PME 3380 - Modelagem de sistemas dinâmicos

/ /

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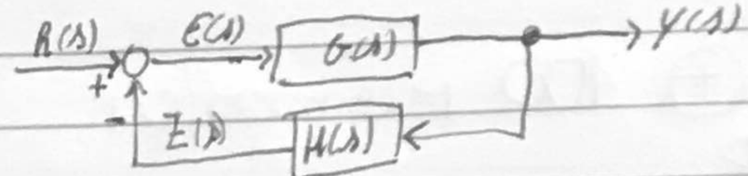
1) Provar que $(I+GH)^{-1}G = G(I+\underbrace{HG}_{L})^{-1} = G(I+L)^{-1}$.

Pelo slide tem-se que $Y(s) = (I+GH)^{-1}GR(s) = TR(s)$

Pelo diagrama de blocos tem-se:




Então: $Z(s) = H(s)Y(s)$

Como $E(s) = R(s) - Z(s)$, então: $H(s)Y(s) = R(s) - E(s)$

Do diagrama $G(s)E(s) = Y(s) \to E(s) = G^{-1}(s)Y(s)$, Logo:

$$H(s)Y(s) = R(s) - G^{-1}(s)Y(s) \to R(s) = (H(s) + G^{-1}(s))Y(s)$$

$$R(s) = (I + H(s)G(s))G^{-1}(s)Y(s) \to Y(s) = \underbrace{G(s)(I+H(s)G(s))^{-1}}_{T}R(s)$$

Então $(I+GH)^{-1}G = G(I+HG)^{-1} = G(I+L)^{-1}$ para $L = HG$

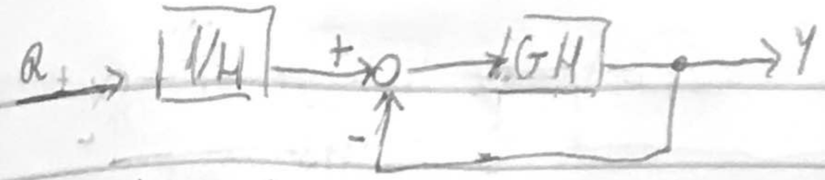
2) Do diagrama: $\left.\begin{array}{l} Z = HY \\ E = R - Z \\ Y = GE \end{array}\right\}$ Então: $Z = HGE = HG(R-Z) \to HGR = (I+HG)Z$

$$\frac{Z}{R} = (I+HG)^{-1}HG = \frac{HG}{I+HG}$$

De $Y = GE \to H^{-1}Z = GR - GZ$ $\quad (H^{-1}+G)Z = GR \to (I+GH)H^{-1}Z = GR$

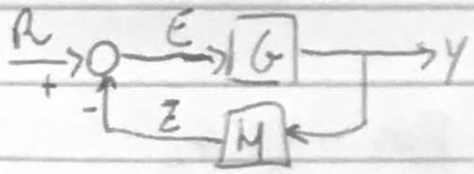
$$\frac{Z}{R} = H(I+GH)^{-1}G = \frac{HG}{I+GH}$$

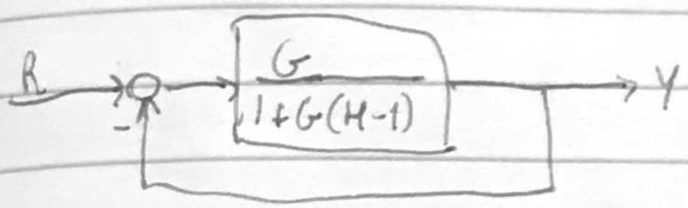
Como $L = GH = HG$, então $\dfrac{HG}{I+HG} = \dfrac{GH}{I+GH} = \dfrac{L}{I+L}$

