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22/10

Ex 1) $2\ddot{x} + 7\dot{x} + 3x = 0$; $x(0) = x_0$
$\dot{x}(0) = 0$

aplicando $\mathcal{L}\{\}$

$\ddot{x} \rightarrow s^2 X(s) - s\,x(0) - \dot{x}(0)$

$\dot{x} \rightarrow s X(s) - x(0)$

$x \rightarrow X(s)$

$2s^2 X(s) - 2s x_0 + 7s X(s) - 7x_0 + 3X(s) = 0$

$X(s) \cdot (2s^2 + 7s + 3) = (7 + 2 \cdot s) x_0$

$X(s) = \left(\dfrac{7 + 2s}{2s^2 + 7s + 3}\right) x_0 \rightarrow 49 - 4 \cdot 2 \cdot 3 = 25$

$\dfrac{-7 \pm 5}{4} \rightarrow \begin{cases} -3 \\ -1/2 \end{cases}$

$X(s) = \dfrac{(7+2s)\,x_0}{2(s+\frac{1}{2})(s+3)}$

Separando:

$\dfrac{A x_0}{2s+1} + \dfrac{B x_0}{s+3} = \dfrac{[(s+3)A + (2s+1)B] x_0}{2(s+1/2)(s+3)} = \dfrac{(7+2s) x_0}{2(s+1/2)(s+3)} = s(A+2B) + (3A+B) = 2s + 7$

$\begin{cases} A + 2B = 2 \\ 3A + B = 7 \end{cases}$

$6A + 2B = 14$
$-A - 2B = -2$
$5A = 12 \rightarrow A = \dfrac{12}{5} = 2{,}4$

$\dfrac{12}{5} + 2B = 2 \rightarrow B = -0{,}2 = \dfrac{-1}{5}$

$X(s) = \left[\dfrac{-1}{(s+3)} + \dfrac{12}{(2s+1)}\right] \dfrac{x_0}{5}$

aplicando $\mathcal{L}^{-1}\{\} \rightarrow \boxed{-\dfrac{x_0}{5} e^{-3t} + \dfrac{6x_0}{5} e^{-1/2 t} = X(t)}$

$\mathcal{L}^{-1}\left\{\dfrac{1}{s-a}\right\} = e^{at}$

ou

$\boxed{\dfrac{x_0}{5}\left(6e^{-0{,}5t} - e^{-3t}\right) = X(t)}$

2) $\dddot{X} + 2\ddot{X} + 7\dot{X} = \ddot{u} + 7\dot{u} + 5u$; $\left\{ \begin{matrix} u(t) = 1 & ; X(0) = 9 \\ \ddot{X}(0) = 2 & ; u(0) = 0 \\ \dot{X}(0) = 1 & ; \dot{u}(0) = 0 \end{matrix} \right\}$

$$\left[S^3 X_{(s)} - S^2 \overset{9}{X(0)} - S \overset{1}{\dot{X}(0)} - \overset{2}{\ddot{X}(0)}\right] + 2\left(S^2 X_{(s)} - S \overset{9}{X(0)} - \overset{1}{\dot{X}(0)}\right) + 7\left(S X_{(s)} - \overset{9}{X(0)}\right) =$$

$$= \left(S^3 X_{(s)} - 9S^2 - S - 2\right) + \left(S^2 X_{(s)} \cdot 2 - 18S - 2\right) + \left(7 \cdot S X_{(s)} - 63\right) =$$

$$= X_{(s)}\left(S^3 + 2S^2 + 7S\right) - 9S^2 - 19S - 67 = \ddot{u} + 7\dot{u} + 5u = \left[S^2 U_{(s)} - S\underset{0}{u(0)} - \underset{0}{\dot{u}(0)}\right] + 7\left[S U_{(s)} - \underset{0}{u(0)}\right]$$

$$+ 5U_{(s)} = U_{(s)} \cdot \left[S^2 + 7S + 5\right]$$

$$X_{(s)} = U_{(s)} \frac{S^2 + 7S + 5}{S^3 + 2S^2 + 7S} + \frac{(9S^2 + 19S + 67)}{S^3 + 2S^2 + 7S} \rightarrow \text{Como } U_{(t)} = 1 \rightarrow U_{(s)} = \frac{1}{S}$$

$$X_{(s)} = \frac{1}{S^2} \frac{9S^3 + 20S^2 + 74S + 5}{S^2 + 2S + 7} = \frac{1}{S^2} \frac{9S^3 + 20S^2 + 74S + 5}{(S^2 + 2S + 1) + 6} = \frac{9S^3 + 20S^2 + 74S + 5}{S^2\left[(S+1)^2 + 6\right]}$$

Por frações parciais se isola: $\frac{508}{49} \cdot \frac{1}{S} + \frac{5}{7} \cdot \frac{1}{S^2} - \frac{67}{49} \frac{(S+1)}{(S+1)^2 + 6} - \frac{4}{49} \frac{1}{(S+1)^2 + 6}$

aplicando transf. inversa:

$$\boxed{\frac{508}{49} \cdot 1 + \frac{5}{7} \cdot t - \frac{67}{49} e^{-t} \cos(\sqrt{6}\, t) - \frac{4}{49} \frac{e^{-t}}{\sqrt{6}} \operatorname{sen}(\sqrt{6}\, t)}$$