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- Exercícios do dia / -

1- $2\ddot{x} + 7\dot{x} + 3x = 0$

$x(0) = x_0$ e $\dot{x}(0) = 0$

Tranformada de Laplace:

$$2\mathcal{L}\{\ddot{x}\} + 7\mathcal{L}\{\dot{x}\} + 3\mathcal{L}\{x\} = 0$$

$$\Rightarrow 2\left(s^2 X(s) - s\,x(0) - \dot{x}(0)\right) + 7\left(s X(s) - x(0)\right) +$$
$$+ 3X(s) = 0$$

$$\Rightarrow 2s^2 X(s) + 7 \cdot s X(s) + 3X(s) = 2s x_0 + 7x_0$$

$$\Rightarrow X(s) = \frac{x_0(2s+7)}{2s^2+7s+3} = \frac{x_0(2s+7)}{2\left(s^2 + \frac{7s}{2} + \frac{3}{2}\right)} = \frac{x_0(2s+7)}{2(s+\frac{1}{2})(s+3)}$$

Método das frações parciais:

$$\frac{x_0(2s+7)}{(2s+1)(s+3)} = \frac{A}{(2s+1)} + \frac{B}{(s+3)} = \frac{As + 3A + 2sB + B}{(2s+1)(s+3)}$$

$$\begin{cases} A + 2B = 2x_0 \\ 3A + B = 7x_0 \end{cases} \Rightarrow 3A + 6B = 6x_0$$

$$5B = -x_0 \Rightarrow B = -\frac{x_0}{5}$$

$$A = 2x_0 - 2B = 2x_0 + \frac{2x_0}{5} \Rightarrow A = \frac{12x_0}{5}$$

$$\therefore X(s) = \frac{12x_0}{5(2s+1)} - \frac{x_0}{(s+3)}$$

Transformada inversa:

$$\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$$

$$x(t) = \frac{6x_0}{5}\mathcal{L}^{-1}\left\{\frac{1}{s+\frac{1}{2}}\right\} - \frac{x_0}{5}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$$

$$= \frac{6x_0}{5}\cdot e^{-\frac{1}{2}t} - \frac{x_0}{5}\cdot e^{-3t}$$

2- $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$

$\ddot{x}(0) = 2 \mid \dot{x}(0) = 1 \mid x(0) = 9$

$\dot{u}(0) = 0 \mid u(0) = 0$

Transformada de Laplace

$$(s^3X(s) - \ddot{x}(0) - s\dot{x}(0) - s^2x(0)) + 2(s^2X(s) - \dot{x}(0) - sx(0))$$
$$+ 7(sX(s) - x(0)) = (s^2U(s) - su(0) - \dot{u}(0))$$
$$+ 7(sU(s) - u(0)) + 5U(s)$$

$$\Rightarrow s^3X(s) + 2s^2X(s) + 7sX(s) - 2 - s - 9s^2 - 2 - 18s - 63$$
$$= s^2U(s) + 7sU(s) + 5U(s)$$

$$\Rightarrow X(s)\cdot(s^3 + 2s^2 + 7s) = U(s)\cdot(s^2 + 7s + 5) + 9s^2 + 19s + 67$$

$$\Rightarrow X(s) = U(s)\cdot\frac{s^2+7s+5}{s^3+2s^2+7s} + \frac{9s^2+19s+67}{s^3+2s^2+7s}$$

$$= \frac{s^2+7s+5 + 9s^3 + 19s^2 + 67s}{s^2(s^2+2s+7)}$$

$$= \frac{9s^3 + 20s^2 + 74s + 5}{s^2(s^2+2s+7)}$$

Método das frações parciais:

$$X(s) = \frac{5}{7s^2} + \frac{508}{49s} + \frac{(-67s-71)}{49\left[(s+1)^2+6\right]}$$

Transformada inversa:

$$e^{-t} \cdot \cos(\sqrt{6}t) \quad \frac{1}{\sqrt{6}} e^{-t} \operatorname{sen}(\sqrt{6}t) = \mathcal{L}^{-1}$$

$$X(t) = \frac{5}{7}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + \frac{508}{49}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{67}{49}\mathcal{L}^{-1}\left\{\frac{(s+1)}{(s+1)^2+6}\right\}$$

$$- \frac{4}{49}\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+6}\right\}$$

$$X(t) = \frac{5}{7}t + \frac{508}{49} - \frac{67}{49} \cdot e^{-t}\cos(\sqrt{6}t) - \frac{4}{49\sqrt{6}} \cdot e^{-t}\operatorname{sen}(\sqrt{6}t)$$