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Modelagem

1) $2\ddot{x} + 7\dot{x} + 3x = 0$; $x(0) = x_0$; $\dot{x}(0) = 0$

↳ transformada de Laplace

$$2(S^2 X(s) - S\,x(0)) + 7(S\,X(s) - X_0) + 3X(s) = 0$$

$$(2S^2 + 7S + 3)X(s) = (2.S + 7)X_0 \rightarrow \boxed{X(s) = \frac{(2S+7)X_0}{2S^2+7S+3}}$$

• Podemos escrever: $X(s) = \dfrac{X_0(2S+7)}{(S+3)(2S+1)} = \dfrac{A}{S+3} + \dfrac{B}{2S+1} = \dfrac{S(2A+B)+(A+3B)}{(S+3)(2S+1)}$

$$\begin{cases} 2A + B = 2X_0 \\ A + 3B = 7X_0 \end{cases} \Rightarrow \begin{matrix} B = (12/5)X_0 \\ A = (-1/5)X_0 \end{matrix} \rightarrow \boxed{X(s) = \frac{-X_0}{5(S+3)} + \frac{12X_0}{5(2S+1)}}$$

• Transformada inversa: $x(t) = \dfrac{-x_0}{5}\mathcal{L}^{-1}\left\{\dfrac{1}{S+3}\right\} + \dfrac{6X_0}{5}\mathcal{L}^{-1}\left\{\dfrac{1}{S+1/2}\right\}$

$$\boxed{X(t) = -\frac{X_0}{5}e^{-3t} + \frac{6X_0}{5}e^{-t/2}}$$

2) $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$; $\ddot{x}(0) = 2$; $\dot{x}(0) = 1$; $x(0) = 9$; $\dot{u}(0) = 0 = u(0)$

↳ transformada de Laplace

$$S^3X(s) - S^2x(0) - S\dot{x}(0) - \ddot{x}(0) + 2(S^2X(s) - Sx(0) - \dot{x}(0)) + 7(SX(s) - X(0)) = S^2U(s) - S\cancel{u(0)}^{0} - \cancel{\dot{u}(0)}^{0} + 7(SU(s) - \cancel{u(0)}^{0}) + 5U(s)$$

$$\therefore X(s)(S^3 + 2S^2 + 7S) = U(s)(S^2 + 7S + 5) + 9S^2 + 19S + 67$$

↳ $U(s) = \dfrac{1}{S}$

foi utilizado método das frações parciais

$$X(s) = \frac{9S^3 + 20S^2 + 74S + 5}{S^2(S^2 + 2S + 7)} = \boxed{\frac{508}{49S} + \frac{5}{7S^2} - \frac{(67S + 71)}{49[(S+1)^2 + 6]}}$$

• Transformada inversa:

$$X(t) = \frac{508}{49}\mathcal{L}^{-1}\{\underbrace{1/S}_{1}\} + \frac{5}{7}\mathcal{L}^{-1}\{\underbrace{1/S^2}_{t}\} - \frac{67}{49}\mathcal{L}^{-1}\left\{\frac{S+1}{(S+1)^2+6}\right\} - \frac{4}{49}\mathcal{L}^{-1}\left\{\frac{1}{(S+1)^2+6}\right\}$$

$$\therefore \boxed{X(t) = \frac{508}{49} + \frac{5t}{7} - \frac{67}{49}e^{-t}.\cos(t\sqrt{6}) - \frac{4}{49\sqrt{6}}e^{-t}\,\mathrm{sen}(t\sqrt{6})}$$