3) 

$$Y = \left(\frac{R}{H} - Y\right) GH \rightarrow (I+GH)\,Y = \frac{GHR}{H}$$

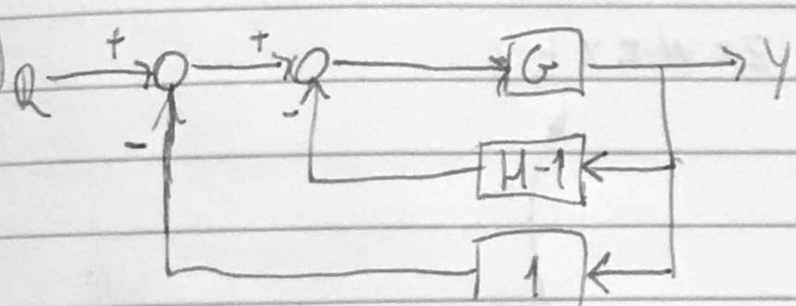
$$Y = \frac{G}{I+GH} R$$ Em blocos:



4) 

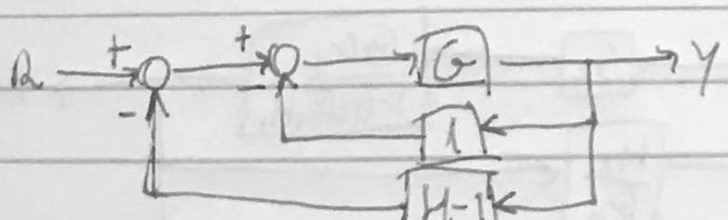
$$Y = \frac{GR}{1+G(H-1)} - \frac{GY}{1+G(H-1)} \rightarrow \frac{1+G(H-1)+G}{1+G(H-1)}\,Y = \frac{GR}{1+G(H-1)}$$

$$Y = \frac{G}{1+GH} R$$

5) 

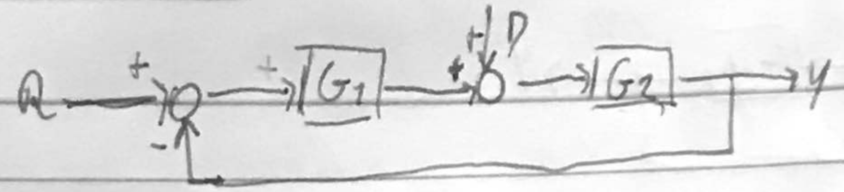
$$Y = GR - GY - G(H-1)Y$$

$$(1+GH)Y = GR \longrightarrow Y = \frac{GR}{1+GH}$$

6) 

$$Y = GR - G(H-1)Y - GY$$

$$Y = \frac{GR}{1+GH}$$

7) 

para D=0 $Y_1 = RG_1G_2 - Y_1G_1G_2$

$$Y_1 = \frac{RG_1G_2}{1+G_1G_2}$$

8) para R=0

$$Y_2 = DG_2 - Y_2G_1G_2$$

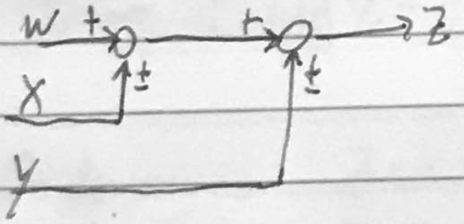
$$Y_2 = \frac{DG_2}{1+G_1G_2}$$

9) para $R \neq 0$ e $D \neq 0$:

$$Y = (RG_1 + D)G_2 - YG_1G_2$$

$$Y = RG_1G_2 + DG_2 - YG_1G_2$$

$$Y = \frac{(RG_1+D)G_2}{1+G_1G_2} = \frac{RG_1G_2}{1+G_1G_2} + \frac{DG_2}{1+G_1G_2} = Y_1 + Y_2$$

10) 

