Aula do dia 22/10/2020

① · transformada de Laplace

$2\ddot{x} + 7\dot{x} + 3x = 0, \; x(0) = x_0, \; \dot{x}(0) = 0$

$2(s^2 X_s - s \cdot x_0) + 7(s X_s - x_0) + 3 X_s = 0$

$(2 \cdot s^2 + 7s + 3) X_s = (2 \cdot s + 7) x_0$

$$X_s = \frac{(2 \cdot s + 7) x_0}{2s^2 + 7s + 3} = \frac{(2s+7) x_0}{(s+3)(2s+1)} = \frac{A}{s+3} + \frac{B}{2s+1}$$

$$X_s = \frac{s(2A+B) + (A+3B)}{(s+3)(2s+1)} ; \begin{cases} 2A + B = 2x_0 \\ A + 3B = 7x_0 \end{cases} \Rightarrow \begin{array}{l} B = 12/5 \cdot x_0 \\ A = -1/5 \cdot x_0 \end{array}$$

$$\boxed{X_s = \frac{-x_0}{5(s+3)} + \frac{12 x_0}{5(2s+1)}}$$

· transformada inversa

$$x(t) = -x_0/5 \; \mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} + 6x_0/5 \; \mathcal{L}^{-1}\left\{\frac{1}{s+1/2}\right\}$$

$$\boxed{x(t) = -\frac{x_0}{5} e^{-3t} + \frac{6 x_0}{5} \cdot e^{-t/2}}$$

② transformada de laplace

$$\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u \; \begin{cases} \ddot{x}(0) = 2, \; \dot{x}(0) = 1, \; x(0) = 9 \\ \dot{u}(0) = 0, \; u(0) = 0 \end{cases}$$

$s^3 X_s - s^2 x_0 - s \dot{x}_0 - \ddot{x}_0 + 2(s^2 X_s - s x_0 - \dot{x}_0) + 7(s X_s - x_0) =$

$= s^2 U_s - s u_0 - \dot{u}_0 + 7(s U_s - u_0) - s U_s$

$X_s(s^3 + 2s^2 + 7s) = U_s(s^2 + 7s + 5) + 9s^2 + 19s + 67$

$$\cdot \; U_s = Y_s : \; X_s = \frac{9s^3 + 20s^2 + 74s + 5}{s^2(s^2 + 2s + 7)} = \frac{508}{49s} + \frac{5}{7s^2} - \frac{(67s + 71)}{49((s+1)^2 + 6)}$$

· transformado inverso:

$$x(t) = \frac{508}{49} \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{5}{7} \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \frac{67}{49} \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2 + 6}\right\} - \frac{4}{49} \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2 + 6}\right\}$$

$$\boxed{x(t) = \frac{508}{49} + \frac{5t}{7} - \frac{67}{49} e^{-t} \cos(t\sqrt{6}) - \frac{4}{49\sqrt{6}} e^{-t} \operatorname{sen}(t\sqrt{6})}$$