

Exercício aula 03/11/2020

① $(I + GH)^{-1} G = G(I + HG)^{-1}$

$$(I + GH)(I + GH)^{-1} G (I + HG) = (I + GH) G (I + HG)^{-1} (I + HG)$$

$$G(I + HG) = (I + GH)G$$

$$G + GHG = G + GHG$$

$$\therefore (I + GH)^{-1} G = G(I + HG)^{-1}$$

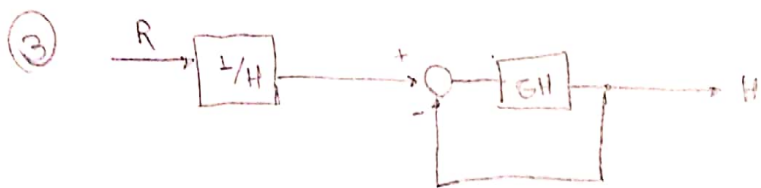
② $Z = HG(R - Z) \Rightarrow (I + HG)Z = HGR$

$$Z = (I + HG)^{-1} HGR$$

$$HG(I + GH)^{-1} = (I + HG)^{-1} HG$$

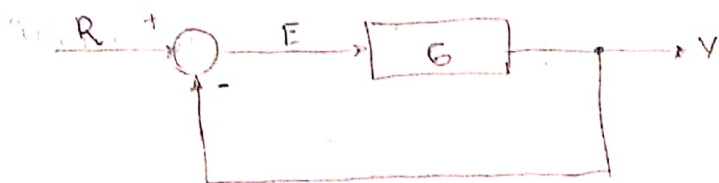
$$(I + HG)HG = HG(I + GH)$$

$$HG + HGHG = HG + HGHG$$



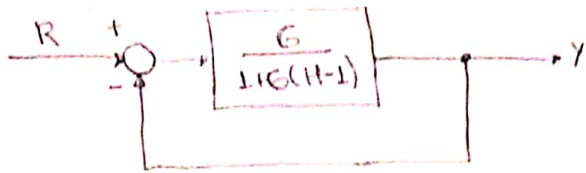
$$Y = GH \left(\frac{R}{H} - Y \right) \Rightarrow Y = GR - GHY \Rightarrow (I + GH)Y = GR$$

$$\frac{Y}{R} = \frac{G}{I + GH}$$



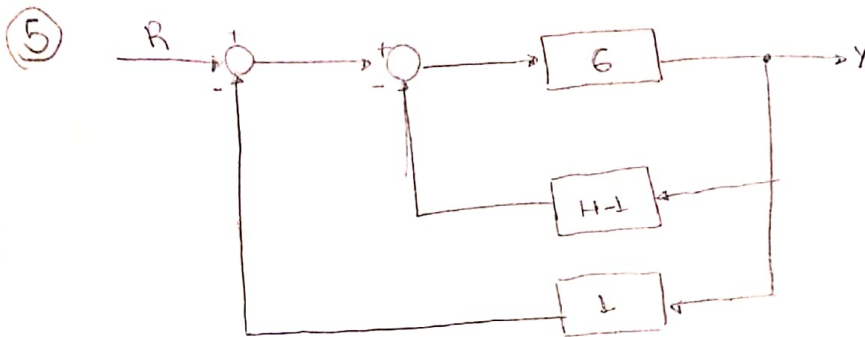
$$Y = GH \left(\frac{1}{H} R - Y \right) \Rightarrow (I + GH)Y = GR \Rightarrow \frac{Y}{R} = \frac{G}{I + GH}$$