$Z = W \pm X \pm Y$

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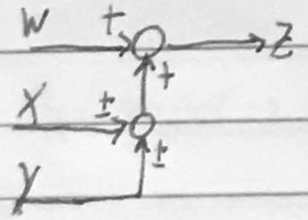
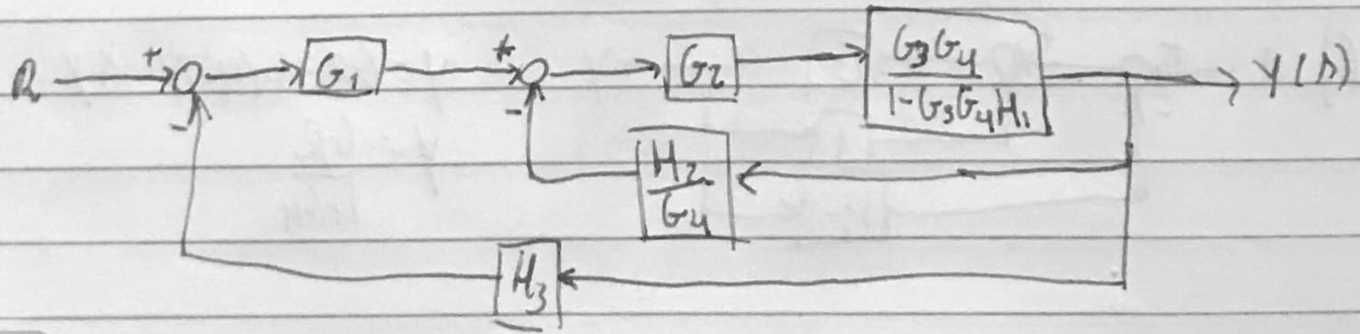


$Z = W \pm X \pm Y$

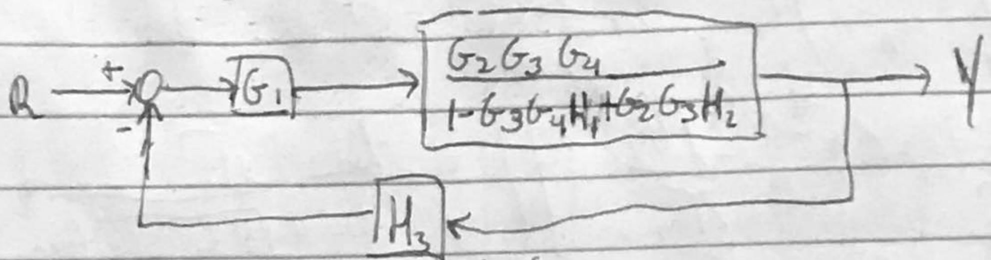
11) 

$$Y = \left[(R - H_3 Y)G_1 - \frac{H_2}{G_4} Y\right] G_2 \cdot \frac{G_3 G_4}{1 - G_3 G_4 H_1}$$

$$Y = \frac{G_1 G_2 G_3 G_4 R}{1 - G_3 G_4 H_1} - \frac{H_3 G_1 G_2 G_3 G_4 Y}{1 - G_3 G_4 H_1} - \frac{H_2 G_2 G_3}{1 - G_3 G_4 H_1} Y$$

$$Y = \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + H_3 G_1 G_2 G_3 G_4 + H_2 G_2 G_3} R$$

12)

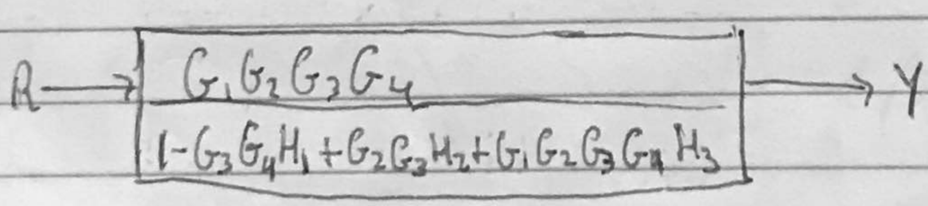



$$Y = (R - H_3 Y) \cdot G_1 \cdot \frac{G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2}$$

$$Y \left( \frac{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + H_3 \cdot G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2} \right) = \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2} R$$

$$Y = \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3} R$$

13)




$$Y = \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3} R$$