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ex aula 22/10

1) $2\ddot{x} + 7\dot{x} + 3x = 0$; $x(0) = x_0$; $\dot{x}(0) = 0$

$\downarrow \mathcal{L}$

$$2s^2X - 2x_0 s + 7sX - 7x_0 + 3X = 0$$

$$(2s^2 + 7s + 3)X = (2s+7)x_0$$

$$X(s) = \frac{2s+7}{2s^2+7s+3} x_0 \qquad G(s) = \frac{X(s)}{U(s)}$$

$G(s) = 0$ ∴ não há entrada

Polos: $2s^2 + 7s + 3 = 0$

$$s = \frac{-7 \pm \sqrt{49-24}}{4} = \frac{-7 \pm 5}{4}$$

$s_1 = -3$ e $s_2 = -\frac{1}{2}$

Transformada inversa:

$$X(s) = \frac{(2s+7)x_0}{2.(s+0,5)(s+3)} = \left(\frac{\alpha_1}{s+0,5} + \frac{\alpha_2}{s+3}\right)\frac{x_0}{2}$$

$$a = \frac{2s+7}{s+3}\bigg|_{s=-3} = -\frac{2}{5} \qquad b = \frac{2s+7}{s+3}\bigg|_{s=-\frac{1}{2}} = \frac{12}{5}$$

$$X(s) = \left[-\frac{2}{5(s+3)} + \frac{12}{5(s+\frac{1}{2})}\right]\frac{x_0}{2} = \frac{x_0}{5(s+3)} + \frac{6x_0}{5(s+\frac{1}{2})}$$

$\downarrow \mathcal{L}^{-1}$

$$x(t) = -\frac{x_0}{5}e^{-3t} + \frac{6}{5}x_0 e^{-t/2}$$

(2) $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$ com $\ddot{x}(0) = 2$; $\dot{x}(0) = 1$; $x(0) = 9$; $u(0) = 1$; $\dot{u}(0) = 0$; $u(t) = 1$

Aplicando $\mathcal{L}$ na EDO, temos:

$$s^3X - s^2x(0) - s\dot{x}(0) - \ddot{x}(0) + 2(s^2X - sx(0) - \dot{x}(0)) + 7(sX - x(0))$$
$$= s^2U - su(0) - \dot{u}(0 + 7(sU - u(0)) + 5U$$

$$(s^3+2s^2+7s)X(s) - s^2x(0) - s(\dot{x}(0) + 2x(0)) - (\ddot{x}(0) + 2\dot{x}(0) + 7x(0))$$
$$= (s^2+7s+5)U - su(0) - (\dot{u}(0) + 7u(0))$$
$$\therefore \quad X(s)(s^3+2s^2+7s) = U(s)(s^2+7s+5) + 9s^2 + 18s + 60$$

$$X(s) = \frac{U(s)(s^2+7s+5)}{(s^3+2s^2+7s)} + \frac{9s^2+18s+60}{(s^3+2s^2+7s)}$$

Função de transferência

$$G(s) = \frac{X(s)}{U(s)} = \frac{s^2+7s+5}{s(s^2+2s+7)}$$

Polos do sistema: $s_1 = 0$; $s_2 = -1 + \sqrt{5}\,i$ e $s_3 = -1 - \sqrt{5}\,i$, com isso o sistema é estável.

Pelo método das frações parciais.

$$X(s) = \frac{9s^3 + 19s^2 + 67s + 5}{s^2(s^2+2s+7)} \Rightarrow X(s) = \frac{a}{s} + \frac{b}{s^2} + \frac{cs+d}{s^2+2s+7}$$

$$b = s^2X(s)\Big|_{s=0}$$
$$b = \frac{9s^3 + 19s^2 + 67s + 5}{s^2+2s+7}\Bigg|_{s=0}$$
$$b = \frac{5}{7}$$

$$a = \frac{d}{ds}\left[s^2X(s)\right]_{s=0}$$
$$a = \frac{(27s^2+38s+67)(s^2+2s+7) - (9s^3+19s^2+67s+5)(2s+2)}{(s^2+2s+7)^2}\Bigg|_{s=0}$$
$$a = \frac{459}{49}$$

$$X(s) = \frac{459}{49s} + \frac{5}{7s^2} + \frac{cs+d}{s^2+2s+7} = \frac{459\, s(s^2+2s+7) + 35(s^2+2s+7) + 49s^2(cs+d)}{49s^2(s^2+2s+7)}$$

$$(459+49c)s^3 + (918+35+49d)s^2 + (32.13+70)s + 245$$

$$= 49(9s^3 + 19s^2 + 67s + 5)$$

$$\left| \begin{array}{l} 459 + 49c = 49\cdot 9 \\ 918 + 35 + 49d = 49\cdot 19 \end{array} \right. \Rightarrow \left| \begin{array}{l} c = \frac{-18}{49} \\ d = -\frac{22}{49} \end{array} \right.$$

$$\therefore\ X(s) = \frac{1}{49}\left[\frac{459}{s} + \frac{245}{7s^2} - \frac{18s+22}{s^2+2s+7}\right] = \frac{1}{49}\left[\frac{459}{s} + \frac{245}{7s^2} - \frac{18(s+1)}{(s+1)^2+6} - \frac{4}{(s+1)^2+6}\right]$$

$$\mathcal{L}^{-1}[X(s)] = x(t) = \frac{459}{9} + \frac{5}{7}t - \frac{18}{49}e^{-t}\cos(\sqrt{6}t) - \frac{4}{49\sqrt{6}}e^{-t}\operatorname{sen}(\sqrt{6}t)$$