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1) $2\ddot{x} + 7\dot{x} + 3x = 0$

$x(0) = x_0 ; \dot{x}(0) = 0$

$2\mathcal{L}\{\ddot{x}\} + 7\mathcal{L}\{\dot{x}\} + 3\mathcal{L}\{x\} = 0$

$2s^2X(s) + 7sX(s) + 3X(s) = 2sx_0 + 7x_0$

$X(s) = \frac{x_0(2s+7)}{2(s+3)(s+0,5)} =$

$= \frac{A}{s+3} + \frac{B}{2s+1} = \frac{2sA + A + Bs + 3B}{(s+3)(2s+1)}$

$\begin{cases} 2A + B = 2x_0 \\ A + 3B = 7x_0 \end{cases} \quad | \quad A = -x_0/5 \quad ; \quad B = 12x_0/5$

$X(s) = -\frac{x_0}{s(s+3)} + \frac{12x_0}{s(2s+1)}$

Aplicando a transf. inversa

$\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$

$x(t) = -\frac{x_0}{5}e^{-3t} + \frac{6x_0}{5}e^{-\frac{t}{2}}$

2) $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$

$\ddot{x}(0) = 2$; $\dot{x}(0) = 1$; $x(0) = 0$;

$\dot{u}(0) = 0$; $u(0) = 0$

Aplicando a transf. de Laplace:

$$s^3X(s) - x''(0) - s\dot{x}(0) - s^2x(0) + 2\left(s^2X(s) - \dot{x}(0) - sx(0)\right) + 7\left(sX(s) - x(0)\right) = s^2 \cdot U(s) + 7sU(s) + 5U(s)$$

$$X(s) = U(s)\frac{s^2+7s+5}{s^3+2s+7s} + \frac{9s^2+19s+67}{s^3+2s^2+7s}$$

Como $U(s) = \frac{1}{s}$: $X(s) = \frac{9s^3+20s^2+74s+5}{s^2(s^2+2s+7)}$

Por frações parciais:

$$X(s) = \frac{508}{49s} + \frac{5}{7s^2} - \frac{67s+71}{49\left[(s+1)^2+6\right]}$$

Aplicando a trans. inversa:

$e^t \cdot \cos(\sqrt{6}t)$ $\frac{1}{\sqrt{6}}e^{-t}\operatorname{sen}(t\sqrt{6})$

$$x(t) = \frac{508}{49} + \frac{5}{7}t - \frac{67}{49}e^{-t}\cos(t\sqrt{6}) - \frac{4}{49\sqrt{6}}e^{-t}\operatorname{sen}(t\sqrt{6})$$