

Exercício da aula 22/10

①. Transformada de Laplace:

$$2\ddot{x} + 7\dot{x} + 3x = 0 \quad ; \quad x(0) = x_0 \quad ; \quad \dot{x}(0) = 0$$

$$2(s^2 \cdot X_s - s x_0) + 7(s X_s - x_0) + 3 \cdot X_s = 0$$

$$(2s^2 + 7s + 3) X_s = (2s + 7) x_0$$

$$X_s = \frac{(2s + 7) x_0}{2s^2 + 7s + 3} = \frac{(2s + 7) x_0}{(s + 3)(2s + 1)} = \frac{A}{s + 3} + \frac{B}{2s + 1}$$

$$X_s = \frac{s(2A + B) + (A + 3B)}{(s + 3)(2s + 1)}$$

$$\begin{cases} 2A + B = 2 \cdot x_0 \\ A + 3B = 7 \cdot x_0 \end{cases} \Rightarrow \begin{cases} B = 12/5 \cdot x_0 \\ A = -1/5 \cdot x_0 \end{cases}$$

$$X_s = \frac{-x_0}{5(s + 3)} + \frac{12x_0}{5(2s + 1)}$$

• Transformada inversa:

$$x(t) = -x_0/5 \mathcal{L}^{-1}\{1/s + 3\} + 6x_0/5 \mathcal{L}^{-1}\{1/s + 1/2\}$$

$$x(t) = -\frac{x_0}{5} \cdot e^{-3t} + \frac{6x_0}{5} \cdot e^{-t/2}$$

② Transformada Laplace: $\ddot{x} + 2\dot{x} + 7x = \ddot{u} + 7\dot{u} + 5u$ com $\ddot{x}(0) = 2$
 $\dot{x}(0) = 1$
 $x(0) = 9$ ①
 $\dot{u}(0) = 0$
 $u(0) = 0$

$$s^3 X_s - s^2 x_0 - s \dot{x}_0 - \ddot{x}_0 + 2(s^2 X_s - s x_0 - \dot{x}_0) + 7(s X_s - x_0) = s^3 U_s - s^2 u_0 - s \dot{u}_0 - \ddot{u}_0 + 7(s U_s - u_0) + 5 U_s$$

$$X_s (s^3 + 2s^2 + 7s) = U_s (s^2 + 7s + 5) + 9s^2 + 19s + 67$$

• $U_s = 1/s$ e utilizando o método das frações parciais:

$$X_s = \frac{9s^3 + 20s^2 + 74s + 5}{s^2(s^2 + 2s + 7)} = \frac{508}{49s} + \frac{5}{7s^2} - \frac{(67s + 71)}{49((s+1)^2 + 6)}$$

• Transformada inversa:

$$x(t) = \frac{508}{49} \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{5}{7} \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \frac{67}{49} \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2 + 6}\right\} - \frac{4}{49} \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2 + 6}\right\}$$

$$x(t) = \frac{508}{49} + \frac{5t}{7} - \frac{67}{49} e^{-t} \cos(t\sqrt{6}) - \frac{4}{49\sqrt{6}} e^{-t} \sin(t\sqrt{6})$$