

Exercícios aula 22/10/2020

$$1) \quad 2\ddot{x} + 7\dot{x} + 3x = 0; \quad x(0) = x_0; \quad \dot{x}(0) = 0$$

$$2s^2 X(s) + 7s X(s) + 3X(s) = 1 \quad (\text{degrau unitário})$$

$$\Rightarrow \boxed{\frac{X(s)}{U(s)} = \frac{1}{2s^2 + 7s + 3}}$$

•) Resolvendo a EDO:

$$\mathcal{L}(f(t)) = 2(s^2 X(s) - s x(0) - \dot{x}(0)) + 7(s X(s) - x(0)) + 3X(s) = 0$$

$$\Rightarrow 2s^2 X(s) - 2s x_0 + 7s X(s) - 7x_0 + 3X(s) = 0$$

$$\Rightarrow \underline{X(s) = \frac{2s + 7}{2s^2 + 7s + 3} \cdot x_0}$$

Separando as frações parciais, obtém-se para os polos -3 e $-0,5$:

$$\Rightarrow X(s) = \left[\frac{a}{s+3} + \frac{b}{s+0,5} \right] \cdot \frac{x_0}{2}$$

$$\rightarrow a = (s+3) \cdot X(s) \Big|_{s=-3} \Rightarrow a = \frac{2 \cdot (-3) + 7}{(-3 + 0,5)} = -\frac{2}{5}$$

$$\rightarrow b = (s+0,5) \cdot X(s) \Big|_{s=-0,5} \Rightarrow b = \frac{-0,5 \cdot 2 + 7}{(3 - 0,5)} = \frac{12}{5} \quad (1)$$

$$\Rightarrow X(s) = \left[\frac{-2}{5(s+3)} + \frac{12}{5(s+0.5)} \right] \cdot \frac{x_0}{2}$$

$$\Rightarrow X(s) = \frac{-x_0}{5(s+3)} + \frac{6x_0}{5(s+0.5)}$$

Aplicando a transformada inversa:

$$x(t) = -\frac{x_0}{5} \cdot e^{-3t} + \frac{6x_0}{5} \cdot e^{-t/2}$$

$$2) \ddot{x} + 2\dot{x} + 7x = \ddot{u} + 7\dot{u} + 5u$$

$$\ddot{x}(0) = 2; \dot{x}(0) = 1; x(0) = 9; u(t) = 1; u(0) = \dot{u}(0) = 0.$$

a) Determinando a FT:

$$s^3 \cdot X(s) + 2s^2 \cdot X(s) + 7s \cdot X(s) = s^2 \cdot U(s) + 7s \cdot U(s) + 5U(s)$$

$$\Rightarrow X(s) \cdot (s^3 + 2s^2 + 7s) = U(s) \cdot (s^2 + 7s + 5)$$

$$\Rightarrow \boxed{\frac{X(s)}{U(s)} = \frac{s^2 + 7s + 5}{s^3 + 2s^2 + 7s}}$$

b) Resolvendo a EDO:

$$+ 7(sX(s) - x(0))$$

$$L(f(t)) = s^3 \cdot X(s) - s^2 x(0) - s \dot{x}(0) - \ddot{x}(0) + 2(s^2 X(s) - s x(0) - \dot{x}(0))$$

$$\Rightarrow = U(s) \cdot s^2 - s \cdot u(s) - \dot{u}(0) + 7(s \cdot U(s) - u(s)) + 5 \cdot U(s)$$

$$\Rightarrow s^3 \cdot X(s) - s^2 \cdot 9 - s - 2 + 2s^2 \cdot X(s) - 2 \cdot s \cdot 9 - 2 + 7s \cdot X(s) - 7 + 5 \cdot U(s)$$

$$- 63 = U(s) \cdot s^2 - s + 7s \cdot U(s) - 7 + 5 \cdot U(s) = 0$$

$$\Rightarrow X(s) = \frac{U(s) \cdot (s^2 + 7s + 5)}{s^3 + 2s^2 + 7s} + \frac{9s^2 + 18s + 60}{s^3 + 2s^2 + 7s} - \frac{s+9}{s^2+2s+7}$$

•) Frações parciais

$$\Rightarrow X(s) = \frac{5}{7s^2} + \frac{39}{49s} - \frac{\left(\frac{39}{49}s + \frac{64}{49}\right)}{s^2 + 2s + 7} + \frac{60}{7s} + \frac{\left(\frac{3}{7}s + \frac{6}{7}\right)}{s^2 + 2s + 7}$$

$$\Rightarrow X(s) = \frac{\frac{5}{7}}{s^2} + \frac{\frac{459}{49}}{s} - \frac{18(s+1)}{49[(s+1)^2 + 6]}$$

•) Domínio do tempo:

$$x(t) = \frac{5}{7} \cdot t + \frac{459}{49} - \frac{18}{49} \cdot e^{-t} \cdot \cos(\sqrt{6} \cdot t)$$