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① $2\ddot{x} + 7\dot{x} + 3x = 0$

$x(0) = x_0$

$\dot{x}(0) = 0$

$2\ddot{x} = 2\left(s^2X(s) - sx(0) - \dot{x}(0)\right) = 2\left(s^2X(s) - sx_0\right)$

$7\dot{x} = 7\left(sX(s) - x_0\right)$

$3x = 3(X(s))$

$\Rightarrow 2s^2X(s) - 2sx_0 + 7sX(s) - 7x_0 + 3X(s) = 0$

$X(s) = \dfrac{x_0(7 + 2s)}{2s^2 + 7s + 3}$

$G(s) = 0$

- $2s^2 + 7s + 3 = 0$

$s = \dfrac{-7 \pm \sqrt{49 - 4.2.3}}{2.2} \longrightarrow -0,5$ ou $\hookrightarrow -3$

- Fazendo a transf. inversa: $X(s) = \dfrac{x_0}{2} \cdot \left(\dfrac{\alpha}{s+0,5} + \dfrac{\beta}{s+3}\right)$

- $\alpha = \dfrac{2.(-0,5) + 7}{-0,5 + 3} = 2,4$ - $\beta = \dfrac{2(-3) + 7}{-3 + 0,5} = -0,4$

$\Rightarrow X(s) = x_0\left(\dfrac{1,2}{s+0,5} - \dfrac{0,2}{s+3}\right)$

$\therefore x(t) = 1,2 \cdot x_0 \cdot e^{-0,5t} - 0,2 \cdot e^{-3t}$

② $\dddot{x} + 2\ddot{x} + 7\dot{x} = \dot{u} + 7u + 5u$

$x(0) = 9$; $\dot{x}(0) = 1$; $\ddot{x}(0) = 2$; $u(0) = 1$; $\dot{u}(0) = 0$

$\dddot{x} = s^3X(s) - s^2x(0) - s\dot{x}(0) - \ddot{x}(0) = s^3X(s) - 9s^2 - s - 2$

$2\ddot{x} = 2\left(s^2X(s) - sx(0) - \dot{x}(0)\right) = 2\left(s^2X(s) - 9s - 1\right)$

$7\dot{x} = 7\left(sX(s) - 9\right)$

$$\ddot{u} = s^2 U(s) - sU(0) - \dot{u}(0) = s^2 U(s) - s$$
$$7\dot{u} = 7(sU(s) - 1)$$
$$5u = 5U(s)$$

$$\Rightarrow s^3 X(s) - 9s^2 - 5 - 2 + 2s^2 X(s) - 18s - 2 + 7sX(s) - 63 = s^2 U(s) - s + 7sU(s) - 7 + 5U(s)$$

$$\therefore X(s) = \frac{U(s)(s^2 + 7s + 5) + 9s^2 + 18s + 60}{s^3 + 2s^2 + 7s} \quad \rightarrow U(s) = \frac{1}{s}$$

$$X(s) = \frac{s^2 + 7s + 5 + 9s^3 + 18s^2 + 60s}{s^4 + 2s^3 + 7s^2} = \frac{9s^3 + 19s^2 + 67s + 5}{s^4 + 2s^3 + 7s^2}$$

- $s^4 + 2s^3 + 7s^2 = s^2(s^2 + 2s + 7) = 0$

$$s = 0 \quad ; \quad s = \frac{-2 \pm \sqrt{4 - 4.7}}{2.1} = \frac{-2 \pm 2i\sqrt{6}}{2} \rightarrow -1 + i\sqrt{6}$$
$$\hookrightarrow -1 - i\sqrt{6}$$

$$X(s) = \frac{\alpha}{s} + \frac{\beta}{s^2} + \frac{(\delta s + \psi)}{s^2 + 2s + 7}$$