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$2\ddot{x} + 7\dot{x} + 3x = 0$

sabendo que $\begin{cases} x(0) = x_0 \\ \dot{x}(0) = 0 \end{cases}$

Pela transformada de Laplace:

$2(s^2X - sx_0 - x'(0)) + 7(sX - x_0) + 3X = 0$

$2s^2X + 7sX + 3X = 2sx_0 + 7x_0$

Método das frações parciais:

$$X = \frac{A}{s+3} + \frac{B}{2s+1} = \frac{2As + A + Bs + 3B}{(s+3)(2s+1)}$$

$\begin{cases} A = -x_0/5 \\ B = 12x_0/5 \end{cases}$

Transformada inversa:

$$x(t) = -\frac{x_0}{5}e^{-3t} + \frac{6}{5}x_0^{-\frac{t}{2}}$$

(2)

$$\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$$

sabendo que
$$\begin{cases} \ddot{x}(0) = 2 & \dot{u}(0) = 0 \\ \dot{x}(0) = 1 & u(0) = 1 \\ x(0) = 9 & u(t) = 1 \end{cases}$$

$$s^3X - 2 - s - 9s^2 + 2(s^2X - 1 - 9s) + 7(sX - 9) = s^2U + 7sU + 5U$$
$$X(s^3 + 2s^2 + 7s) = U(s^2 + 7s + 5) + 9s^2 + 19s + 67$$

$$X(s) = \frac{s^2 + 7s + 5}{s^3 + 2s^2 + 7s} \cdot U(s) + \frac{9s^2 + 19s + 67}{s^3 + 2s^2 + 7s}$$

$$G(s) = \frac{s^2 + 7s + 5}{s^3 + 2s^2 + 7s}$$

Com pólos $(s^3 + 2s^2 + 7s) = s(s^2 + 2s + 7)$
$$\begin{cases} s = 0 \\ s = -1 + i\sqrt{6} \\ s = -1 - i\sqrt{6} \end{cases}$$

Com $U(t) = 1 \Rightarrow U(s) = 1/s$
$$X(s) = \frac{9s^3 + 19s^2 + 67s + 5}{s^2(s^2 + 2s + 7)} = \frac{k_1}{s} + \frac{k_2}{s^2} + \frac{k_3 s + k_4}{s^2 + 2s + 7}$$

Achando $k_{1,2,3,4}$

$$k_1 = \frac{d}{ds}\left[s^2X(s)\right]_{s=0} = \frac{(27s^2 + 38s + 67)(s^2 + 2s + 7) - (9s^3 + 19s^2 + 67s + 5)(2s + 2)}{(s^2 + 2s + 7)^2} = \frac{459}{49}$$

$$k_2 = X(s) . s^2 \big|_{s=0} = 5/7$$

$$(459 + 49k_3)s^3 + (918 + 35 + 49k_4)s^2 + (3213 + 70)s + 245 = 49(9s^3 + 19s^2 + 67s + 5)$$

$$\begin{cases} 459 + 49k_3 = 49 \cdot 9 & k_3 = -18/49 \\ 918 + 35 + 49k_4 = 49 \cdot 19 & k_4 = -22/49 \end{cases}$$

$$X(s) = \frac{459}{49} \cdot \frac{1}{s} + \frac{5}{7} \cdot \frac{1}{s^2} - \frac{18s/49 + 22/49}{s^2 + 2s + 7}$$

$$X(s) = \frac{1}{49} \left[459 \cdot \frac{1}{s} + 35 \cdot \frac{1}{s^2} - \frac{18(s+1)}{(s+1)^2 + 6} - \frac{4}{(s+1)^2 + 6} \right]$$

$$X(t) = \frac{1}{49} \left[459 + 35t - 18 e^{-t} \cos \sqrt{6}\, t - \frac{4}{\sqrt{6}} e^{-t} \operatorname{sen} \sqrt{6}\, t \right]$$