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1- $2\ddot{x} + 7\dot{x} + 3x = 0$; $x(0) = x_0$; $\dot{x}(0) = 0$

Transformada de Laplace

$2\mathcal{L}\{\ddot{x}\} + 7\mathcal{L}\{\dot{x}\} + 3\mathcal{L}\{x\} = 0$

$2(s^2X(s) - sx(0) - \dot{x}(0)) + 7(sX(s) - x(0)) + 3X(s) = 0$

$(2s^2 + 7s + 3)X(s) = (2s + 7)x_0$

$X(s) = \dfrac{x_0(2s+7)}{2s^2+7s+3} = \dfrac{x_0(2s+7)}{2(s+3)(s+\frac{1}{2})}$

Temos que:

$\dfrac{x_0(2s+7)}{(s+3)(2s+1)} = \dfrac{A}{s+3} + \dfrac{B}{2s+1} = \dfrac{2sA + A + Bs + 3B}{(s+3)(2s+1)}$

$2A + B = 2x_0$
$A + 3B = 7x_0$

$A = -\frac{1}{5}x_0$ $B = \frac{12}{5}x_0$

$X(s) = \dfrac{-x_0}{5(s+3)} + \dfrac{12x_0}{5(2s+1)}$

Pela transformada inversa:

$\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$

$x(t) = \dfrac{-x_0}{5}\mathcal{L}^{-1}\left\{\dfrac{1}{s+3}\right\} + \dfrac{6}{5}x_0\mathcal{L}^{-1}\left\{\dfrac{1}{s+\frac{1}{2}}\right\}$

$x(t) = -\dfrac{x_0}{5}e^{-3t} + \dfrac{6}{5}x_0e^{-\frac{t}{2}}$

2- $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$

$\ddot{x}(0) = 2$ $\dot{u}(0) = 0$

$\dot{x}(0) = 1$ $u(0) = 1$

$x(0) = 9$

Transformada de Laplace:

$$s^3X(s) - s^2x(0) - s\dot{x}(0) - \ddot{x}(0) + 2(s^2X(s) - sx(0) - \dot{x}(0)) + 7(sX(s) - x(0))$$

$$= s^2U(s) - su(0) - \dot{u}(0) + 7(sU(s) - u(0)) + 5U(s)$$

$$X(s)(s^3 + 2s^2 + 7s) = U(s)(s^2 + 7s + 5) + 9s^2 + 18s + 60$$

$$X(s) = \frac{U(s)(s^2 + 7s + 5)}{s^3 + 2s^2 + 7s} + \frac{9s^2 + 18s + 60}{(s^3 + 2s^2 + 7s)}$$

$$G(s) = \frac{X(s)}{U(s)} \rightarrow G(s) = \frac{s^2 + 7s + 5}{s(s^2 + 2s + 7)}$$

$s_1 = 0$ $s_2 = -1 + \sqrt{6}i$ $s_3 = -1 - \sqrt{6}i$

$$X(s) = \frac{9s^3 + 19s^2 + 67s + 5}{s^2(s^2 + 2s + 7)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs + D}{s^2 + 2s + 7}$$

$$X(s) = \frac{1}{49}\left(\frac{459}{s} + \frac{245}{7s^2} - \frac{18s + 22}{s^2 + 2s + 7}\right) = \frac{1}{49}\left(\frac{459}{s} + \frac{245}{7s^2} - \frac{18(s+1)}{(s+1)^2 + 6} - \frac{4}{(s+1)^2 + 6}\right)$$

Transformada inversa

$$x(t) = \frac{459}{9} + \frac{5t}{7} - \frac{18}{49}e^{-t}\cos(\sqrt{6}t) - \frac{4}{49\sqrt{6}}e^{-t}\sin(\sqrt{6}t)$$