Modelagem 22/10

1. $2\ddot{x} + 7\dot{x} + 3x = 0$; $x(0) = x_0$ $\dot{x}(0) = 0$

$\mathcal{L}[2\ddot{x}] = 2(s^2 X(s) - s.x_0 - \cancel{\dot{x}(0)}^{0})$

$\mathcal{L}[7\dot{x}] = 7(sX(s) - x_0)$

$\mathcal{L}[3x] = 3X(s)$

Dom. da freq.: $2s^2X(s) - 2sx_0 + 7sX(s) - 7x_0 + 3X(s) = 0$

$(2s^2 + 7s + 3)X(s) = (2s+7)x_0$

$$X(s) = \frac{(2s+7)x_0}{2s^2+7s+3} \rightarrow D(s)$$

F.T. (supondo $u(t) = \delta(t) \rightarrow$ impulso)

$$\boxed{G(s) = \frac{X(s)}{U(s)} = \frac{1}{2s^2+7s+3}} \rightarrow \text{F.T.}$$

Polos do sistema.

$D(s) = 2s^2 + 7s + 3 = 0$ $S_1 = -3$; $S_2 = -0{,}5$

$$\frac{2s+7}{2s^2+7s+3} = \frac{\alpha_1}{(s+3)} + \frac{\alpha_2}{(s+0{,}5)} = \frac{(2s+7)}{(s+3)(s+0{,}5)}$$

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$$\alpha_1 = \left[\frac{(2s+7)}{\cancel{(s+3)}(s+0{,}5)} \cdot \cancel{(s+3)}\right]_{s=s_1} \Rightarrow \alpha_1 = -0{,}4$$

$$\alpha_2 = \frac{(2(-0{,}5)+7)}{(-0{,}5+3)} = 2{,}4$$

$$\therefore X(s) = \left(\frac{-0{,}4}{s+3} + \frac{2{,}4}{s+0{,}5}\right) x_0 \Rightarrow \boxed{x(t) = (2{,}4e^{-0{,}5t} - 0{,}4e^{-3t}) \cdot x_0}$$

→ EDO resolvida

2\. $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$ $\qquad \ddot{x}(0) = 2,\ \dot{x}(0) = 1;\ x(0) = 9$

$\mathcal{L}[\dddot{x}] = s^3 X(s) - s^2 9 - s - 2$ $\qquad u(0) = 0;\ \dot{u}(0) = 0$

$\mathcal{L}[2\ddot{x}] = (s^2 X(s) - s.9 - 1)\,2$ $\qquad u(t) = 1$

$\mathcal{L}[7\dot{x}] = 7(s.X(s) + 9)$

$\mathcal{L}[\ddot{u}] = s^2 U(s) - s.0 - 0$ $\qquad \mathcal{L}[5u] = 5.U(s)$

$\mathcal{L}[7\dot{u}] = (s.U(s) - 0).7$

Domínio da frequência.

$$s^3X(s) - 9s^2 - s - 2 + 2s^2X(s) - 18s - 2 + 7sX(s) - 63 = s^2U(s) + 7sU(s)$$

$$X(s)(s^3 + 2s^2 + 7s) = 9s^2 + 19s + 67 + U(s)(s^2 + 7s)$$

$$X(s) = \underbrace{\left(\frac{9s^2 + 19s + 67}{s^3 + 2s^2 + 7s}\right)}_{A(s)} + \underbrace{\left(\frac{s^2 + 7s}{s^3 + 2s^2 + 7s}\right)}_{B(s)} U(s)$$

F.T. $\Rightarrow$ $U(s) = \frac{1}{s}$ (degrau unitário)

$$\boxed{G(s) = \frac{X(s)}{U(s)} = \frac{s^2 + 7s}{s(s^3 + 2s^2 + 7s)}}$$

$D(s) = s^3 + 2s^2 + 7s = 0 \Rightarrow$ polos: $s_1 = 0$, $s_2 = -1 - 2{,}4j$, $s_3 = -1 + 2{,}4j$

$$A(s) = \frac{c_1}{s} + \frac{c_2 s + c_3}{s^2 + 2s + 7}$$

$$c_1 = A(s).s\Big|_{s=0} = \frac{(9s^2 + 19s + 67)\cancel{s}}{\cancel{s}(s^2 + 2s + 7)}\Bigg|_{s=0} = \frac{67}{7}$$

$$A(s) = \frac{67}{7}\cdot\frac{1}{s} + \frac{c_2 s + c_3}{s^2+2s+7} = \frac{9s^2 + 19s + 67}{s(s^2+2s+7)} \Rightarrow$$

$$\Rightarrow \frac{67(s^2+2s+7) + 7s(c_2 s + c_3)}{\cancel{7\, s(s^2+2s+7)}} = \frac{(9s^2 + 19s + 67)7}{\cancel{s(s^2+2s+7)}}$$

$$67s^2 + 134s + \cancel{469} + 7c_2\cdot s^2 + 7c_3 s = 63s^2 + 133s + \cancel{469}$$

$$(67 + 7c_2)\cdot s^2 + (134 + 7\cdot c_3)s = 63s^2 + 133s \Rightarrow \begin{cases} 67 + 7c_2 = 63 \\ 134 + 7c_3 = 133 \end{cases}$$

$$c_2 = -4/7 \qquad c_3 = -1/7$$

$$A(s) = \frac{1}{7}\left(67\left(\frac{1}{s}\right) - \frac{4s+1}{s^2+2s+7}\right)$$

$$B(s) = \frac{b_1}{s} + \frac{b_2\cdot s + b_3}{s^2+2s+7} \Rightarrow b_1 = B(s=0)\cdot s\Big|_{s=0} = 0$$

$$\therefore \frac{(b_2 s + b_3)s}{\cancel{s^2+2s+7}} = \frac{s^2 + 7s}{\cancel{s(s^2+2s+7)}} \Rightarrow \begin{cases} b_2 = 1 \\ b_3 = 7 \end{cases}$$

$$B(s) = \frac{s+7}{s^2+2s+7}$$

Rearranjando A e B e aplicando a transformada inversa.

$$A(s) = \frac{1}{7}\left[67\cdot\left(\frac{1}{s}\right) - 4\cdot\frac{s+1}{(s+1)^2+6} + 3\cdot\frac{\sqrt{6}}{(s+1)^2+6}\left(\frac{1}{\sqrt{6}}\right)\right]$$

$$a(t)\ \frac{1}{7}\left(67 - 4\cdot e^{-t}\cos(\sqrt{6}\,t) + \frac{\sqrt{6}}{2}\, e^{-t}\,\mathrm{sen}(\sqrt{6}\,t)\right)$$

$$B(s) = \frac{s+1}{(s+1)^2+6} + \frac{6}{\sqrt{6}}\cdot\frac{\sqrt{6}}{(s+1)^2+6}$$

$$b(t) = e^{-t}\cdot\cos(\sqrt{6}\,t) + \sqrt{6}\cdot e^{-t}\cdot\mathrm{sen}(\sqrt{6}\,t)$$

Finalmente: Resposta no domínio do tempo (EDO resolvida)

$$x(t) = a(t) + b(t)\cdot \underbrace{u(t)}_{1} \Rightarrow x(t) = \underbrace{a(t)}_{\text{sol. homogênea}} + \underbrace{b(t)}_{\text{sol particular}}$$