(1) $2\ddot{x} + 7\dot{x} + 3x = 0$

Transformada de Laplace:

$$2\mathcal{L}\{\ddot{x}\} + 7\mathcal{L}\{\dot{x}\} + 3\mathcal{L}\{x\} = 0$$

$$2(s^2X(s) - sX(0)) + 7(sX(s) - X(0)) + 3X(s) = 0$$

$$(2s^2 + 7s + 3)X(s) = (2s+7)x_0 \Rightarrow X(s) = \frac{x_0(2s+7)}{2(s+3)(s+1/2)}$$

$$X(s) = \frac{x_0(2s+7)}{(s+3)(2s+1)} = \frac{A}{s+3} + \frac{B}{2s+1} = \frac{s(2A+B) + (A+3B)}{(s+3)(2s+1)}$$

$$\begin{cases} 2A + B = 2x_0 \\ A + 3B = 7x_0 \end{cases} \quad \begin{matrix} \longrightarrow B = \frac{12}{5}x_0 \\ \longrightarrow A = -\frac{1}{5}x_0 \end{matrix}$$

$$X(s) = \frac{-x_0}{5(s+3)} + \frac{12x_0}{5(2s+1)}$$

Transformada inversa:

$$X(t) = \frac{-x_0}{5} \cdot \mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} + \frac{6}{5}x_0 \mathcal{L}^{-1}\left\{\frac{1}{s+1/2}\right\}$$

$$X(t) = \frac{-x_0}{5} e^{-3t} + \frac{6x_0}{5} \cdot e^{-t/2}$$

(2) $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$; $\ddot{x}(0) = 2$, $\dot{x}(0) = 1$, $x(0) = 9$, $\dot{u}(0) = 0$, $u(0) = 0$

Transformada de Laplace:

$$(s^3X(s) - \ddot{x}(0) - s\dot{x}(0) - s^2x(0)) + 2(s^2X(s) - \dot{x}(0) - sx(0)) + 7(sX(s) - x(0)) =$$

$$= (s^2U(s) - \dot{u}(0) - su(0)) + 7(sU(s) - u(0) + 5U(s)$$

$$s^3X(s) - 2 - s - 9s^2 + 2s^2X(s) - 2 - 18s + 7sX(s) - 63 = s^2U(s) + 7sU(s) + 5U(s)$$

$$(s^3 + 2s^2 + 7s)X(s) = 9s^2 + 19s + 67 + (s^2 + 7s + 5)U(s) \Rightarrow U(s) = \frac{1}{s}$$

$$X(s) = \frac{508}{49s} + \frac{5}{7s^2} - \frac{67s + 71}{49[(s+1)^2 + 6]}$$

Transformada inversa:

$$x(t) = \frac{500}{49} + \frac{5t}{7} - \frac{67}{49} \cdot e^{-t} \cdot \cos(t\sqrt{6}) - \frac{4}{49\sqrt{6}} \cdot e^{-t} \cdot \mathrm{sen}(t\sqrt{6})$$