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Exercícios da Aula 22/10

① $2\ddot{x} + 7\dot{x} + 3x = 0 \; ; \; x(0) = x_0 \; , \; \dot{x}(0) = 0$

$\xrightarrow{L} 2(s^2X(s) - sx(0) - \dot{x}(0)) + 7(sX(s) - x(0)) + 3X(s) = 0 \Rightarrow X(s) = \frac{2s+7}{2s^2+7s+3}x_0 \Rightarrow$

$\Rightarrow X(s) = \frac{2s+7}{2(s+3)(s+\frac{1}{2})}x_0 = \left(\frac{A}{s+3} + \frac{B}{s+\frac{1}{2}}\right)\frac{x_0}{2} = \left(\frac{2SA + A + Bs + 3B}{(s+3)(s+\frac{1}{2})}\right)\frac{x_0}{2}$

$\begin{cases} 2A + B = 2x_0 \\ A + 3B = 7x_0 \end{cases} \Rightarrow \begin{matrix} A = -\frac{x_0}{5} \\ B = \frac{12}{5}x_0 \end{matrix} \Rightarrow \boxed{X(s) = \frac{12x_0}{5(s+\frac{1}{2})} - \frac{x_0}{5(s+3)}}$

$\xrightarrow{L^{-1}} \boxed{x(t) = \frac{6}{5}x_0 e^{-\frac{1}{2}t} - \frac{x_0}{5}e^{-3t}}$

② $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u \; ; \; u(t) = 1 \; ; \; \begin{cases} \ddot{x}(0) = 2 \; ; \; \dot{x}(0) = 1 \; ; \; x(0) = 9 \\ \dot{u}(0) = 0 \; ; \; u(0) = 0 \end{cases}$

$\xrightarrow{L} s^3X(s) - s^2x(0) - s\dot{x}(0) - \ddot{x}(0) + 2(s^2X(s) - sx(0) - \dot{x}(0)) + 7(sX(s) - x(0)) =$

$= s^2U(s) - su(0) - \dot{u}(0) + 7(sU(s) - u(0)) + 5U(s) \Rightarrow$

$\Rightarrow (s^3 + 2s^2 + 7s)X(s) - 9s^2 - 19s - 67 = (s^2 + 7s + 5)U(s) \Rightarrow$

$\Rightarrow X(s) = \frac{(s^2+7s+5)}{(s^3+2s^2+7s)}U(s) + \frac{(9s^2+19s+67)}{(s^3+2s^2+7s)}$, com $u(t) = 1 \rightarrow U(s) = \frac{1}{s}$

$X(s) = \frac{s^2+7s+5+s(9s^2+19s+67)}{s^2(s^2+2s+7)} = \frac{9s^3+20s^2+74s+5}{s^2(s^2+2s+7)}$

$X(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+2s+7} \longrightarrow A = \frac{508}{49} \; ; \; B = \frac{5}{7} \; ; \; C = -\frac{67}{49} \; ; \; D = -\frac{71}{49}$

$\boxed{X(s) = \frac{508}{49s} + \frac{5}{7s^2} - \frac{67s+71}{49((s+1)^2+6)}}$

$\xrightarrow{L^{-1}} L^{-1}[X(s)] = \frac{508}{49}L^{-1}\left[\frac{1}{s}\right] + \frac{5}{7}L^{-1}\left[\frac{1}{s^2}\right] - \frac{67}{49}L^{-1}\left[\frac{(s+1)}{(s+1)^2+6}\right] - \frac{4}{49}L^{-1}\left[\frac{1}{(s+1)^2+6}\right]$

$\boxed{x(t) = \frac{508}{49} + \frac{5}{7}t - \frac{67}{49}e^{-t}\cos(\sqrt{6}t) - \frac{4}{49\sqrt{6}}e^{-t}\operatorname{sen}(\sqrt{6}t)}$