Portanto, como as expressões são iguais as estruturas são equivalentes



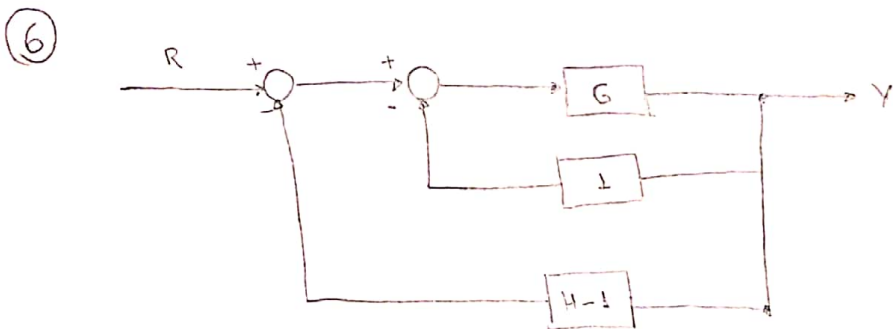
$$Y = \frac{G}{1 + G(H-1)} (R-1) \Rightarrow \left[1 + \frac{G}{1 + G(H-1)} \right] Y = \frac{GR}{1 + G(H-1)}$$

$$\frac{Y(1 + GH - G + G)}{1 + G(H-1)} = \frac{GR}{1 + G(H-1)} \Rightarrow \frac{Y}{R} = \frac{G}{1 + GH}$$



$$G[(R-Y) - (H-1)Y] = Y \Rightarrow G(R - Y - HY + Y) = Y \Rightarrow GR - GHY = Y$$

$$Y(1 + GH) = GR \Rightarrow \frac{Y}{R} = \frac{G}{1 + GH}$$



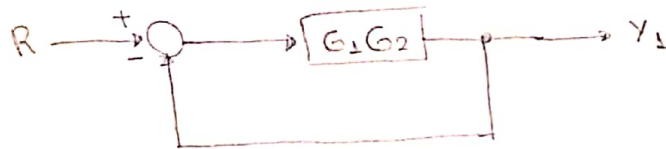
$$G[R - (H-1)Y - Y] = Y \Rightarrow G(R - HY + Y - Y) = Y \Rightarrow GR - GHY = Y$$

$$Y(1 + GH) = GR \Rightarrow \frac{Y}{R} = \frac{G}{1 + GH}$$

7) Com $D=0$, o diagrama é dado por



Para o caso escalar temos $G_2 G_1 = G_1 G_2$ com uma forma com:



$$(R - Y_1) G_1 G_2 = Y_1 \Rightarrow G_1 G_2 R = Y_1 (1 + G_1 G_2) \Rightarrow Y_1 = \frac{G_1 G_2 R}{1 + G_1 G_2}$$

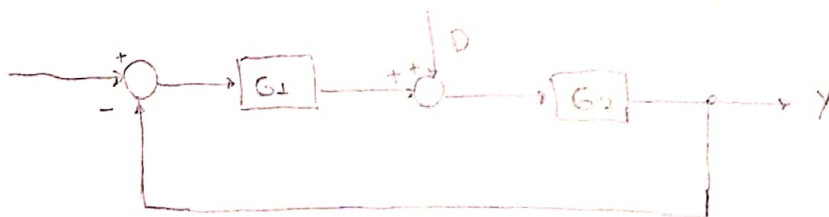
8) Com $R=0$, o diagrama é dado por:



$$G_2(D - G_1 Y_2) - Y_2 = 0 \Rightarrow G_2 D - G_2 G_1 Y_2 = Y_2 \Rightarrow Y_2 (1 + G_2 G_1) = G_2 D$$

$$\Rightarrow Y_2 = \frac{G_2 D}{1 + G_1 G_2}$$

9)



