \\ \ddot{u}(0) = 0; \dot{u}(0) = 1 \end{cases}$$

$$\Rightarrow s^3 X(s) - s^2 x(0) - s\dot{x}(0) - \ddot{x}(0) + 2(s^2 X(s) - s\dot{x}(0) - \dot{x}(0)) + 7(sX(s) - x(0)) = s^3 U(s) - s\dot{u}(0) - \ddot{u}(0) + 7(sU(s) - u(0)) + 5U(s)$$

$$\Rightarrow X(s)(s^3 + 2s^2 + 7s) = U(s)(s^2 + 7s + 5) + 9s^2 + 18s + 60$$

$$\Rightarrow X(s) = \frac{U(s)(s^2 + 7s + 5)}{s^3 + 2s^2 + 7s} + \frac{9s^2 + 18s + 60}{s^3 + 2s^2 + 7s}$$

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