Ex 22/10 - Henrique Kuhlmann - 10772672

1) $2\ddot{x} + 7\dot{x} + 3x = 0$ com $x(0) = x_0$ e $\dot{x}(0) = 0$

$\downarrow \mathcal{L}\{x\}$

$2(s^2X - sx_0) + 7(sX - x_0) + 3X = 0$

$2s^2X + 7sX + 3X = 2sx_0 + 7x_0$

$\downarrow$

$$X = \frac{x_0(2s+7)}{2s^2+7s+3} = \frac{x_0(2s+7)}{2(s+3)(s+1/2)}$$

$$X = \frac{A}{s+3} + \frac{B}{2s+1} = \frac{2As + A + Bs + 3B}{(s+3)(2s+1)}$$

$A = -x_0/5$

$B = 12x_0/5$

Sabendo que $\mathcal{L}\{e^{ax}\} = \frac{1}{s-a}$, basta aplicar a transformada inversa:

$$\boxed{x(t) = -\frac{x_0}{5}e^{-3t} - \frac{6x_0}{5}e^{-t/2}}$$

2) $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$ com

$\ddot{x}(0) = 2$ | $u(0) = 0$

$\dot{x}(0) = 1$ | $\dot{u}(0) = 0$

$x(0) = 9$ | $u(1) = 1$

$\downarrow \mathcal{L}$

$(s^3X - 2 - s - 9s^2) + 2(s^2X - 1 - 9s) + 7(sX - 9) = (s^2U) + 7(sU) + 5U$

$X(s^3 + 2s^2 + 7s) = U(s^2 + 7s + 5) + (9s^2 + 19s + 67)$

$$X(s) = U(s) \cdot \frac{s^2+7s+5}{s^3+2s^2+7s} + \frac{9s^2+19s+67}{s^3+2s^2+7s}$$

$9s^3 + 19s^2 + 67s$

Mas se $u(t) = 1$, $U(s) = 1/s$

Substituindo $U(s) = 1/s$:

$$X(s) = \frac{9s^3 + 20s^2 + 74s + 5}{s^2(s^2+2s+7)}.$$

Separando em frações parciais, chegamos em:

$$X(s) = \frac{508}{49s} + \frac{5}{7s^2} - \frac{67s + 71}{49[(s+1)^2+6]}$$

Sabendo que: $\mathcal{L}\{1/s\} = 1$, $\mathcal{L}\{1/s^2\} = t$, $\mathcal{L}\left\{\frac{s+1}{(s+1)^2+6}\right\} = e^{-t}\cos(\sqrt{6}t)$ e $\mathcal{L}\left\{\frac{e^{-t}}{\sqrt{6}}\operatorname{sen}(\sqrt{6}t)\right\}$

Logo:

$$\boxed{x(t) = \frac{508}{49} + \frac{5}{7}t - \frac{67}{49}e^{-t}\cos(\sqrt{6}t) - \frac{4}{96\sqrt{6}}e^{-t}\operatorname{sen}(\sqrt{6}t)}$$