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① $2\ddot{x} + 7\dot{x} + 3x = 0$ ; $x(0) = x_0$ e $\dot{x}(0) = 0$

- TRANSFORMADA DE LAPLACE

$$2L\{\ddot{x}\} + 7L\{\dot{x}\} + 3L\{x\} = 0$$

$$2\left(s^2X(s) - sX(s) - \cancel{x'(s)}^{0}\right) + 7\left(sX(s) - x(s)\right) + 3X(s) = 0$$

$$\therefore (2s^2 + 7s + 3)X(s) = (2s+7)x_0 \Rightarrow X(s) = \frac{x_0(2s+7)}{2(s+3)(s+1/2)}$$

REESCREVENDO, $X(s) = \frac{x_0(2s+7)}{(s+3)(2s+1)} = \frac{A}{s+3} + \frac{B}{2s+1} = \frac{s(2A+B) + (A+3B)}{(s+3)\cdot(2s+1)}$

$$\begin{cases} 2A+B = 2x_0 \longrightarrow B = 12/5\, x_0 \\ A+3B = 7x_0 \longrightarrow A = -1/5\, x_0 \end{cases} \longrightarrow X(s) = \frac{-x_0}{5(s+3)} + \frac{1}{5}\frac{2(x_0)}{(2s+1)}$$

~> TRANSFORMADA INVERSA:

$$X(t) = \frac{-x_0}{5}\cdot L^{-1}\left\{\frac{1}{s+3}\right\} + \frac{6\cdot x_0}{5}\cdot L\left\{\frac{1}{s+1/2}\right\}$$

$$\therefore \boxed{X(t) = \frac{-x_0}{5}\cdot e^{-3t} + \frac{6x_0}{5}\cdot e^{-t/2}}$$

② $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$ ; $\ddot{x}(0) = 2$ | $\dot{x}(0) = 1$ | $x(0) = 9$ | $\dot{u}(0) = 0$ | $u(0) = 0$

~> T. Laplace: $\left[s^3X(s) - \ddot{x}(0) - s\dot{x}(0)\right] + 2\left[s^2X(0) - x(0) - sX(0)\right] +$

$$7\left[sX(0) - x(0)\right] = \left[s^2U(0) - \cancel{\dot{u}(0)}^{0} - s\cancel{u(0)}^{0}\right] + 7\left[sU(0) - \cancel{u(0)}^{0}\right] + 5U(0)$$

$$\therefore s^3X(s) - 2 - s - 9s^2 + 2s^2X(s) - 2 - 18s + 7sX(0) - 63 = s^2U(0) + 7sU(s) + 5U(s)$$

$$(s^3 + 2s^2 + 7s)X(s) = 9s^2 + 19s + 67 + (s^2 + 7s + 5)U(s) \longrightarrow \boxed{U(s) = 1/s}$$

$$\therefore X(s) = \frac{9s^3 + 20s^2 + 74s + 5}{s^2(s^2 + 2s + 7)} = \frac{508}{49\cdot s} + \frac{5}{7s^2} - \frac{(67s + 71)}{49[(s+1)^2 + 6]}$$

~> T. INVERSA: $X(t) = \frac{508}{49}\cdot \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{5}{7}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \frac{4}{49}\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+6}\right\} - \frac{67}{49}\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+6}\right\}$

$$\therefore X(t) = \frac{508}{49} + \frac{5t}{7} - \frac{67}{49}e^{-t}\cos(t\sqrt{6}) - \frac{4}{49\sqrt{6}}e^{-t}\operatorname{sen}(t\sqrt{6})$$

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