$$G_2 [G_1 (R - Y) + D] = Y$$

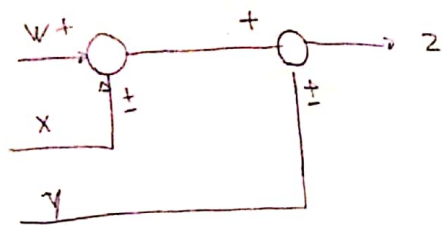
$$G_2 G_1 R - G_2 G_1 Y + G_2 D = Y$$

$$Y (1 + G_1 G_2) = G_2 G_1 R + G_2 D$$

$$Y = \frac{G_1 G_2 R}{1 + G_1 G_2} + \frac{G_2 D}{1 + G_1 G_2} = Y_1 + Y_2$$

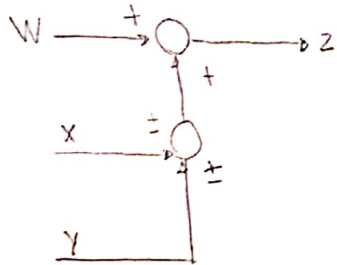
L2

10



$$(W \pm X) \pm Y = Z$$

$$Z = W \pm X \pm Y$$

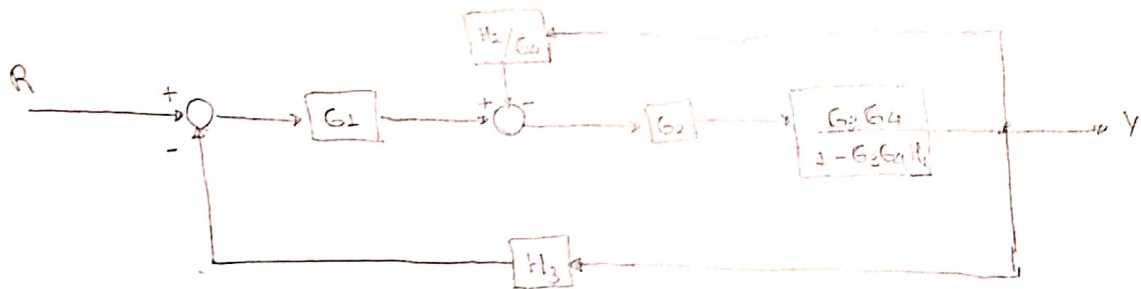


$$(\pm X \pm Y) + W = Z$$

$$Z = W \pm X \pm Y$$

Portanto, os diagramas são equivalentes

11



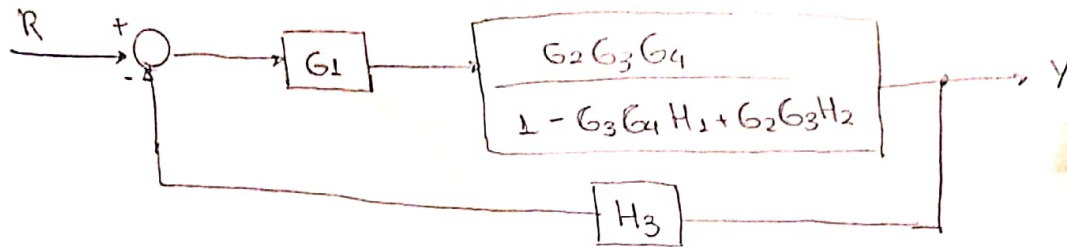
$$\left[G_2 \left[G_1 (R - H_3 Y) - \frac{H_2 Y}{G_4} \right] \right] \frac{G_3 G_4}{1 - G_3 G_4 H_1} = Y$$

$$\frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1} \cdot R = Y \left[1 + \frac{G_2 G_3 G_4}{1 - G_3 G_4 H_1} \left(G_1 H_3 + \frac{H_2}{G_4} \right) \right]$$

$$G_1 G_2 G_3 G_4 R = Y (1 - G_3 G_4 H_1 + G_1 G_2 G_3 G_4 H_3 + G_2 G_3 H_2)$$

$$\frac{Y}{R} = \frac{G_1 G_2 G_3 G_4}{(1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3)}$$

12



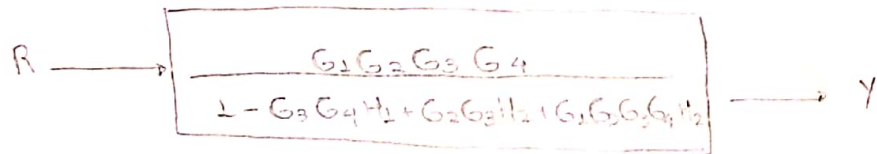
$$\frac{G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2} [G_1 (R - H_3 Y)] = Y$$

$$G_1 G_2 G_3 G_4 (R - H_3 Y) = Y (1 - G_3 G_4 H_1 + G_2 G_3 H_2)$$

$$G_1 G_2 G_3 G_4 R = Y (1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3)$$

$$\frac{Y}{R} = \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3}$$

13



$$\frac{Y}{R} = \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3}$$