PME 3380 - Modelagem de Sistemas Dinâmicos

Exercícios Aula 22/10/2020

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Ex. 1

$$2\ddot{x} + 7\dot{x} + 3x = 0, \quad x(0) = x_0 \; ; \; \dot{x}(0) = 0$$

$$2s^2X(s) + 7sX(s) + 3X(s) = 1 \rightarrow \text{Degrau unitário}$$

Assim:

$$\boxed{\frac{X(s)}{U(s)} = \frac{1}{2s^2 + 7s + 3}}$$

Resolução da E.D.O

$$\mathcal{L}(f(t)) = 2(s^2X(s) - sX(0) - \dot{x}(0)) + 7(sX(s) - X(0)) + 3X(s) = 0$$

$$2s^2X(s) - 2sX_0 + 7sX_s - 7x_0 + 3X(s) = 0$$

$$\therefore \quad X(s) = \frac{2s + 7}{2s^2 + 7s + 3} \cdot x_0$$

Separando em frações parciais, têm-se:

Pólos: -3 e $-0,5$ $\therefore$

$$X(s) = \left[\frac{a}{(s+3)} + \frac{b}{(s + 1/2)}\right] \cdot \frac{x_0}{2} \quad \therefore$$

$$a = (s+3) \cdot X(s)\Big|_{s=-3} \rightarrow a = \frac{2(-3) + 7}{(-3 + 1/2)} = -2/5$$

$$b = \left(s + \tfrac{1}{2}\right) X(s) \Big|_{s=-1/2} \rightarrow b = \frac{2(-1/2) + 7}{(-1/2 + 3)} = 12/5$$

$$X(s) = \left[\frac{-2}{5(s+3)} + \frac{12}{5(s + 1/2)} \right] \frac{x_0}{2} \therefore$$

$$X(s) = \frac{-x_0}{5(s+3)} + \frac{6x_0}{5(s+1/2)}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$\boxed{x(t) = \frac{-x_0}{5} \cdot e^{-3t} + \frac{6x_0}{5} \cdot e^{-t/2}}$$

Ex.2

$$\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$$

Dado degrau unitário $u(t) = 1$

$\ddot{x}(0) = 2$; $\dot{x}(0) = 1$, $x(0) = 9$; $u(t) = 1$; $u(0) = 0$; $\dot{u}(0) = 0$;

Determinação da FT

$$s^3 X(s) + 2s^2 X(s) + 7s X(s) = s^2 U(s) + 7s U(s) + 5 U(s) \therefore$$

$$X(s) \cdot (s^3 + 2s^2 + 7s) = U(s) (s^2 + 7s + 5) \therefore$$

$$\boxed{\frac{X(s)}{U(s)} = \frac{s^2 + 7s + 5}{s^3 + 2s^2 + 7s}}$$

Resolução da EDO

$$\mathcal{L}(\mathcal{L}(t)) = s^3X(s) - s^2x(0) - s\dot{x}(0) - s\dot{x}(0) - \ddot{x}(0) + 2(s^2X(s) - sX(0) - \dot{x}(0)) +$$

$$+7(sX(s) - X(0)) = s^2U(s) - sU(s) - \dot{U}(0) + 7(sU(s) - U(0)) + 5U(s)$$

$\therefore$

$$s^3X(s) - s^2 9 - s.1 - 2 + 2s^2X(s) - 2s\,9 - 2 + 7sX(s) - 7\,9 = s^2U(s) -$$

$$-s + 7sU(s) - 7 + 5U(s) \quad \therefore$$

$$X(s)(s^3 + 2s^2 + 7s) - (9s^2 + 19s + 67) = U(s)(s^2 + 7s + 5) - (s+7) \therefore$$

$$X(s) = \frac{U(s)(s^2+7s+5)}{s^3+2s^2+7s} + \frac{9s^2+19s+67}{s^3+2s^2+7s} - \frac{s+7}{s^3+2s^2+7s}$$

Com $u(t) = 1$ e $U(s) = 1/s$, têm-se:

$$X(s) = \frac{s^2+7s+5}{s^2(s^2+2s^3+7s^2)} + \frac{9s^2+18s+60}{s^3+2s^2+7s}$$

Separando em frações parciais, têm-se:

$$X(s) = \frac{5}{7s^2} + \frac{39}{49s} - \frac{\left(\frac{39}{49}s + \frac{64}{49}\right)}{s^2+2s+7} + \frac{60}{7s} + \frac{\frac{3}{7}s + \frac{6}{7}}{s^2+2s+7} \quad \therefore$$

$$X(s) = \frac{5}{7s^2} + \frac{459}{49s} - \frac{18(s+1)}{49((s+1)^2+6)}$$

Aplicando a transformada Inversa, têm-se:

$$\boxed{X(t) = \frac{459}{49} + \frac{5}{7}t - \frac{18}{49}.e^{-t}.\cos(\sqrt{6}\,t)}$$