André Kenzo Nakamura 9787112

Modelagem - Ex aula (22/10)

1) $2\ddot{x} + 7\dot{x} + 3x = 0$ $\quad \dot{x}(0) = 0 \quad x(0) = x_0$

(I) (II) (III)

I - $2\ddot{x} \rightarrow 2[s^2X(s) - sx_0]$

II - $7\dot{x} \rightarrow 7[sX(s) - x_0]$

III - $3x \rightarrow 3X(s)$

$X(s)(2s^2 + 7s + 3) - x_0(2s + 7) = 0$ ▷ Pólos: $s = -\frac{1}{2}$, $s = -3$

$X(s) = \frac{(2s+7)x_0}{(2s^2+7s+3)}$, $G(s) = \frac{X(s)}{U(s)} = 0$

Transformada inversa:

$$X(s) = \frac{(2s+7)x_0}{2(s+0,5)(s+3)} = \left(\frac{\alpha_1}{s+0,5} + \frac{\alpha_2}{s+3}\right)\frac{x_0}{2}$$

$$\alpha_1 = \left.\frac{2s+7}{s+3}\right|_{s=-\frac{1}{2}} = 2,4 \qquad \alpha_2 = \left.\frac{2s+7}{s+0,5}\right|_{s=-3} = -0,4$$

$$X(s) = \left(\frac{1,2}{s+0,5} - \frac{0,2}{s+3}\right)x_0$$

$$x(t) = 1,2\,x_0 e^{-0,5t} - 0,2e^{-3t}$$

---

2) $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$

(I) (II) (III) (IV) (V) (VI)

$\ddot{x}(0) = 2 \quad \dot{x}(0) = 1 \quad x(0) = 9$

$u(0) = 1 \quad \dot{u} = 0 \quad u(t) = 1$

I - $\dddot{x} = s^3X(s) - 9s^2 - s - 2$

II - $2\ddot{x} = 2(s^2X(s) - 9s - 1)$

III - $7\dot{x} = 7(sX(s) - 9)$

IV - $\ddot{u} = (s^2U(s) - s)$

V- $7\dot{u} = 7(sU(s) - 1)$ VI- $5u = 5U(s)$

$$X(s) = \frac{(s^2 + 7s + 5)U(s)}{(s^3 + 2s^2 + 7s)} + \frac{(9s^2 + 18s + 60)}{(s^3 + 2s^2 + 7s)}$$

$$G(s) = \frac{s^2 + 7s + 5}{s^3 + 2s^2 + 7s}$$

$s^3 + 2s^2 + 7s \rightarrow s = 0 \quad s = -1 \pm i\sqrt{6}$ (Polos)

$$X(s) = \frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3 s + a_4}{s^2 + 2s + 7}$$??